Available online at www.sciencedirect.com
ScienceDirect
Structural Safety 30 (2008) 236–252
STRUCTURAL SAFETY
www.elsevier.com/locate/strusafe

Uncertainty propagation in probabilistic seismic loss estimation

Jack W. Baker *, C. Allin Cornell

Department of Civil and Environmental Engineering, Stanford University, Stanford, CA 94305-4020, USA

Received 18 December 2005; received in revised form 23 September 2006; accepted 28 November 2006
Available online 16 January 2007

Abstract

Probabilistic estimation of losses in a building due to earthquake damage is a topic of interest to decision makers and an area of active research. One promising approach to the problem, proposed by the Pacific Earthquake Engineering Research (PEER) Center, involves breaking the analysis into separate components associated with ground motion hazard, structural response, damage to components and repair costs. Each stage of this method has both inherent (aleatory) randomness and (epistemic) model uncertainty, and these two sources of uncertainty must be propagated through the analysis in order to determine the total uncertainty in the resulting loss estimates. In this paper, the PEER framework for seismic loss estimation is reviewed and options for both characterizing and propagating the various sources of uncertainty are proposed. Models for correlations (among, e.g., element repair costs) are proposed that may be useful when empirical data is lacking. Several options are discussed for propagating uncertainty, ranging from flexible but expensive Monte Carlo simulation to closed form solutions requiring specific functional forms for relationships between variables to be assumed. A procedure that falls between these two extremes is proposed, which integrates over the discrete element damage states, and uses the first-order second-oment method to collapse several conditional random variables into a single conditional random variable representing total repair cost given the ground motion intensity. Numerical integration is then used to incorporate the ground motion hazard. Studies attempting to characterize epistemic uncertainty or develop specific elements of the framework are referenced as an aid for users wishing to implement this loss-estimation procedure.

Keywords: Seismic loss estimation; Propagation of uncertainty; First-order second-moment; Aleatory; Epistemic; Generalized equi-correlated model

1. Introduction

Decision makers are interested in estimates of annual losses in a building due to earthquake damage (both financial and in terms of casualties). Determination of loss estimates, as well as the uncertainty in these estimates, is a topic of current study in performance-based earthquake engineering. Among the quantities to be determined are the uncertainty in the annual losses and the contribution of each individual source of

* Corresponding author. Tel.: +1 650 725 2573; fax: +1 650 723 7514.
E-mail address: bakerjw@stanford.edu (J.W. Baker).

0167-4730/$ - see front matter
doi:10.1016/j.strusafe.2006.11.003

uncertainty to the total uncertainty in annual losses. Current efforts in this field consider the ground motion hazard, building response, damage to building elements, element repair costs, and total repair cost as individual random variables, and then propagate uncertainty through each step to find a final result. This paper provides an overview of the Pacific Earthquake Engineering Research (PEER) Center loss estimation framework, and describes approaches for characterizing and propagating the many sources of uncertainty associated with loss estimation. For brevity, derivations of some equations and descriptions of model generalizations are omitted and referenced in a related report [1] and other relevant documents.

To calculate total uncertainty in the output variables of interest, it is necessary to incorporate both epistemic and aleatory uncertainty and account for their effects on the output variables. An approach to this problem is proposed here that uses a combination of numerical integrations and first-order second-moment (FOSM) approximations. The FOSM method is used to collapse the large vectors of conditional random variables into a single conditional random variable, total repair cost given ground motion intensity. Numerical integration is then used to combine this random variable with the ground motion hazard. This last step is treated accurately because it is believed to be the dominant contributor to the results. The combination of total repair cost given ground motion intensity with the ground motion hazard also does not involve vectors of random variables, meaning that the numerical integration is practicable. If the random variables associated with this final integration have specific functional forms, an analytic solution is also available, as will be illustrated. The quantities to be studied are the expected value and variance of the mean annual loss (due to epistemic uncertainty), mean frequency of collapse, and annual rate of exceeding a given cost.

Alternative approaches for propagation of uncertainty are briefly mentioned where appropriate. Because this framework involves large vectors of dependent random variables, and because at least one stage of the analysis is computationally expensive, however, the approach proposed here is believed have advantages relative to alternatives such as Monte Carlo simulation and complete numerical integration. Regardless of the approach used to propagate uncertainty, it is necessary to represent and estimate what may be a complex variance and covariance structure among the many random variables present in the problem. This paper presents several models which may be adopted for these estimates, and the models are equally useful for any propagation method. Some uncertainties, such as record-to-record variability of structural response at a given IM level, may be empirically estimated for specific structures. More generic quantification is needed, however, for many other types of uncertainties if this procedure is to be practically performed. References to relevant uncertainty estimates are provided here to aid those wishing to implement the procedure.

2. Loss estimation framework

The proposed procedure uses the loss estimation framework from the Pacific Earthquake Engineering Research (PEER) Center [2,3]. There are several components in this loss estimation model, consisting of quantifying the seismic ground motion hazard, structural response, damage to the building and contents, and resulting consequences (financial losses, fatalities, and business interruption). The process is modular, allowing the stages to be modeled and executed independently, and then linked back together using intermediate output variables. Similar multi-stage methodologies have had success in other complex Probabilistic Risk Assessment problems [4,5], and a significant effort has been made by the PEER Center to develop this approach for practical earthquake risk assessment applications. The framework for estimating total repair costs (TC) is built around the following equation

$$\lambda_{\mathrm{TC}}(z) = \int_{\mathbf{u}} \int_{\mathbf{v}} \int_{\mathbf{y}} \int_{x} G_{\mathrm{TC}|\mathbf{DVE}}(z|\mathbf{u}) f_{\mathbf{DVE}|\mathbf{DM}}(\mathbf{u}|\mathbf{v}) f_{\mathbf{DM}|\mathbf{EDP}}(\mathbf{v}|\mathbf{y}) f_{\mathbf{EDP}|\mathrm{IM}}(\mathbf{y}|x) \left| \frac{\mathrm{d}\lambda_{\mathrm{IM}}(x)}{\mathrm{d}x} \right| \mathrm{d}\mathbf{u}\, \mathrm{d}\mathbf{v}\, \mathrm{d}\mathbf{y}\, \mathrm{d}x \quad (1)$$

with terms defined as follows. $\lambda_{\mathrm{TC}}(z)$ is the annual rate of exceeding a total repair cost of z. $G_{\mathrm{TC}|\mathbf{DVE}}(z|\mathbf{u})$ is the Complimentary Cumulative Distribution Function (CCDF) of TC, conditioned on the vector of damage values of each element (DVE_j is the damage value of element j). Boldface notation is used to denote vector-valued variables. $f_{\mathbf{DVE}|\mathbf{DM}}(\mathbf{u}|\mathbf{v})$ is the Probability Density Function (PDF) of the vector of damage values of each element, given the vector of damage states of each element (DM_j is the damage state of element j). $f_{\mathbf{DM}|\mathbf{EDP}}(\mathbf{v}|\mathbf{y})$ is the PDF of the vector of (discrete) damage states, given the vector of engineering demand parameters.

$f_{\mathbf{EDP}|\mathrm{IM}}(\mathbf{y}|x)$ is the PDF of the vector of engineering demand parameters, given the intensity measure. $|\mathrm{d}\lambda_{\mathrm{IM}}(x)|$ is the absolute value of the derivative of the annual rate of exceeding a given value of the intensity measure, denoted IM; this information comes from a seismic hazard curve. Eq. (1) is based on the Total Probability Theorem, which is used to aggregate all of these conditional probability distributions.

A similar equation is used to compute the annual rate of collapse. Its form is simpler because it is not necessary to track individual element behavior when predicting global collapse

$$\lambda_{\mathrm{Collapse}} = \int_x P(C|\mathrm{IM}=x)\left|\frac{\mathrm{d}\lambda_{\mathrm{IM}}(x)}{\mathrm{d}x}\right|\mathrm{d}x \tag{2}$$

where $\lambda_{\mathrm{Collapse}}$ is the annual rate of collapse and $P(C|\mathrm{IM}=x)$ is the probability that a record with intensity $\mathrm{IM}=x$ will cause collapse of the structure. The amount of time that a building is out of service due to earthquake damage also of interest, but is not considered here because the relevant models are not well defined [6]. The various model components and assumptions typically used in this framework are described in the following paragraphs.

All damage is assumed to occur on an element level, if the structure does not collapse. Total repair cost (denoted TC) is then the sum of the damage cost of each element in the structure [7]. If the structure collapses, then the repair cost is simply a single random variable representing replacement cost.

At high IM levels, the potential exists for a structure to experience collapse. Global side-sway collapse is indicated in computer analyses by extreme deflections at one or more story levels, and local collapses due to column axial failures can also be incorporated [8]. These collapses are important, both for directly computing the annual rate of collapse as well for modeling repair costs due to collapses. The probability of collapse is typically estimated by scaling records up until they cause collapse [9], and then counting the fraction of records at a given IM level that cause collapse or fitting a function to the observed fractions of collapse over a range of IM levels [10,11].

Means and variances of repair costs for each possible damage state are needed for all element types under consideration. Mean repair costs can be estimated from sources such as R.S. Means Company's published materials on construction cost estimating [12], making appropriate adjustments for differences between the costs of post-disaster repair and new construction. As might be expected, uncertainties in repair costs can be very large [8].

Damage measures are typically not continuous, but rather a discrete set of damage states [7,13]. Occurrence of damage is described by fragility functions, which return the probability of an element exceeding given damage states as a function of structural response level. The damage state classification of an element is termed a Damage Measure, or DM. One fragility function is needed for each potential damage state of each element type.

Structural response parameters (e.g., maximum interstory drifts and peak floor accelerations for each floor) are termed Engineering Demand Parameters, or EDPs. A probabilistic model is needed for the distribution of EDPs, conditioned on the level of IM. This conditional distribution is usually estimated from results of nonlinear dynamic analysis performed on a finite element structural model. This is typically the most computationally expensive step of the analysis procedure, so minimizing the required number of analyses is desirable.

It is necessary to determine the annual rate of exceeding various levels of ground motion intensity, i.e. the hazard curve, for the predictor intensity measure (IM) at the location of interest. This is done using either site-specific Probabilistic Seismic Hazard Analysis (PSHA) or seismic hazard maps. The PSHA procedure has been described in detail elsewhere [14,15], and is not further discussed in this study.

Several assumptions are made in the analysis framework. They are believed to be consistent with state-of-the-art loss estimation efforts, but most could be relaxed without any formal difficulty. Markovian dependence is assumed for all conditional distributions in the framework. For example, it is assumed that the distribution of the DM vector can be conditioned solely on the EDP vector, and that knowledge of the IM provides no additional information. Second, all relations in the framework are assumed to be scalar functions. For example, the conditional distribution of the Damage Measure of element j is a function of only the ith Engineering Demand Parameter. Or alternatively, $f_{\mathrm{DM}_j|\mathbf{EDP}}(v_j \mid \mathbf{y}) = f_{\mathrm{DM}_j|\mathrm{EDP}_i}(v_j \mid y_i)$. This assumption keeps the functional relationships between individual variables simple, which aids in estimating functions from data. Finally, all damage is assumed to occur on the element level. Generalizations to account for, e.g., contractor efficiencies of scale or demand surge (the tendency for construction prices to increase after a disaster due to an increase in demand for construction services) can be incorporated in the model framework once research progresses to the stages where their effects can be quantified.

3. Uncertainty propagation

Explicit numerical evaluation of Eq. (1) is not practically possible in most cases, due to the high dimensionality of several of the random variables, so other solution methods are needed. Alternatives include Monte Carlo simulation and approximate methods such as first-order second-moment analysis. There are several stages of propagation, and different methods might be used at different steps.

The procedure proposed in detail here uses a combination of first-order second-moment analysis and explicit integration. The FOSM method is used to calculate the mean and variance of TC given IM. That is,

$$G_{\mathrm{TC|IM}}(z|x) = \int_{\mathbf{u}} \int_{\mathbf{v}} \int_{\mathbf{y}} G_{\mathrm{TC}|\mathbf{DVE}}(z|\mathbf{u}) f_{\mathbf{DVE}|\mathbf{DM}}(\mathbf{u}|\mathbf{v}) f_{\mathbf{DM}|\mathbf{EDP}}(\mathbf{v},\mathbf{y}) f_{\mathbf{EDP}|\mathrm{IM}}(\mathbf{y}|x)\,\mathrm{d}\mathbf{u}\,\mathrm{d}\mathbf{v}\,\mathrm{d}\mathbf{y} \tag{3}$$

is evaluated by estimating the mean and variance of TC|IM. A complementary cumulative distribution function $G_{\mathrm{TC|IM}}(z|x)$ is fit to this mean and variance, and integrated numerically or analytically over the derivative of the hazard curve, $|\mathrm{d}\lambda_{\mathrm{IM}}(x)|$, to generate the mean annual rate of exceeding a given repair cost.

The FOSM approximations used to obtain moments of TC|IM from **EDP**, **DM** and **DVE** are justified by the assumption that the uncertainty in the IM hazard curve is the most significant contributor to variance of the total loss. Therefore, the full distribution for IM is retained, but FOSM approximations are used for all (first and second) moments conditioned on IM. In addition, information about the full distributions of some variables such as repair costs is typically not available, and so using only the first two moments of these distributions does not result in a significant loss of available information. Details of the steps required to perform the proposed procedure are presented in the following sections.

3.1. Specify EDP given IM

The proposed model for structural response used in this study is $\mathrm{EDP}_i|\mathrm{IM} = h_i(\mathrm{IM})\varepsilon_i(\mathrm{IM})$, where $h_i(\mathrm{IM})$ is the (deterministic) mean value of EDP_i given IM, and $\varepsilon_i(\mathrm{IM})$ is a random variable with mean of one and conditional variance adjusted to model the variance in EDP_i. (The notation $X|Y$ denotes that the random variable X is conditioned on Y.) Taking logarithms gives a random variable of the form $\ln \mathrm{EDP}_i|\mathrm{IM} = \ln h_i(\mathrm{IM}) + \ln \varepsilon_i(\mathrm{IM})$. Note that the expected value of $\ln \mathrm{EDP}_i|\mathrm{IM}$ is $\ln h_i(\mathrm{IM})$, and that the variance of $\ln \mathrm{EDP}_i|\mathrm{IM}$ is equal to the variance of $\ln \varepsilon_i(\mathrm{IM})$. The mean and variance of $\ln \mathrm{EDP}_i|\mathrm{IM}$ for each EDP, as well as the correlations between EDPs (all as a function of the IM level), can be determined from Incremental Dynamic Analysis. This information will be needed for the propagation procedure that follows. The mean of parameter EDP_i is denoted $\mu_{\ln \mathrm{EDP}_i|\mathrm{IM}}$, or equivalently, $E[\ln \mathrm{EDP}_i|\mathrm{IM}]$. Similarly, variances and correlations are denoted $\sigma^2_{\ln \mathrm{EDP}_i|\mathrm{IM}}$ and $\rho_{\ln \mathrm{EDP}_i|\mathrm{IM}, \ln \mathrm{EDP}_j|\mathrm{IM}}$. The correlation between variables can equivalently be represented by the covariance, denoted $\sigma_{\ln \mathrm{EDP}_i|\mathrm{IM}, \ln \mathrm{EDP}_j|\mathrm{IM}}$. (Variances will be denoted by σ^2_X or $\mathrm{Var}[X]$ and similarly, covariances will be denoted $\sigma_{X,Y}$ or $\mathrm{Cov}[X, Y]$.)

As an alternative to estimating only the means and variances as proposed above, one could use Monte Carlo simulation. A large number of dynamic analyses would be performed, and each resulting vector of structural response values used as input for the later stages of the assessment [7]. Monte Carlo simulation has the advantage of more directly incorporating epistemic model uncertainty; one can treat assumed structural model parameters such as element stiffnesses as having epistemic uncertainty, and include simulated values for these parameters in parallel with the dynamic analysis simulations.

3.2. Collapse out the intermediate variables for element damage states

The discrete states of the Damage Measure variable used in current loss estimation are not compatible with the FOSM approach, which requires continuous functions for the moments. To deal with the discrete states, we take advantage of the fact that one can always "collapse" the two distributions **DM**|**EDP** and **DVE**|**DM** into one continuous distribution **DVE**|**EDP** by integrating over the intermediate conditioning variable

$$f_{\mathbf{DVE}|\mathbf{EDP}}(\mathbf{u}|\mathbf{y}) = \int_{\mathbf{v}} f_{\mathbf{DVE}|\mathbf{DM}}(\mathbf{u}|\mathbf{v}) f_{\mathbf{DM}|\mathbf{EDP}}(\mathbf{v}|\mathbf{y})\,\mathrm{d}\mathbf{v} \tag{4}$$

For a given element with n possible damage states, we use a set of element fragility functions $F_1, F_2 \ldots F_n$, such that $F_i(y) = P(\mathrm{DM} > d_i|\mathrm{EDP} = y)$, as illustrated in Fig. 1a. These functions will have a corresponding set of distributions $c_1, c_2 \ldots c_n$ of element repair costs such that $c_i(v)$ is a probability distribution of DVE, given that the damage state equals d_i, as illustrated in Fig. 1b. These models provide information sufficient to determine the first two moments of the collapsed distributions.

From the total probability theorem, Eq. (4) can be written in scalar form for each individual DVE parameter as $f_{\mathrm{DVE|EDP}} = \sum_i f_{\mathrm{DVE|DM}=d_i} P_{\mathrm{DM}=d_i|\mathrm{EDP}}$. For later FOSM purposes, furthermore, it is sufficient to find simply the conditional means, variances, and covariances of the DVEs given the EDPs. Taking the mean of this PDF gives the result

$$E[\mathrm{DVE}|\mathrm{EDP}] = \sum_i E[\mathrm{DVE}|\mathrm{DM} = d_i]P(\mathrm{DM} = d_i) = \sum_i \mu_i(F_i(\mathrm{EDP}) - F_{i+1}(\mathrm{EDP})) \tag{5}$$

Recognizing that $\sigma_X^2 = E[X^2] - \mu_X^2$, one can similarly obtain the following result

$$\begin{aligned}\sigma^2_{\mathrm{DVE|EDP}} &= E_{\mathrm{DM}}[\mathrm{Var}_{\mathrm{DVE}}[\mathrm{DVE}|\mathrm{DM}]] + \mathrm{Var}_{\mathrm{DM}}[E_{\mathrm{DVE}}[\mathrm{DVE}|\mathrm{DM}]] \\ &= \sum_i \sigma^2_{\mathrm{DVE}_i}(F_i(\mathrm{EDP}) - F_{i+1}(\mathrm{EDP})) + \sum_i (\mu_i - \bar{\mu})^2 (F_i(\mathrm{EDP}) - F_{i+1}(\mathrm{EDP}))\end{aligned} \tag{6}$$

where $\bar{\mu}$ is the conditional expectation computed in Eq. (5). Fig. 2 shows an example of the mean and standard deviation of DVE|EDP, computed using the distributions shown in Fig. 1. Note that no first-order approximation was needed to obtain these results.

Monte Carlo simulation could also be used to evaluate Eq. (4), and this would be a natural choice if simulation was being used to evaluate other steps of the model. The computational expense of Monte Carlo will be minimal at this step, because simulation only involves repeatedly evaluating the simple analytical equations associated with Fig. 1.

3.3. A model for element correlations

To complete the characterization of DVE given EDP, correlations among the DVE values of all elements are needed. Properly accounting for these correlations is important: one study found that neglecting correlations among element damage values resulted in a 25% underestimation of variance in total repair cost, relative to the case where best estimates of correlations were used [16]. Note that correlations will be defined for the continuous distribution of DVE given EDP, rather than the previous distributions with discrete DM states where a correlation coefficient is a less meaningful measure of stochastic dependence. Estimation of these

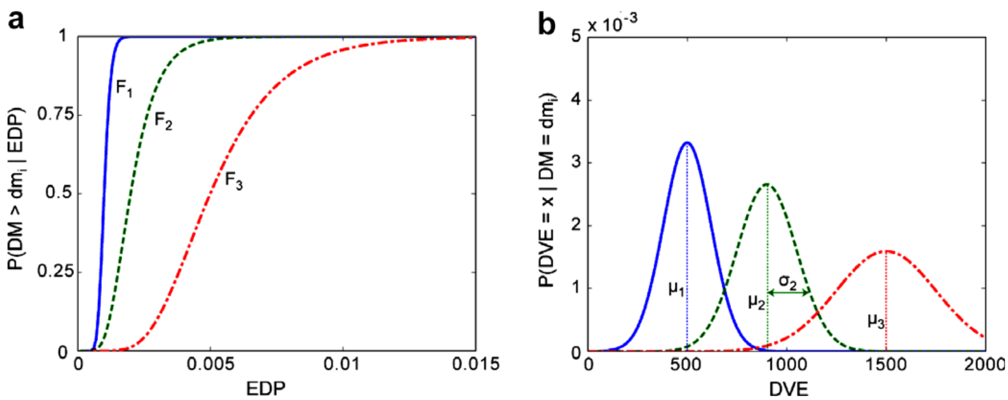

Fig. 1. Illustration of element damage and cost models. (a) Element fragility functions and (b) element repair costs.

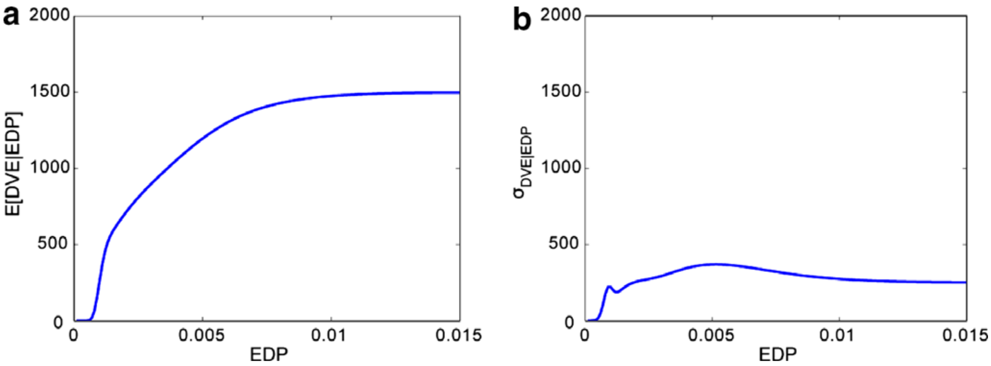

Fig. 2. (a) Mean and (b) standard deviation of DVE|EDP.

correlations is a difficult task due to a lack of data, but the following characterization scheme may be helpful. Assume a model of the form: $\ln \mathrm{DVE}_k | \ln \mathrm{EDP}_i = g_k(\ln \mathrm{EDP}_i) + \ln \varepsilon_{\mathrm{Struc}} + \ln \varepsilon_{\mathrm{ElClass}_m} + \ln \varepsilon_{\mathrm{El}_k}$, where $\varepsilon_{\mathrm{Struc}}$ is a random variable representing sources of uncertainty common to the entire structure, $\varepsilon_{\mathrm{ElClass}_m}$ represents uncertainty common only to elements of class "m" (e.g., drywall partitions, moment connections, etc.), and $\varepsilon_{\mathrm{El}_k}$ represents uncertainty unique to element k. All of these ε's are assumed to be mutually uncorrelated. Note that logarithms have been taken so that instead of a product of random variables, the model contains a (more tractable) sum of random variables. The variances of these random variables are then defined as $\mathrm{Var}[\ln \varepsilon_{\mathrm{Struc}} \mid \ln \mathrm{EDP}_i] = \beta^2_{\mathrm{Struc}}$, $\mathrm{Var}[\ln \varepsilon_{\mathrm{ElClass}_m} \mid \ln \mathrm{EDP}_i] = \beta^2_{\mathrm{ElClass}}$ for all m, and $\mathrm{Var}[\ln \varepsilon_{\mathrm{El}_k} \mid \ln \mathrm{EDP}_i] = \beta^2_{\mathrm{El}}$ for all k. The total variance of $\ln \mathrm{DVE}_k | \ln \mathrm{EDP}_i$ is the sum of these variances.

For this special case, a simple closed-form solution exists for the correlation coefficient. If two elements are in the same class (e.g. drywall partitions), then the correlation in their repair costs, DVE_k and DVE_j, is:

$$\rho_{\ln \mathrm{DVE}_k, \ln \mathrm{DVE}_j | \ln \mathrm{EDP}_i} = \frac{\beta^2_{\mathrm{Struc}} + \beta^2_{\mathrm{ElClass}}}{\beta^2_{Struc} + \beta^2_{\mathrm{ElClass}} + \beta^2_{\mathrm{El}}} \tag{7}$$

If two elements are in the different classes (e.g. a drywall partition and a moment connection), then the correlation in their repair costs, DVE_k and DVE_j, is:

$$\rho_{\ln \mathrm{DVE}_k, \ln \mathrm{DVE}_j | \ln \mathrm{EDP}_i} = \frac{\beta^2_{\mathrm{Struc}}}{\beta^2_{\mathrm{Struc}} + \beta^2_{\mathrm{ElClass}} + \beta^2_{\mathrm{El}}} \tag{8}$$

Loosely speaking, the correlation coefficient between two DVE's can be said to be the ratio of their shared variances to their total variance. A more general analytic solution also exists that allows $\beta^2_{\mathrm{ElClass}}$ to vary by class, allows β^2_{El} to vary by element, and allows both to be functionally dependent on the EDP value [1, Appendix A] . The model can also be expanded to more than three ε terms if desired.

The use of more than two uncertain terms, and the use of β^2 terms that vary by class or element is termed a *generalized equi-correlated model*. The correlation matrix for this model has off-diagonal terms that vary from term to term, as opposed to the strict equi-correlated model, where all off-diagonal correlation coefficients are identical [17]. Note that correlation matrices developed from any generalized equi-correlated model will be non-negative definite; this is a necessary property for correlation matrices, and is not guaranteed to hold if the analyst simply estimates individual correlations and aggregates them into a matrix. This correlation model can be used for both the procedure proposed here as well as for Monte–Carlo-based procedures (where they would be incorporated by simulating correlated random variables [18]).

3.4. Calculate element damage values as a function of IM

Using information from above, it is possible to calculate the first and second moments of $\ln \mathbf{DVE}|\mathrm{IM}$. To maintain tractability, this calculation is performed in an approximate way referred to in structural reliability literature as first-order second-moment, or FOSM. With this method, a first-order Taylor expansion of

$\ln \mathrm{DVE}_k|\ln \mathrm{EDP}_i$ is used. Denote $\ln \mathrm{DVE}_k|\ln \mathrm{EDP}_i$ as $g_k(\ln \mathrm{EDP}_i)$. Then the FOSM estimate of the mean of ln DVE_k given IM is

$$\mu_{\ln \mathrm{DVE}_k|\mathrm{IM}} \cong g_k(\mu_{\ln \mathrm{EDP}_i|\mathrm{IM}}) \tag{9}$$

That is, the mean of $\ln \mathrm{DVE}_k|\mathrm{IM}$ is the mean of $\ln \mathrm{DVE}_k|\ln \mathrm{EDP}_i$, evaluated at the mean value of $\ln \mathrm{EDP}_i$ predicted at level IM. Using a similar approach with conditional moments, it can be shown that the variance of $\ln \mathrm{DVE}_k$ given IM is

$$\sigma^2_{\ln \mathrm{DVE}_k|\mathrm{IM}} \cong \sigma^2_{\ln \mathrm{EDP}_i|\mathrm{IM}} \left(\frac{\partial g_k}{\partial \ln \mathrm{EDP}_i}\right)^2 \Bigg|_{\mu_{\ln \mathrm{EDP}_i|\mathrm{IM}}} + v_k(\mu_{\ln \mathrm{EDP}_i|\mathrm{IM}}) \tag{10}$$

where $v_k(\ln \mathrm{EDPt})$ is the variance of $\ln \mathrm{DVE}_k$ given $\ln \mathrm{EDP}_i$. The covariance in DVE values between two components can be found similarly

$$\begin{aligned}\sigma^2_{\ln \mathrm{DVE}_k,\ln \mathrm{DVE}_l|\mathrm{IM}} \cong{} & \sigma_{\ln \mathrm{DVE}_k|\ln \mathrm{EDP}_i,\ln \mathrm{DVE}_l|\ln \mathrm{EDP}_j}(\mu_{\ln \mathrm{EDP}_i|\mathrm{IM}}, \mu_{\ln \mathrm{EDP}_j|\mathrm{IM}}) \\ & + \left(\frac{\partial g_k}{\partial \ln \mathrm{EDP}_i}\right)\left(\frac{\partial g_l}{\partial \ln \mathrm{EDP}_j}\right)\Bigg|_{\substack{\mu_{\mathrm{EDP}_i|\mathrm{IM}} \\ \mu_{\mathrm{EDP}_j|\mathrm{IM}}}} \sigma_{\ln \mathrm{EDP}_i|\mathrm{IM},\ln \mathrm{EDP}_j|\mathrm{IM}}\end{aligned} \tag{11}$$

where the notation $\sigma_{\ln \mathrm{DVE}_k|\ln \mathrm{EDP}_i,\ln \mathrm{DVE}_l|\ln \mathrm{EDP}_j}(\mu_{\ln \mathrm{EDP}_i|IM}, \mu_{\ln \mathrm{EDP}_j|IM})$ is used to denote that the covariance $\sigma_{\ln \mathrm{DVE}_k|\ln \mathrm{EDP}_i,\ln \mathrm{DVE}_l|\ln \mathrm{EDP}_j}$ should be evaluated at $\ln \mathrm{EDP}_i = \mu_{\ln \mathrm{EDP}_i|IM}$ and $\ln \mathrm{EDP}_j = \mu_{\ln \mathrm{EDP}_j|\mathrm{IM}}$.

These results can now be switched to their non-log form using first-order methods

$$\mu_{\mathrm{DVE}_k|\mathrm{IM}} \cong \mathrm{e}^{\mu_{\ln \mathrm{DVE}_k|\mathrm{IM}}} \tag{12}$$

$$\sigma^2_{\mathrm{DVE}_k|\mathrm{IM}} \cong \mathrm{e}^{2h_k(\mu_{\ln \mathrm{EDP}_i|\mathrm{IM}})}\sigma^2_{\ln \mathrm{DVE}_k|\mathrm{IM}} \tag{13}$$

$$\sigma^2_{\mathrm{DVE}_k,\mathrm{DVE}_l|\mathrm{IM}} \cong \mathrm{e}^{v_k(\mu_{\ln \mathrm{EDP}_i|\mathrm{IM}})+v_l(\mu_{\ln \mathrm{EDP}_i|\mathrm{IM}})}\sigma^2_{\ln \mathrm{DVE}_k,\ln \mathrm{DVE}_l|\mathrm{IM}} \tag{14}$$

These values specify the first two moments of the element damage costs for a given IM.

The results from individual elements can then be used to compute the expectation and variance for the total cost of damage to the entire building

$$\mu_{\mathrm{TC}|\mathrm{IM}} = \sum_{k=1}^{n} \mu_{\mathrm{DVE}_k|\mathrm{IM}} \tag{15}$$

$$\sigma^2_{\mathrm{TC}|\mathrm{IM}} = \sum_{k=1}^{n} \sigma^2_{\mathrm{DVE}_k|\mathrm{IM}} + 2\sum_{k=1}^{n}\sum_{l=k+1}^{n} \sigma^2_{\mathrm{DVE}_k,\mathrm{DVE}|\mathrm{IM}} \tag{16}$$

where n is the number of elements in the structure.

3.5. Account for structural collapse

As noted earlier, the analysis method should account for collapses. If the large deflections associated with the collapse cases are statistically combined with the deflections of the non-collapse cases, the estimated means and variances of some EDPs such as interstory drifts may be particularly sensitive to the few collapse cases, and this have undesirable effects on the above FOSM calculations. A more robust method to account for the possibility of collapse is described here. In the following calculations, a collapse indicator variable, "C" is used. The condition of no collapse is denoted by $\sim C$, the complement of the collapse indicator variable.

First, at each IM level, compute the probability of collapse. Next, perform the above procedure using only the structural analysis results that did not cause collapse. The results from Eqs. (15) and (16) computed using only non-collapse results are denoted $E[\mathrm{TC}|\mathrm{IM}, \sim C]$ and $\mathrm{Var}[\mathrm{TC}|\mathrm{IM}, \sim C]$. Finally, define an expected value and variance of total cost given that collapse has occurred, denoted $E[\mathrm{TC}|C]$ and $\mathrm{Var}[\mathrm{TC}|C]$ (note that these terms are not functions of IM, although this is not necessary in what follows). The expected value accounting for collapses is then simply the sum of the collapse and no collapse TC values, weighted by their respective probabilities of occurrence

$$\mu_{\mathrm{TC|IM}} = P(\sim C|\mathrm{IM})E[\mathrm{TC}|\mathrm{IM}, \sim C] + P(C|\mathrm{IM})E[\mathrm{TC}|C] \tag{17}$$

The variance of TC for a given IM level can be shown to equal

$$\begin{aligned}\sigma^2_{\mathrm{TC|IM}} =& P(\sim C|\mathrm{IM})(\mathrm{Var}[\mathrm{TC}|\mathrm{IM}, \sim C] + (E[\mathrm{TC}|\mathrm{IM}] - E[\mathrm{TC}|\mathrm{IM}, \sim C])^2) \\ &+ P(C|\mathrm{IM})(\mathrm{Var}[\mathrm{TC}|C] + (E[\mathrm{TC}|\mathrm{IM}] - E[\mathrm{TC}|C])^2)\end{aligned} \tag{18}$$

The procedure can now be implemented as before, with this conditional mean and variance replacing the equivalent results from Eqs. (15) and (16). This modification is likely needed, unless the building of interest is a ductile earthquake-resistant building and repair costs are dominated by contents damage.

3.6. Incorporate the ground motion hazard

Using the mean and variance of TC given IM and the derivative of the ground motion hazard curve, $d\lambda_{\mathrm{IM}}(x)$, the mean and variance of TC per annum can be calculated by numerical integration

$$\mu_{\mathrm{TC}} = \int_x \mu_{\mathrm{TC|IM}}(x)|\,\mathrm{d}\lambda_{\mathrm{IM}}(x)| \tag{19}$$

$$\begin{aligned}\sigma^2_{\mathrm{TC}} &= E[\sigma^2_{\mathrm{TC|IM}}] + \mathrm{Var}[\mu_{\mathrm{TC|IM}}] \\ &= \int_x \sigma^2_{\mathrm{TC|IM}}(x)|\,\mathrm{d}\lambda_{\mathrm{IM}}(x)| + \int_x \mu^2_{\mathrm{TC|IM}}(x)|\,\mathrm{d}\lambda_{\mathrm{IM}}(x)| - \mu^2_{\mathrm{TC|IM}}\end{aligned} \tag{20}$$

where $\mu_{\mathrm{TC|IM}}(x)$ and $\sigma^2_{\mathrm{TC|IM}}(x)$ denote the conditional mean and variance, respectively, of TC|IM evaluated at IM=x. Note that the first term of Eq. (20) accounts for uncertainty in the cost function *given* IM, and the second two terms account for uncertainty in the IM.

The first- and second-moment information for TC|IM can also be combined with a site hazard to compute $\lambda_{\mathrm{TC}}(z)$, the annual frequency of exceeding a given Total Cost z. For this calculation, it is necessary to assume a probability distribution for TC|IM that has a conditional mean and variance equal to the values calculated previously. By evaluating the integral for several values of z, a plot can be generated relating damage values to rates of exceedance. Details will be presented after epistemic uncertainty is incorporated in the framework.

4. Characterizing epistemic uncertainty

Eqs. (19) and (20) are valid for the case when there is no epistemic uncertainty in the ground motion hazard curve or TC|IM. The calculation must now be extended to account for this uncertainty, which is expected to be significant.

4.1. Epistemic uncertainty in TC given IM

Earlier, a model was assumed that can be written $\mathrm{TC|IM} = E[\mathrm{TC|IM}]\varepsilon_{\mathrm{R}}$, where ε_{R} is a random variable representing aleatory uncertainty. That model is now extended to incorporate epistemic uncertainty. We assume a simplified (first-order) model of epistemic uncertainty, in which that uncertainty is attributed only to the central or mean value of a random variable and not for example, its variance or distribution shape. (In practice, one may slightly inflate this uncertainty in the mean to reflect these second-order elements of epistemic uncertainty.) The total uncertainty in TC|IM is thus represented as $\mathrm{TC} \mid \mathrm{IM} = \overline{E[\mathrm{TC} \mid \mathrm{IM}]}\varepsilon_{\mathrm{R}}\varepsilon_{\mathrm{U}}$, where $\overline{E[\mathrm{TC} \mid \mathrm{IM}]}$ is the best estimate of the (conditional) mean and ε_{R} and ε_{U} are uncorrelated random variables representing aleatory uncertainty and epistemic uncertainty, respectively. Note that $\overline{E[\mathrm{TC} \mid \mathrm{IM}]}\varepsilon_{\mathrm{U}}$ is a random variable representing the (uncertain) estimate of the mean value of TC|IM, with variance $\mathrm{Var}[E[\mathrm{TC|IM}]]$.

Taking logarithms of this model for TC|IM gives $\ln \mathrm{TC} \mid \mathrm{IM} = \ln \overline{E[\mathrm{TC} \mid \mathrm{IM}]} + \ln \varepsilon_{\mathrm{R}}(\mathrm{IM}) + \ln \varepsilon_{\mathrm{U}}(\mathrm{IM})$. The random variables ε_{R} and ε_{U} are uncorrelated, and thus may be dealt with in separate steps. The procedure described earlier find the computed the variance of ε_R due to aleatory uncertainty. Now the variance due to ε_U is needed. Logarithmic forms are used again here to allow use of sums rather than products. The change can be made using the following relationship

$$\mathrm{Var}[\ln \varepsilon_R(\mathrm{IM})] \cong \ln\left(1+\frac{\mathrm{Var}_R[\mathrm{TC}|\mathrm{IM}]}{E_R[\mathrm{TC}|\mathrm{IM}]^2}\right) \tag{21}$$

The terms $\mathrm{Var}[\ln \varepsilon_R]$ and $\mathrm{Var}[\ln \varepsilon_U]$ will be denoted β_R^2 and β_U^2, respectively. Note that in the previous sections, the uncertainty that denoted as $\beta_{\mathrm{TC|IM}}^2$ is now referred to as β_R^2, to distinguish it from the new β_U^2 term.

4.2. Representation of conditional variables

To distinguish between aleatory and epistemic uncertainties of various conditional random variables, an additional notation is introduced. For example, the aleatory uncertainty of $\ln \mathrm{EDP}|\mathrm{IM}$ is denoted as

$$\mathrm{Var}[\ln \mathrm{EDP}|\mathrm{IM}] = \beta^2_{\mathrm{R;EDP|IM}} \tag{22}$$

and epistemic uncertainty in the estimate of the mean of $\ln \mathrm{EDP}|\mathrm{IM}$ is denoted as

$$\mathrm{Var}[\mu_{\ln \mathrm{EDP|IM}}] = \beta^2_{\mathrm{U;EDP|IM}} \tag{23}$$

These values are equivalent to $\sigma^2_{\ln \mathrm{EDP|IM}}$. This notation is introduced simply to distinguish between aleatory and epistemic uncertainty.

Estimation of epistemic uncertainties is a challenging task. References that provide guidance for estimation of, or report estimated values for, the various epistemic uncertainties present in this framework include [1,7,19–27]. When separating epistemic and aleatory uncertainties, care should be taken to avoid double-counting any source of uncertainty by including it in both categories.

4.3. Modeling correlations

Estimates of correlations need to be made at each step of the PEER equation (i.e. **EDP**|**IM**, **DVE**|**EDP** after *DM* has been collapsed out, and TC|**DVE**). Here a model is proposed for this purpose, and its use is demonstrated for correlations in **EDP**|IM. The same model is generally applicable to the other variables as well. Consider the following model

$$\ln \mathbf{EDP}|\mathrm{IM} = \overline{E[\ln \mathbf{EDP}|\mathrm{IM}]} + \varepsilon_{\mathrm{R};\mathbf{EDP}|\mathrm{IM}} + \varepsilon_{\mathrm{U};\mathbf{EDP}|\mathrm{IM}} \tag{24}$$

where $\overline{E[\ln \mathbf{EDP} \mid \mathrm{IM}]}$ is the mean estimate of $E[\ln \mathbf{EDP}|\mathrm{IM}]$ and $\varepsilon_{\mathrm{R;EDP|IM}}$ and $\varepsilon_{\mathrm{U;EDP|IM}}$ are random variables representing aleatory and epistemic uncertainty, respectively. Both random variables have an expected value of zero. The aleatory uncertainty term ($\varepsilon_{\mathrm{R;EDP|IM}}$) can be estimated directly from data obtained using Incremental Dynamic Analysis, but a model for the epistemic uncertainty term ($\varepsilon_{\mathrm{U;EDP|IM}}$) is needed.

Some epistemic uncertainty comes from uncertainty about the accuracy of the computer model used to represent the building's behavior. Another source is "estimation uncertainty" that comes from estimating the moments of $\ln \mathbf{EDP}|\mathrm{IM}$ from a finite sample of data. This is famously seen in the result that the sample mean of n independent samples, each with variance σ^2, has variance $\sigma^2_{\hat{\mu}} = \sigma^2/n$. The epistemic uncertainty is split into two terms representing these sources

$$\varepsilon_{\mathrm{U;EDP|IM}} = \varepsilon_{\mathrm{U_{model};EDP|IM}} + \varepsilon_{\mathrm{U_{estimate};EDP|IM}} \tag{25}$$

where $\varepsilon_{\mathrm{U_{model};EDP|IM}}$ is a random variable representing model uncertainty and $\varepsilon_{\mathrm{U_{estimate};EDP|IM}}$ is a random variable representing estimation uncertainty, and both random variables have means of zero and can be assumed uncorrelated (so they can be analyzed separately).

When calculating epistemic uncertainty, correlations must be calculated between estimates of means at differing IM levels (e.g., correlation of estimates of the expected value of $\ln \mathrm{EDP}$ at $\mathrm{IM} = im_1$ and $\mathrm{IM} = im_2$: $\rho_{E[\ln \mathrm{EDP|IM}=im_1],E[\ln \mathrm{EDP|IM}=im_2]}$). While there is no correlation between aleatory uncertainties, epistemic uncertainty (representing our uncertainty about the mean values) will potentially be correlated. The modeling uncertainty, represented by $\varepsilon_{\mathrm{U_{model}};\mathrm{EDP|IM}}$, may presumably, to a first approximation, be assumed to have a perfect correlation at two IM levels, because the models tend be common at least within the linear and nonlinear ranges. The same perfect correlation could be applied to two different $E[\ln \mathrm{EDP}]$'s at a single given IM level. Estimation uncertainty, represented by $\varepsilon_{\mathrm{U_{estimate}};\mathrm{EDP|IM}}$, may also be correlated at two IM levels. For instance, if the same set of ground motion records is used to estimate the $E[\ln \mathrm{EDP}]$'s at more than one IM level by using scaling, the estimates at varying IM levels will be correlated. A procedure referred to as the bootstrap provides an effective method to measure this correlation as well as the variance of $\varepsilon_{\mathrm{U_{estimate}};\mathrm{EDP|IM}}$ [28].

Once the variance and correlation of $\varepsilon_{\mathrm{U_{model}};\mathrm{EDP|IM}}$ and $\varepsilon_{\mathrm{U_{estimate}};\mathrm{EDP|IM}}$ at two IM levels has been defined, they can be combined to find the correlation of $\varepsilon_{U;\mathrm{EDP|IM}}$ at two IM levels. If the variance of $\varepsilon_{\mathrm{U_{model}};\mathrm{EDP|IM}}$ (denoted $\beta^2_{\mathrm{U_{model}};\mathrm{EDP|IM}}$) is equal at both IM levels, and the variance of $\varepsilon_{\mathrm{U_{estimate}};\mathrm{EDP|IM}}$ (denoted $\beta^2_{\mathrm{U_{estimate}};\mathrm{EDP|IM}}$) is equal at both IM levels, then the correlation of $\varepsilon_{U;\mathrm{EDP|IM}}$ at two IM levels is

$$\rho_{\mathrm{U;EDP|IM_1,IM_2}} = \frac{\beta^2_{\mathrm{U_{model}};\mathrm{EDP|IM}} + \rho \cdot \beta^2_{\mathrm{U_{estimate}};\mathrm{EDP|IM}}}{\beta^2_{\mathrm{U_{model}};\mathrm{EDP|IM}} + \beta^2_{\mathrm{U_{estimate}};\mathrm{EDP|IM}}} \tag{26}$$

where ρ is the correlation between $E[\ln \mathrm{EDP|IM}]$ at two IM levels due to estimation uncertainty (the correlation measured from the bootstrap). Note however that if ρ is expected to be near one, or if $\beta^2_{\mathrm{U_{model}};\mathrm{EDP|IM}}$ is much greater than $\beta^2_{\mathrm{U_{estimate}};\mathrm{EDP|IM}}$, then $\rho_{\mathrm{U;EDP|IM_1,IM_2}}$ will be nearly one. Under these conditions it reasonable to simply assume a perfect correlation. It is also necessary to find correlations for other pairs of random variables due to epistemic uncertainties (e.g. $\rho_{E[\ln \mathrm{EDP}_i|\mathrm{IM}=im_1],E[\ln \mathrm{EDP}_j|\mathrm{IM}=im_1]}$ and $\rho_{E[\ln \mathrm{DVE}_i|\ln \mathrm{EDP}_1],E[\ln \mathrm{DVE}_i|\ln \mathrm{EDP}_2]}$); a similar approach can be used for those correlations, although the simpler assumption of perfect correlation may be appropriate in many cases.

The presence of correlations between $E[\ln \mathrm{EDP|IM}]$ values at two IM levels will result in a non-zero correlation (or equivalently, covariance) between $E[\mathrm{TC|IM}]$ values at two IM levels. The FOSM estimate of this covariance is

$$\mathrm{Cov}[\mu_{\mathrm{TC|IM_1}}, \mu_{\mathrm{TC|IM_2}}] = \sum_k \mathrm{Cov}[\mu_{\mathrm{DVE}_k|\mathrm{IM_1}}, \mu_{\mathrm{DVE}_k|\mathrm{IM_2}}] + 2\sum_k \sum_{l<k} \mathrm{Cov}[\mu_{\mathrm{DVE}_k|\mathrm{IM_1}}, \mu_{\mathrm{DVE}_l|\mathrm{IM_2}}] \tag{27}$$

where

$$\mathrm{Cov}[\mu_{\mathrm{DVE}_k|\mathrm{IM_1}}, \mu_{\mathrm{DVE}_l|\mathrm{IM_2}}] \cong \frac{\partial e^{\mu_{\ln \mathrm{DVE}_k|\mathrm{IM_1}}}}{\partial \mu_{\ln \mathrm{DVE}_k|\mathrm{IM_1}}} \frac{\partial e^{\mu_{\ln \mathrm{DVE}_l|\mathrm{IM_2}}}}{\partial \mu_{\ln \mathrm{DVE}_l|\mathrm{IM_2}}}\Bigg|_{\substack{\overline{\mu_{\ln \mathrm{DVE}_k|\mathrm{IM_1}}} \\ \overline{\mu_{\ln \mathrm{DVE}_l|\mathrm{IM_2}}}}} \mathrm{Cov}[\mu_{\ln \mathrm{DVE}_k|\mathrm{IM_1}}, \mu_{\ln \mathrm{DVE}_l|\mathrm{IM_2}}] \tag{28}$$

and

$$\begin{aligned}\mathrm{Cov}[\mu_{\ln \mathrm{DVE}_k|\mathrm{IM_1}}, \mu_{\ln \mathrm{DVE}_l|\mathrm{IM_2}}] \cong{} & \frac{\partial \mu_{\ln \mathrm{DVE}_k|\ln \mathrm{EDP}_i}}{\partial \ln \mathrm{EDP}_i} \frac{\partial \mu_{\ln \mathrm{DVE}_l|\ln \mathrm{EDP}_j}}{\partial \ln \mathrm{EDP}_j}\Bigg|_{\substack{\overline{\mu_{\ln \mathrm{EDP}_i|\mathrm{IM_1}}} \\ \overline{\mu_{\ln \mathrm{EDP}_j|\mathrm{IM_2}}}}} \mathrm{Cov}[\mu_{\ln \mathrm{EDP}_i|\mathrm{IM_1}}, \mu_{\ln \mathrm{EDP}_j|\mathrm{IM_2}}] \\ & + \mathrm{Cov}[\mu_{\ln \mathrm{DVE}_k|\ln \mathrm{EDP}_i}, \mu_{\ln \mathrm{DVE}_l|\ln \mathrm{EDP}_j}]\end{aligned} \tag{29}$$

where, as above, μ_X is treated as a random variable due to epistemic uncertainty, and the notation $\overline{\mu_X}$ is used to denote the mean estimate of μ_X [1].

4.4. Epistemic uncertainty in the ground motion hazard

Epistemic uncertainty in the ground motion hazard is often displayed qualitatively using fractile uncertainty bands about the mean estimate of the hazard curve, as shown in Fig. 3. More formally, the ground motion hazard at a given IM level can be represented as

$$\lambda_{\mathrm{IM}}(x) = \overline{\lambda_{\mathrm{IM}}(x)}\varepsilon_{\mathrm{UIM}}(x) \tag{30}$$

where $\overline{\lambda_{\mathrm{IM}}(x)}$ is the best estimate or *mean estimate* of $\lambda_{\mathrm{IM}}(x)$, and $\varepsilon_{\mathrm{UIM}}(x)$ is a random variable with a mean of one. Considering the entire range of IM levels implies that $\varepsilon_{\mathrm{UIM}}(x)$ is in fact a random *function* of IM that is

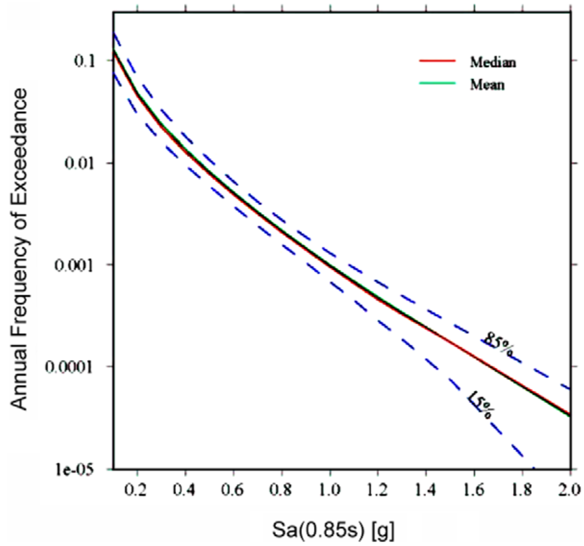

Fig. 3. IM hazard curve for an example site near Los Angeles, California.

correlated across the range of *IM* values. Note, however, that in practice the integral of Eq. (19) will be calculated as a summation

$$E[\mathrm{TC}] = \int_{\mathrm{IM}} E[\mathrm{TC}|\mathrm{IM} = x]|d\lambda_{\mathrm{IM}}(x)| \cong \sum_{i=1}^{n} E[\mathrm{TC}|\mathrm{IM} = x_i] \cdot \left(-\frac{\lambda_{\mathrm{IM}}(x_i) - \lambda_{\mathrm{IM}}(x_{i-1})}{x_i - x_{i-1}}\right) \tag{31}$$

where $x_0 < x_1 < \ldots < x_n$ are the discrete integration points. Here it is only important to recognize that a *discrete* set of $\lambda_{\mathrm{IM}}(x)$ is sufficient to characterize the hazard curve. Therefore we define a new random vector

$$\Delta\lambda_{\mathrm{IM}}(x_i) = -\frac{\lambda_{\mathrm{IM}}(x_i) - \lambda_{\mathrm{IM}}(x_{i-1})}{x_i - x_{i-1}} \tag{32}$$

The mean and covariance of the array $\Delta\lambda_{\mathrm{IM}}(x_i)$, $i = 1, \ldots, n$ can be computed from the mean and covariance of the array of $\lambda_{\mathrm{IM}}(x_i)$, $i = 1, \ldots, n$. The mean value of this array, $\overline{\lambda_{\mathrm{IM}}(x)}$, is a standard output from a PSHA analysis and was used earlier. The variances can be estimated from the fractile uncertainty often displayed in a graph of the seismic hazard curve (e.g. Fig. 3). Covariances of the array are potentially available from the output of PSHA software. Using this formulation, the random variable $E[\mathrm{TC}]$ can be represented as

$$E[\mathrm{TC}] = \sum_{i=1}^{n} E[\mathrm{TC}|\mathrm{IM} = x_i] \cdot \Delta\lambda_{\mathrm{IM}}(x_i) \tag{33}$$

where there is now epistemic uncertainty in $E[\mathrm{TC}|\mathrm{IM} = x_i]$ and $\Delta\lambda_{\mathrm{IM}}(x_i)$. Further, it is assumed that there is no stochastic dependence between the epistemic aspects of $E[\mathrm{TC}|\mathrm{IM} = x_i]$ and $\Delta\lambda_{\mathrm{IM}}(x_i)$. This model can then be used to compute the resulting epistemic uncertainty in $E[\mathrm{TC}]$, $\lambda_{\mathrm{collapse}}$ and $\lambda_{\mathrm{TC}}(z)$.

5. Total repair costs, accounting for epistemic uncertainty

Consider first the effect of epistemic uncertainty on the mean estimate ($\overline{E[\mathrm{TC}]}$) and epistemic variance (Var $[E[\mathrm{TC}]]$) of $E[\mathrm{TC}]$. Calculation of $\overline{E[\mathrm{TC}]}$ is performed by taking advantage of the independence of $E[\mathrm{TC}|\mathrm{IM} = x_i]$ and $\Delta\lambda_{\mathrm{IM}}(x_i)$, and using the linearity of the expectation operator

$$\overline{E[\mathrm{TC}]} = E\left[\sum_{i=1}^{n} E[\mathrm{TC}|\mathrm{IM} = x_i] \cdot \Delta\lambda_{\mathrm{IM}}(x_i)\right] = \sum_{i=1}^{n} \overline{E[\mathrm{TC}|\mathrm{IM} = x_i]} \cdot \overline{\Delta\lambda_{\mathrm{IM}}(x_i)} \tag{34}$$

This is the discrete analog of Eq. (19) (where the mean hazard curve is used in the calculation). Thus, the estimate of expected annual total repair cost is unchanged when epistemic uncertainty is included in the analysis, provided that the mean estimate of the ground motion hazard curve is used.

Calculation of epistemic variance in $E[\mathrm{TC}]$ involves a summation of products of random variables. Consider Eq. (33). If $E[\mathrm{TC}|\mathrm{IM}=x_i]$ is denoted as X_i, and $\Delta\lambda_{\mathrm{IM}}(x_i)$ as Y_i, then $E[\mathrm{TC}]$ is of the form

$$E[\mathrm{TC}] = \sum_{i=1}^{n} X_i \cdot Y_i \tag{35}$$

where X, and Y are random arrays. There is no correlation between X_i, and Y_i, but there is quite likely a correlation between X_i and X_j, and also between Y_i and Y_j $(i \neq j)$, as discussed above. σ_{X_i,X_j} is calculated in Eq. (27) and σ_{Y_i,Y_j} (the ground motion hazard) is discussed in the previous section. Given that the needed covariance matrices have been calculated, the following result has been derived for a product of random arrays [17]

$$\mathrm{Var}[E[\mathrm{TC}]] = \mathrm{Var}\left[\sum_i X_i Y_i\right] = \sum_i \sum_j (\sigma_{X_i,X_j}\sigma_{Y_i,Y_j} + \mu_{X_i}\mu_{X_j}\sigma_{Y_i,Y_j} + \mu_{Y_i}\mu_{Y_j}\sigma_{X_i,X_j}) \tag{36}$$

For brevity, it is left to the reader to make the simple change of notation $X_i = E[\mathrm{TC}|\mathrm{IM}=x_i]$ and $Y_i = \Delta\lambda_{\mathrm{IM}}(x_i)$ at the time of implementation in a computer program. Unlike some other results in this paper, no analytical solution for Eq. (36) exists.

These calculations for the mean and variance of $E[\mathrm{TC}]$ can be revised to incorporate costs due to collapses, although the resulting equations are lengthy. Readers interested in using the results for a loss analysis are referred to [1].

6. Annual rate of collapse, accounting for epistemic uncertainty

The steps described above provide all of the information necessary to compute the mean and variance in annual probability of collapse – a decision variable of interest to project stakeholders. This process is described below, as a relevant supplement to the repair cost calculations. The mean annual frequency of collapse can be computed as

$$\lambda_{\mathrm{collapse}} = \sum_{i=1}^{n} P[C \mid IM = x_i] \cdot \Delta\lambda_{\mathrm{IM}}(x_i) \tag{37}$$

The mean and covariance $\Delta\lambda_{\mathrm{IM}}(x_i)$ were discussed earlier as part of the Total Cost calculations. What remains is to model the mean and covariance of $P(C|\mathrm{IM})$. Define $P(C|\mathrm{IM}_i)$ as X_i. The mean value of $P(C|\mathrm{IM})$ (i.e., $E[X_i]$) can be estimated as the fraction of records that collapse at a given IM level. The variance of X_i can be estimated using the tools discussed in Section 4. The mean and covariance of the ground motion hazard, Y_i, remain identical to the results needed in Eq. (36). Thus the variance of $\lambda_{\mathrm{collapse}}$ can be computing by numerically evaluating Eq. (36) after substituting $P(C|\mathrm{IM}_i)$ for the X_i term.

An analytical solution for the mean and variance of the annual rate of collapse exists, if one makes several functional form assumptions. Consider a random variable for collapse capacity, Z, of the form $Z = \eta_Z \varepsilon_{UZ} \varepsilon_{RZ}$, where η_Z is the median value of Z (expressed in units of IM), and ε_{UZ} and ε_{RZ} are lognormal random variables. ε_{RZ} accounts for aleatory uncertainty in the capacity, and ε_{UZ} accounts for epistemic uncertainty in the median value of Z. The medians of $\varepsilon_{\mathrm{UZ}}$ and ε_{RZ} are defined to be one, and their logarithmic standard deviations are denoted $\sigma_{\ln(\varepsilon_{\mathrm{RZ}})} = \beta_{\mathrm{RZ}}$ and $\sigma_{\ln(\varepsilon_{\mathrm{UZ}})} = \beta_{\mathrm{UZ}}$. The moments of these random variables can be obtained using information previously calculated. First, note that what was previously called $P(C|\mathrm{IM})$ is in fact the CDF of a random variable for collapse: $F_Z(\mathrm{IM})$. Because Z is lognormally distributed, η_Z and β_{RZ} can be estimated by the mean and standard deviation, respectively, of a sample of logarithmic collapse capacity values [10,25]. The term β_{UZ} can be estimated using the techniques discussed earlier in this section for evaluation of epistemic uncertainty. Further, the ground motion hazard curve is approximated by the function $\lambda_{\mathrm{IM}}(x) = k_0 x^{-k}\varepsilon_{\mathrm{UIM}}$, where $\varepsilon_{\mathrm{UIM}}$ is a lognormal random variable with *mean* equal to one and standard deviation $\sigma_{\ln(\varepsilon_{\mathrm{UIM}})} = \beta_{\mathrm{UIM}}$. This form for the hazard curve has been proposed previously by others [29,30].

Under the above assumptions, the mean estimate of the mean annual frequency of collapse is given by

$$E[\lambda_{\text{collapse}}] = k_0 \eta_Z^{-k} \cdot e^{\frac{1}{2}k^2(\beta_{\text{UZ}}^2 + \beta_{\text{RZ}}^2)} \tag{38}$$

Further, $\lambda_{\text{collapse}}$ is a lognormal random variable, and its logarithmic standard deviation is

$$\sigma_{\ln(\lambda_{\text{collapse}})} = \sqrt{\beta_{\text{UIM}}^2 + k^2\beta_{\text{UZ}}^2} \tag{39}$$

This result is derived from related problems that have previously been solved [30].

7. Rate of exceeding a given TC, accounting for epistemic uncertainty

With the epistemic uncertainty in $E[\text{TC}|\text{IM} = im]$ and $\Delta\,\lambda_{\text{IM}}(im)$ described above, one can also compute the mean annual frequency of exceeding a level of TC denoted z (i.e., $\lambda_{\text{TC}}(z)$). Because the terms $G_{\text{TC}|\text{IM}}(z,x)$ and $\lambda_{\text{IM}}(x)$ are now considered random functions due to epistemic uncertainty, the expected value of $\lambda_{\text{TC}}(z)$ is given by

$$E[\lambda_{\text{TC}}(z)] = \sum_{x_i} \overline{G_{\text{TC}|\text{IM}}(z,x_i)\Delta\lambda_{\text{IM}}(x_i)} \tag{40}$$

where $\overline{G_{\text{TC}|IM}(z,x)} = E[P(\text{TC} > \underline{z \mid IM} = x)]$ is the *mean estimate* of the Complementary Cumulative Distribution Function of TC|IM, and $\overline{\text{d}\lambda_{\text{IM}}(x)}$ is the *mean estimate* of the derivative of the hazard curve.

Calculation the of variance of $\lambda_{\text{TC}}(x)$ first requires that the previously calculated epistemic variance in $E[\text{TC}|\text{IM}]$ be propagated to determine the epistemic variance in $G_{\text{TC}|\text{IM}}(z,x)$. If TC|IM is assumed to have a lognormal distribution (perhaps a reasonable assumption given that repair cost distributions are observed to be skewed [8,31]) with mean $E[\ln \text{TC}|\text{IM} = x]$ and standard deviation β_{R}, then $G_{\text{TC}|\text{IM}}(z,x)$ is defined by

$$G_{\text{TC}|\text{IM}}(z,x) = \Phi\left(\frac{\ln z - E[\ln \text{TC}|\text{IM} = x]}{\beta_{\text{R}}}\right) \tag{41}$$

Modeling of epistemic uncertainty here is limited to uncertainty in $E[\ln \text{TC}|\text{IM} = x]$, which has standard deviation $\beta_{\text{U}}(x)$ as discussed earlier. Using a first-order expansion, the variance in $G_{\text{TC}|\text{IM}}(z,x)$ can be shown to equal

$$\text{Var}[G_{\text{TC}|\text{IM}}(z,x)] \cong \phi\left(\frac{\ln z - E[\ln \text{TC}|\text{IM} = x]}{\beta_{\text{R}}(x)}\right)^2 \cdot \frac{\beta_{\text{U}}^2(x)}{\beta_{\text{R}}^2(x)} \tag{42}$$

where $\beta_{\text{R}}(x)$ is the aleatory dispersion of TC|IM as calculated before, and $\phi(\cdot)$ is the probability density function of the standard normal distribution. Similarly, the covariance between $G_{\text{TC}|\text{IM}}(z,x)$ evaluated at two IM levels is

$$\begin{aligned}\text{Cov}[G_{\text{TC}|\text{IM}}(z,x_1), G_{\text{TC}|\text{IM}}(z,x_2)] \cong\ & \phi\left(\frac{\ln z - E[\ln \text{TC}|\text{IM} = x_1]}{\beta_{\text{R}}(x_1)}\right)\phi\left(\frac{\ln z - E[\ln \text{TC}|\text{IM} = x_2]}{\beta_{\text{R}}(x_2)}\right) \\ & \cdot \frac{\text{Cov}[E[\ln \text{TC}|\text{IM} = x_1], E[\ln \text{TC}|\text{IM} = x_2]]}{\beta_{\text{R}}(x_1)\beta_{\text{R}}(x_2)}\end{aligned} \tag{43}$$

where the needed covariance term is given in Eq. (27). The variance of the mean estimate of $\lambda_{\text{TC}}(x)$ can then be computed by evaluating Eq. (36) after making the substitution $X_i = G_{\text{TC}|\text{IM}}(z,x_i)$, with the means and covariances of this term obtained from the equations in this section. Variance in the mean estimate of $\lambda_{\text{TC}}(x)$ can also be computed while accounting for the effect of collapses, but the resulting equations are lengthy and are perhaps not of interest except at the time of implementation [1].

A simple closed form solution exists for the mean and variance of the annual rate of exceeding a given total repair cost, under the following assumptions. $E[\text{TC}|IM{=}im]$ is approximated by a function of the form $a(im)^b$, where a and b are constants. The conditional random variable TC|IM is characterized as $\text{TC}|\text{IM} = E[\text{TC}|\text{IM} = im]\varepsilon_{\text{TC}|\text{IM}}$, where $\varepsilon_{\text{TC}|\text{IM}}$ is a lognormal random variable with median equal to one and aleatory and epistemic logarithmic standard deviations equal to β_{R} and β_{U} respectively. Finally, a function of the form

$\lambda_{IM}(x) = k_0 x^{-k}$ is fit to the true mean site hazard curve, as before. Then the mean annual rate of exceeding Total Cost z is given by

$$E[\lambda_{TC}(z)] = k_0 \left(\frac{z}{a}\right)^{-k/b} \exp\left(\frac{1}{2}\frac{k^2}{b^2}(\beta_R^2 + \beta_U^2)\right) \tag{44}$$

Making the additional assumption that perfect correlations exist within $\ln E[TC|IM = x]$ and $\lambda_{IM}(x)$ over varying levels of x, the lognormal epistemic standard deviation of $\lambda_{TC}(z)$ can be computed as

$$\beta_{\lambda_{TC}(z)} = \sqrt{\beta_{UIM}^2 + \frac{k^2}{b^2}\beta_U^2} \tag{45}$$

This result allows error bounds to be computed for the TC hazard curve. Although this analytical formulation requires several assumptions, its simplicity may make it useful in some situations.

8. The effect of variance in TC given IM

Most loss estimation studies have focused on $E[TC|IM]$ and neglected $Var[TC|IM]$, with the objective of estimating only $E[TC]$. Here the role of uncertainty in *TC* given IM, is briefly considered to illustrate its effect on results of interest.

Consider a representation for TC|IM as follows. The expected value of TC|IM is modeled by the function

$$E[TC|IM] = 1 - e^{-2IM^2} \tag{46}$$

where TC is expressed as the fraction of the replacement cost, so its expected value varies between zero and one. This function is similar in form to functions found from detailed loss estimation studies (e.g., [16]). This function is plotted in Fig. 4. The logarithmic standard deviation of TC|IM, denoted $\beta_{TC|IM}$, is assumed to be constant. Here no distinction is made between epistemic and aleatory uncertainties, as the distinction would not affect this result. The IM hazard curve is approximated by the functional form $\lambda_{IM}(x) = k_0 x^{-k}$, with constants $k_0 = 0.0002$ and $k = 3$ (these values were obtained by fitting a numerical hazard curve for southern California, using spectral acceleration at one second as the IM). Using these values, the mean annual frequency of exceeding a level of TC is computed numerically using Eq. (40). The results are plotted in Fig. 5 for a range of $\beta_{TC|IM}$ values.

The analytical solution from Eq. (44) also provides some insight. First, the mean value of TC|IM is approximated by the function $E[TC|IM] = 1.4IM^{1.8}$, as shown in Fig. 4. The analytical solution produces results comparable to the numerical integration results for many of the TC and $\beta_{TC|IM}$ values, as seen in Fig. 5. The accuracy of this analytical solution depends upon several conditions. First, here the numerical integration

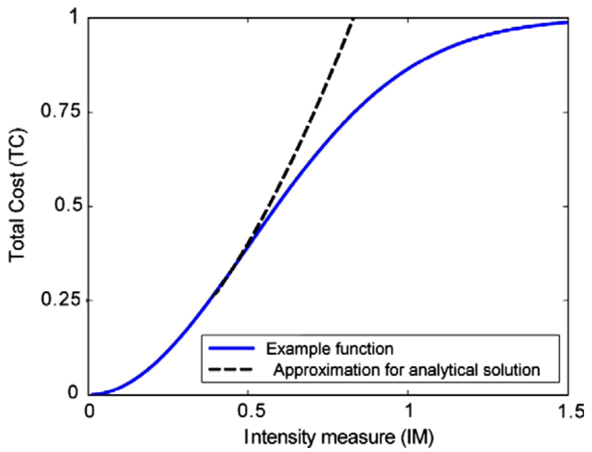

Fig. 4. Expected TC | IM $= 1 - e^{-2IM^2}$ for illustration, and the approximation TC|IM $= 1.4IM^{1.8}$, used for the analytical solution.

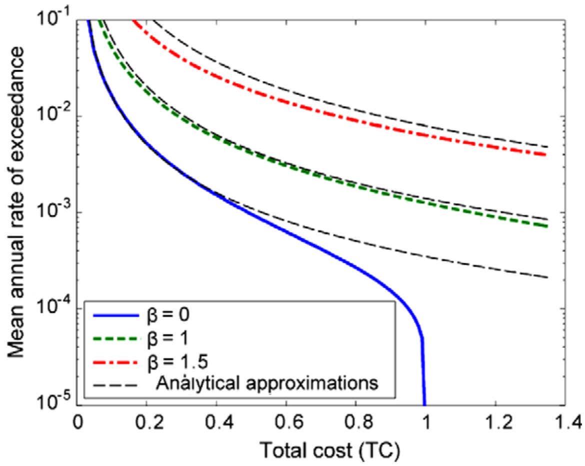

Fig. 5. Analytical and numerical solutions for the mean annual frequency of exceeding a range of Total Cost levels, for several values of $\beta_{TC|IM}$.

solution used the k_0x^{-k} functional form to represent the IM hazard curve, and also used a constant $\beta_{TC|IM}$ value. In situations where these assumptions are not valid, the number of approximations needed for the analytical solution will increase and its accuracy will decrease. Second, the analytical solution is sensitive to the region of the ground motion hazard curve where the $\lambda_{IM}(x) = k_0x^{-k}$ functional fit is performed, so the fitting should be performed over the IM region that is contributing most to repair costs. Finally, the result is sensitive to the effectiveness of the fit illustrated in Fig. 4, so again this fit should be performed in the region that governs total repair cost results. If the functional fits are performed in other regions of these curves, the analytical results can differ greatly. For these reasons, analytical solution results should be calculated and interpreted with care.

A further observation can be made using the analytical solution of Eq. (44). If $\beta_{TC|IM}$ has a value of 0, one obtains a result reflecting no uncertainty in repair costs given *IM*. For a $\beta_{TC|IM}$ value of 0.5 (a value obtained in one application of the PEER framework [8]), the results are multiplied by a factor of 1.4 relative to the $\beta_{TC|IM} = 0$ case. That is, the annual rate of exceeding a given total cost has increased by 40% due to the effects of uncertainty in TC (for a given IM) in this problem. The intuitive reason for this is that because of uncertainty in TC|IM, low *IM* levels have the possibility of causing large repair costs; although high IM levels might conversely cause low repair costs, the two effects do not offset each other because low IM levels occur much more often than high IM levels. If the $\beta_{TC|IM}$ value were increased to 1 or 1.5 (which might be possible considering the many large uncertainties present in this problem [16]), the inflation relative to the $\beta_{TC|IM} = 0$ case would be approximately 4 or 20, respectively. Thus, it may be possible for this uncertainty to have a significant effect on the mean annual frequency of exceeding a given TC level. However, at least for moderate $\beta_{TC|IM}$ values of less than one, this "amplification" due to uncertainty still provides only a second-order correction to the result obtained when $\beta_{TC|IM}$ has a value of 0. It is for this reason that the FOSM approximations used to measure uncertainty in $\beta_{TC|IM}$ are believed to provide a sufficiently accurate result. This also illustrates why numerical integration over the hazard curve is retained, because an explicit hazard curve is needed to evaluate the basic result even when $\beta_{TC|IM}$ has a value of 0.

9. Conclusions

A procedure for propagating uncertainties in seismic loss estimation has been proposed, utilizing the framework proposed by PEER for performance-based earthquake engineering. The calculation procedure combines inputs of aleatory and epistemic uncertainty in ground motion hazard, building response, damage to building elements, and element repair costs to quantify uncertainty in estimated repair costs and rate of collapse.

Potential options for propagating uncertainty through this framework are Monte Carlo simulation, numerical integration and first-order second-moment (FOSM) approximations. The procedure proposed here uses a combination of numerical integrations and FOSM approximations. The FOSM method is used to collapse the large vectors of conditional random variables into a single conditional random variable, Total Cost given IM. Numerical integration is then used to combine this random variable with the ground motion hazard. This numerical integration is treated accurately because it is believed to be the dominant contributor to the final results. This final integration also does not involve vectors of random variables, meaning that numerical integration is possible, unlike the high-dimensional integrations required for the other steps. The quantities that can be computed are the expected value and epistemic variance of the mean annual loss, mean rate of collapse, and mean rate of exceeding a given cost.

Large vectors of random variables are needed to represent engineering demand parameters, element damage measures, and element repair costs in this loss estimation framework. Accounting for these large vectors makes the basic integral equation used for loss estimation (Eq. (1) more difficult to evaluate than it might at first appear. The large vectors also imply that a method is needed to quantify the joint stochastic properties of these element properties, and while these dependencies have a large impact on final results, data needed for characterizing them is sometimes severely lacking. In the context of the proposed procedure, which accounts for dependency through linear correlation coefficients, several simple models for characterizing correlations based on generalizations of the equi-correlated model are proposed. Estimation of correlations using this approach may be more intuitive than simply estimating correlation coefficients individually for each pair of random variables in a vector. It is expected that implementations of this procedure will take the form of simple computer programs, which will aid repeated calculations for vectors of random variables and facilitate sensitivity analyses by allowing input parameters to be varied while quickly observing the end effect of these variations.

Both aleatory and epistemic uncertainties are considered in the analysis. While this aleatory/epistemic treatment is well developed in some areas (e.g., annual frequency of failure calculations in the nuclear industry), it has to date received limited attention in seismic loss estimation efforts. As an aid for those hoping to implement these methods, references to previous studies attempting to characterize epistemic uncertainty are provided. While consideration of epistemic uncertainty leads to increased complexity in estimation and analysis, the authors believe it to be a necessary component in the development of this field. This is especially true because in its current data-poor state, cost analysis can involve large epistemic uncertainties which may significantly affect the total variance of TC|IM.

Acknowledgements

This work was supported primarily by the Earthquake Engineering Research Centers Program of the National Science Foundation, under Award Number EEC-9701568 through the Pacific Earthquake Engineering Research Center (PEER). Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect those of the National Science Foundation.

References

[1] Baker JW, Cornell CA. Uncertainty specification and propagation for loss estimation using FOSM methods. PEER 2003-07. Berkeley, California: Pacific Earthquake Engineering Research Center, University of California at Berkeley; 2003. p. 89.

[2] Cornell CA, Krawinkler H. Progress and challenges in seismic performance assessment. PEER Center News 2000;3(2). http://peer.berkeley.edu/news/2000spring/performance.html.

[3] Deierlein GG. Overview of a comprehensive framework for earthquake performance assessment. PEER 2004/05. In: Fajfar P, Krawinkler H, editors. 2004, International Workshop on Performance-Based Seismic Design Concepts and Implementation: Bled, Slovenia. p. 12. Available from http://peer.berkeley.edu/Products/PEERReports/reports-2004/reports04. html.

[4] NUREG. PRA procedures guide—a guide to the performance of probabilistic risk assessment for nuclear power plants. NUREG/CR-2300. Washington, DC: Division of Engineering Technology Office of Nuclear Regulatory Research U.S. Nuclear Regulatory Commission; 1983.

[5] Garrick B. Recent case studies and advancements in probabilistic risk assessment. Risk Analysis 1984;4(4):267–79.

[6] Comerio M. Downtime modeling for risk management. In: 9th International Conference on Structural Safety and Reliability (ICOSSAR05), Rome, Italy; 2005. p. 8.

[7] Porter KA, Kiremidjian AS. Assembly-based vulnerability and its uses in seismic performance evaluation and risk-management decision-making. In: John A, editor. Stanford: Blume Earthquake Engineering Center; 2001. p. 214.
[8] Aslani H, Miranda E. Probabilistic earthquake loss estimation and loss deaggregation in buildings. Report #154. 2006, John A. Blume Earthquake Engineering Center: Stanford, CA. Available from http://blume.stanford.edu/Blume/Publications.htm.
[9] Vamvatsikos D, Cornell CA. Applied incremental dynamic analysis. Earthquake Spectra 2004;20(2):523–53.
[10] Baker JW, Cornell CA. Vector-valued ground motion intensity measures for probabilistic seismic demand analysis. Report #150. 2005, John A. Blume Earthquake Engineering Center: Stanford, CA. p. 321. Available from http://blume.stanford.edu/Blume/Publications.htm.
[11] Ibarra LF, Krawinkler H. Global collapse of frame structures under seismic excitations. Report #152. 2005, John A. Blume Earthquake Engineering Center: Stanford, CA. p. 324. Available from http://blume.stanford.edu/Blume/Publications.htm.
[12] RS Means Company Inc. Means assemblies cost data. 31st ed. Kingston, MA; 2006.
[13] National Institute of Building Sciences (NIBS). Standardized earthquake loss estimation methodology (HAZUS 99 technical manual). Washington, D.C.: Report prepared for Federal Emergency Management Agency, Federal Emergency Management; 1999.
[14] Kramer SL. Geotechnical earthquake engineering. Upper Saddle River, N.J.: Prentice Hall; 1996.
[15] McGuire RK. Seismic hazard and risk analysis. Berkeley: Earthquake Engineering Research Institute; 2004.
[16] Miranda E, Aslani H, Taghavi S. Assessment of seismic performance in terms of economic losses. PEER 2004/05. In: Fajfar P, Krawinkler H, editors. 2004, International Workshop on Performance-Based Seismic Design Concepts and Implementation: Bled, Slovenia. p. 149–60.
[17] Ditlevsen O. Uncertainty modeling: With applications to multidimensional civil engineering systems. New York; London: McGraw-Hill International Book Co.; 1981.
[18] Gentle JE. Random number generation and Monte Carlo methods. 2nd ed. New York: Springer-Verlag; 2003.
[19] Kennedy RP, Ravindra MK. Seismic fragilities for nuclear power plant risk studies. Nuclear engineering and design: an international journal devoted to the thermal, mechanical and structural problems of nuclear energy 1984:7947–68.
[20] FEMA. 350. Recommended seismic design criteria for new steel moment-frame buildings. SAC Joint Venture. Washington, D.C.: prepared for the Federal Emergency Management Agency; 2000.
[21] FEMA. 355f. State of the art report on performance prediction and evaluation of steel moment-frame buildings. American Society of Civil Engineers. Washington, D.C.: Federal Emergency Management Agency; 2000.
[22] Yun S-Y, Hamburger RO, Cornell CA, Foutch DA. Seismic performance evaluation for steel moment frames. Journal of Structural Engineering 2002;128(4):12.
[23] Bazzurro P, Cornell CA, Menun C, Luco N, Motahari M. Advanced seismic assessment guidelines. Report for Pacific Earthquake Engineering Research (PEER) Center Lifelines Program Project 507; 2004. Available from http://peer.berkeley.edu/lifelines/LL-CEC/reports/final_reports/507-FR. pdf.
[24] Jones AL, Kramer SL, Aduino P. Estimation of uncertainty in geotechnical properties for performance-based earthquake engineering. PEER 2002-16. 2002, Pacific Earthquake Engineering Research Center, University of California at Berkeley: Berkeley, California. p. 100.
[25] Ibarra LF. Global collapse of frame structures under seismic excitations. Ph.D. Thesis. Dept. of Civil and Environmental Engineering. Stanford, CA: Stanford University; 2003.
[26] Wen YK, Ellingwood BR, Veneziano D, Bracci J. Uncertainty modeling in earthquake engineering, ed. MAE Center Project FD-2 Report. 2003. Available from http://mae.ce.uiuc.edu/Research/Whitepaper.pdf.
[27] SSHAC. Recommendations for probabilistic seismic hazard analysis: Guidance on uncertainty and use of experts. NUREG/CR–6372, Senior Seismic Hazard Analysis Committee and U.S. Nuclear Regulatory Commission, editors. Washington, D.C.: 1997.
[28] Efron B, Tibshirani RJ. An introduction to the bootstrap. Monographs on statistics and applied probabilities; 57. New York: Chapman & Hall; 1993.
[29] Kennedy RP, Short SA. Basis for seismic provisions of DOE-STD-1020. UCRL-CR-111478 and BNL-52418. Lawrence Livermore National Laboratory and Brookhaven National Laboratory; 1994.
[30] Cornell CA, Jalayer F, Hamburger RO, Foutch DA. Probabilistic basis for 2000 SAC federal emergency management agency steel moment frame guidelines. Journal of Structural Engineering 2002;128(4):526–33.
[31] Touran A, Suphot L. Rank correlations in simulating construction costs. Journal of Construction Engineering and Management 1997;123(3):297–301.