Illustrating a Bayesian Approach to Seismic Collapse Risk Assessment

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ABSTRACT: In this study, we present a Bayesian method for efficient collapse response assessment of structures. The method facilitates integration of prior information on collapse response with data from nonlinear structural analyses in a Bayesian setting to provide a more informed estimate of the collapse risk. The prior information on collapse can be obtained from a variety of sources, including information on the building design criteria and simplified linear dynamic analysis or nonlinear static (pushover) analysis. The proposed method is illustrated on a four-story reinforced concrete moment frame building to assess its seismic collapse risk. The method is observed to significantly improve the statistical and computational efficiency of collapse risk predictions compared to alternative methods.

1. INTRODUCTION

Building codes achieve seismic performance goals related to life safety of building occupants by controlling collapse risk of structures to acceptable levels. Modeling of structural collapse is challenging due to highly nonlinear structural response under extreme ground shaking, and its simulation requires nonlinear structural analysis tools and models that can capture various sources of cyclic and in-cycle degradation in structural components. Moreover, robust estimation of collapse risk should consider uncertainties in the earthquake ground motions and structural modeling, and propagate these effects from component through to system level response. These uncertainties affect the statistical efficiency of the estimated collapse risk parameters and add to the computational demand associated with collapse risk assessment of structures.

Bayesian statistics facilitate the incorporation of any prior knowledge to inform statistical inference. Singhal and Kiremidjian (1998) proposed using Bayesian statistics to update fragility functions with observational building damage data. Jaiswal et al. (2011) also used a Bayesian approach for computing empirical collapse fragility functions combining expert opinion and field data for global building types. Jalayer et al. (2010) used a Bayesian framework for assessing the effects of structural modeling uncertainty. They incorporated test and inspection results of structures in order to update the prior information on the modeling uncertainties.

In this study, we present a Bayesian method for efficient collapse response assessment of structures combining analysis data and judgment. The method facilitates integration of prior information to estimate collapse fragility parameters with data from nonlinear structural analyses. The combination of nonlinear analysis simulations with prior collapse fragility information aims to improve computational and statistical efficiency.

2. BAYESIAN APPROACH

In this section, we present a Bayesian method to collapse risk assessment. We discuss background to development of the method, and present the methodology along with the analysis rule.

2.1. *Collapse Risk Metrics*

Collapse fragility functions define probability of collapse as a function of ground motion intensity (IM). It is common to use lognormal distribution to represent collapse fragility curves (Bradley and Dhakal, 2008). Using a lognormal distribution, the probability of collapse given a ground motion intensity, $P(C|IM = im)$, is defined as

$$P(C|IM = im) = \Phi\left(\frac{ln(im) - ln(\hat{\theta})}{\hat{\beta}}\right) \quad (1)$$

where $\Phi()$ is the standard normal cumulative distribution function, and $\hat{\theta}$ and $\hat{\beta}$ represent median collapse capacity and logarithmic standard deviation (dispersion), respectively.

Collapse risk is often quantified using mean annual frequency of collapse (λ_c), which is defined as

$$\lambda_c = \int_0^\infty P(C|IM = im)\, |\mathrm{d}\lambda_{IM}(im)| \quad (2)$$

where λ_{IM} is the mean annual frequency of exceedance of IM. This metric, by augmenting structural collapse response with site seismic hazard characteristics, provides a site-specific measure of collapse risk. In this study, our goal is to reliably estimate λ_c for collapse risk assessment.

2.2. *Proposed Method*

In this section, we provide a step-by-step procedure for conducting collapse risk assessment using the Bayesian approach. Figure 1 illustrates the steps. The essence of this approach is to transform an initial estimate of the collapse response to an informed estimate using nonlinear structural analyses data with the goal of efficiently estimating λ_c. The steps are listed as follows:

a) Define an initial estimate of the collapse fragility curve and estimate the uncertainty in the median collapse capacity (Figure 1a).

b) Select two or more IM levels at which to scale and conduct nonlinear dynamic analyses. Quantify the prior distributions at these IM levels (Figure 1b).

c) Select ground motion suites consistent with conditional spectra at the chosen IM levels (Figure 1c).

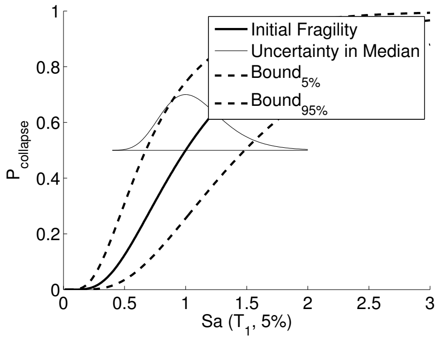
d) Conduct nonlinear time history analyses using the selected ground motions scaled to the IM levels of interest. Incorporate modeling uncertainty by sampling model realizations (Figure 1d).

e) Obtain posterior distributions at the IM levels by updating the prior distributions with data from structural analyses (Figure 1e).

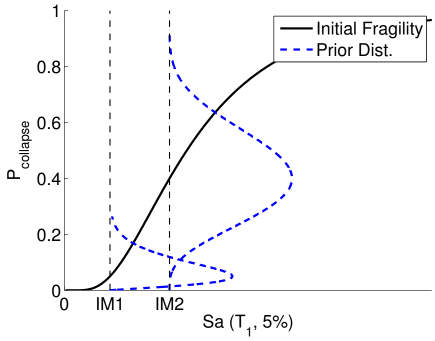
f) Obtain a final estimate of the collapse fragility function using the maximum likelihood method. For this method, likelihood is obtained using the posterior distributions at the IM levels (Figure 1f).

2.3. *Details of the Proposed Method*

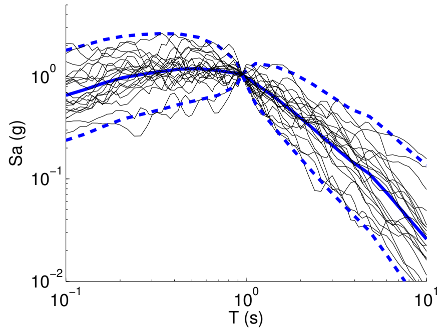
Users of this method are expected to provide prior information on the collapse response of the structure. The prior information on collapse response can be informed by a variety of sources, including information on the building design criteria and simplified linear dynamic analysis or nonlinear static (pushover) analysis. Using a lognormal assumption, we expect the users to provide an estimate of median collapse capacity (θ) and dispersion (β) defining the initial collapse fragility function. To treat epistemic and aleatory uncertainties in collapse fragility functions, median collapse capacity (Θ) is defined to be a Bayesian random variable. Θ is modeled using a lognormal distribution with median θ and dispersion β_θ. In addition to θ and β, users are also expected to provide β_θ. Figure 1a illustrates an initial fragility curve and uncertainty in median collapse capacity using by a probability density function. In cases where information on β_θ is not available, users can make judgment-based assumptions for estimating β_θ considering the limitations in structural idealizations, calibration of



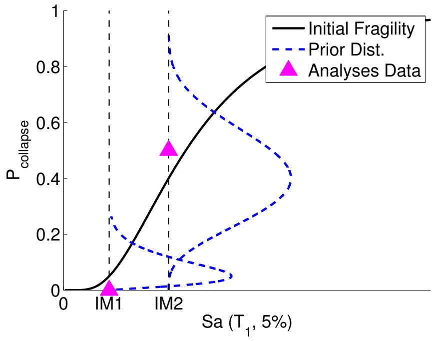
(a) Initial collapse fragility curve and uncertainty in median capacity



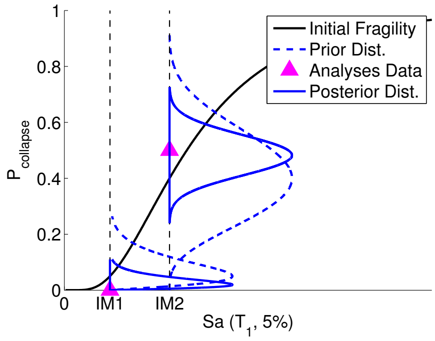
(b) Selected IM levels and prior distributions at these levels



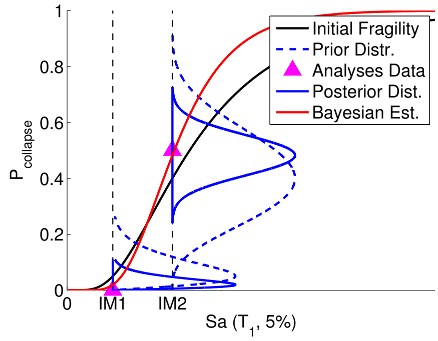
(c) Ground motions selected to match conditional spectra



(d) Data from nonlinear time history analyses at selected IM levels



(e) Posterior distributions at selected IM levels



(f) Final estimate of the collapse fragility function

Figure 1: Steps of the proposed method

model parameters, number of analyses, and software used for structural analyses such as: "Median collapse capacity is estimated as θ within $\pm\Delta\%$ certainty with $(1-\alpha)\%$ confidence." This leads to the median collapse capacity being defined using a lognormal distribution having median at θ and dispersion β_θ as given in the equation below (Ellingwood and Kinali, 2009).

$$\beta_\theta = \sqrt{\ln\left(\left[\frac{\Delta}{\Phi^{-1}(1-\alpha/2)}\right]^2 + 1\right)} \qquad (3)$$

An example of such an assumption is as follows: "Median collapse capacity is estimated as 1 g within $\pm$ 50% certainty with 90% confidence." This statement is translated into a lognormal distribution having a median of 1 g ($\theta = 1$) and dispersion of approximately 0.3 ($\beta_\theta = 0.3$) . The 90% confidence interval for this distribution is 0.61 g to 1.64 g.

Two or more IM levels should be used to select ground motions and conduct structural analyses. A simulation-based grid search is conducted to identify the combinations of IM levels that lead to minimum error of λ_c estimates. Due to space constraints, we do not present the results of the simulation-based grid search. Based on search results, we recommend that IM_1 is selected corresponding to probability of collapse of 10% or lower on the initial collapse fragility curve. Select IM_2 such that it corresponds to probability of collapse between 30% and 80% on the initial fragility curve as an increasing function of Δ.

The prior distribution at the i^{th} IM level (IM_i) defines the probability of collapse at IM_i ($P(\tau_i)$). It is characterized using a beta distribution $P(\tau_i) \sim Beta(\alpha_i, \beta_i)$. In this method, the parameters of the beta distribution, namely α_i, β_i, are calibrated as follows:

We define two curves, namely $Bound_{5\%}$ and $Bound_{95\%}$, which are lognormal cumulative distributions. They have dispersions of β. The medians of $Bound_{5\%}$ and $Bound_{95\%}$ correspond to 5% and

95% quantiles of Θ, respectively. Figure 1a illustrates $Bound_{5\%}$ and $Bound_{95\%}$.

For any IM_i, the 5% quantile of $P(\tau_i)$ is obtained as the point corresponding to IM_i on $Bound_{5\%}$. Similarly, the 95% quantile of $P(\tau_i)$ is obtained as the point corresponding to IM_i on $Bound_{95\%}$. The point on the initial fragility function corresponding to IM_i denotes the mode of $P(\tau_i)$. Using these constraints, one can obtain the parameters, α_i and β_i, defining $P(\tau_i) \sim Beta(\alpha_i, \beta_i)$. Figure 1b illustrates selected IM levels along with the prior distributions at these levels.

Using Bayes theorem, the posterior distribution, $P(\tau_i|X_i)$ is defined as follows:

$$P(\tau_i|X_i) \propto P(\tau_i)P(X_i|\tau_i) \tag{4}$$

where $P(X_i|\tau_i)$ defines the likelihood function. In this method, we use data from structural analyses to define the likelihood function. Nonlinear time history analyses should be conducted for this purpose, preferably incorporating modeling uncertainty and record-to-record variability. To account for record-to-record variability, hazard consistent ground motion suites are selected at each IM level. An example ground motion suite is selected matching the conditional spectra, and is shown in Figure 1c. For ground motion selection, readers are referred to Jayaram et al. (2011). Previous research has shown that neglecting modeling uncertainty results in inconservative estimates of collapse capacity (Liel et al., 2009; Dolsek, 2009; Ugurhan et al., 2014). For robust estimates of collapse risk, we recommend characterizing the uncertainty in modeling parameters, and obtaining samples of model realizations from the characterized probability distributions. Nonlinear time history analyses should be conducted using the ground motion suites scaled to IM levels of interest matched with the sampled model realizations. Analyses results are collected as binomial data (X_i) in the form of number of collapses and no-collapses, $P(X_i|\tau_i) \sim Binomial(\tau_i, X_i)$. Figure 1d illustrates the results from structural analyses in terms of proportions of collapses at the two selected IM levels.

In this method, prior distributions are defined using beta distributions, and likelihood data is collected using a binomial distribution. Since beta and binomial distributions form a conjugate pair, posterior distributions are also defined to have beta distributions $P(\tau_i|X_i) \sim Beta(\hat{\alpha}_i, \hat{\beta}_i)$. Posterior distributions are illustrated for the selected IM levels in Figure 1e.

Using the two posterior distributions at different IM levels as the probability distributions defining a likelihood function, we use a maximum likelihood estimator to find the final parameters of a collapse fragility function, namely $\hat{\theta}, \hat{\beta}$ (Figure 1f).

2.4. *Validation of the Method*

Our current work explores to validate the proposed method. In the interest of computational efficiency, instead of analyzing real structural models we use simulated structural analyses data, which is binomial data drawn from an assumed target fragility function. We apply the Bayesian method to obtain estimates the collapse fragility function and λ_c. This procedure is repeated in a Monte-Carlo simulation framework, which allows us to study the statistical efficiency of the Bayesian approach in terms of the variance and bias of the estimators.

In the interest of space, we will not be presenting the results from the validation analyses. Key observations from the validation analyses are as follows: The variability in collapse risk metrics is significantly reduced using the Bayesian approach compared to an alternative method of maximum likelihood estimate. If used with an initial unbiased estimate of the collapse fragility function, Bayesian method provides an unbiased estimate of λ_c. However, a biased initial guess produces bias in Bayesian estimates. The amount of bias introduced in the prior information heavily affects the amount of bias that will be observed in the Bayesian estimates. Posterior predictive checking (Gelman et al., 2013) is a model checking method to assess the plausibility of the posterior distributions. This method is well suited to provide a check for the estimates obtained using the Bayesian approach.

3. APPLICATION OF BAYESIAN APPROACH

3.1. Structural Modeling and Analysis

Collapse risk assessment of a 4-story reinforced concrete special moment frame is conducted to illustrate the application of the Bayesian approach. The structure was designed by Haselton and Deierlein (2007) in accordance with 2003 International Building Code and ASCE7-02 provisions. It is located at a seismically active site in downtown Los Angeles, CA with $V_{s,30} = 285$ m/s.

We use concentrated plasticity modeling to idealize the structural system. Frame members are modeled as elastic elements with zero-length rotational springs at both ends. The hysteretic behavior and in-cycle and cyclic deterioration rules are governed by Ibarra et al. (2005). The fundamental period of the structure is $T_1 = 0.94$ s and a Rayleigh damping of 3% is applied to it. The structure is modeled and analyzed using Open System for Earthquake Engineering Platform (McKenna et al., 2014).

3.2. Prior Information on Collapse Response

Nonlinear static analysis is a common structural analysis strategy, and is generally conducted to check the nonlinear structural analysis model. Vamvatsikos and Cornell (2005) provide a fast method, SPO2IDA, which uses pushover analysis results to infer nonlinear dynamic response by establishing connections between pushover curves and incremental dynamic analyses (IDA) curves. In this study, we use this method to obtain an initial estimate of the collapse response of the structure. Since nonlinear static analysis is not computationally demanding and is generally conducted before any dynamic analyses, we assume that it does not add to the computational demand of the Bayesian method.

The 4-story frame structure is analyzed using nonlinear static analysis. Using the software for SPO2IDA, we obtain estimates of 16, 50 and 84% fractal IDA curves as shown in Figure 2. Dynamic instability, which is characterized by the zero slope of an IDA curve, is observed at ductility values around 10. These estimated fractals of IDA curves

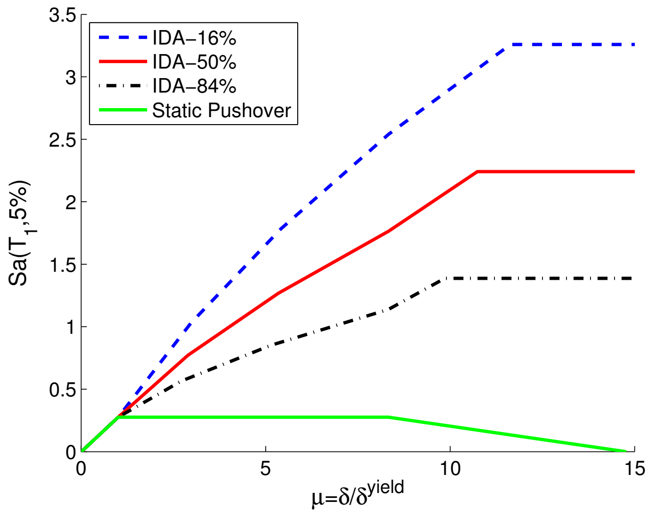


Figure 2: Pushover and 16, 50 and 84% fractal IDA curves obtained using the method by Vamvatsikos and Cornell (2005)

lead to a lognormal collapse fragility function having a median collapse capacity of 2.19 g and dispersion of 0.43. We use $\Delta = 0.4$ to characterize the uncertainty in median collapse capacity estimate.

3.3. Ground Motion and Modeling Uncertainty

To represent record-to-record variability, ground motion suites are selected consistent with conditional spectra at two different ground motion intensity levels. IM is used as 5% damped spectral acceleration at $T_1 = 0.94$ s, $Sa(T_1, 5\%)$. IM_1 is recommended to be selected corresponding to or below 10% probability of collapse on the initial fragility curve. Probability of collapse of 4% on the initial fragility function corresponds to $Sa(T_1, 5\%)$ of 1.05 g. This is the ground motion intensity having a probability of exceedance of 2% in 50 years for the site where the structure is located. Similarly, IM_2 is recommended to be selected such that it corresponds to probability of collapse between 30% and 80% on the initial fragility curve as an increasing function of Δ. Probability of collapse of 40% on the initial fragility function corresponds to $Sa(T_1, 5\%)$ of 1.96 g. This is the ground motion intensity having probability of exceedance of 1% in 200 years for this site. We selected 30 ground motions at each IM level matching the conditional spectra.

Modeling uncertainty is propagated using Latin Hypercube sampling. The parameters that are iden-

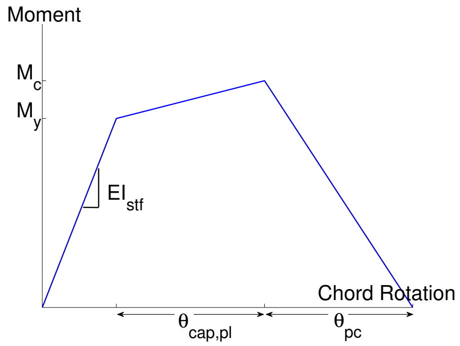


Figure 3: Monotonic behavior of a concentrated plasticity hinge model

tified as random variables are six parameters defining the monotonic backbone and hysteretic rules of a structural component. These parameters are flexural strength (M_y), ratio of maximum moment and yield moment capacity (M_c/M_y), effective initial stiffness which is defined by the secant stiffness to 40% of yield force ($EI_{stf,40}/EI_g$), plastic rotation capacity ($\theta_{cap,pl}$), post-capping rotation capacity (θ_{pc}) and energy dissipation capacity for cyclic stiffness and strength deterioration (γ). The monotonic backbone curve as a function of these parameters is shown in Figure 3. The variability in the modeling parameters is represented using logarithmic standard deviations of 0.73, 0.59, 0.5, 0.31, 0.27 and 0.1 for θ_{pc}, $\theta_{cap,pl}$, γ, M_y, $EI_{stf,40}/EI_g$ and M_c/M_y, respectively. These values are computed using the beam-column element calibration database and the predictive equations developed by Haselton et al. (2008). In total, two different components are assumed to exist in the structural model, namely, beam and column components. Equivalent viscous damping ratio, column footing rotational stiffness and joint shear strength are also treated as random with logarithmic standard deviations of 0.6, 0.3 and 0.1 (Haselton and Deierlein, 2007), respectively.

Correlation structure among the random variables are adopted from Ugurhan et al. (2014). In this correlation model, beam-to-beam and column-to-column correlations are idealized as perfect correlation, whereas beam-to-column and within component correlations are idealized by correlation coefficients that are derived using random effects regression. In total 15 random variables are used in this study and 30 realizations of these variables are obtained using Latin Hypercube sampling.

3.4. Results

Ground motions in each suite are scaled to the corresponding IM level and are matched with model realizations in order to conduct nonlinear time history analyses. The number of collapses observed are 6 and 13 out of 30 analyses at $IM_1 = 1.05$ g and $IM_2 = 1.96$ g, respectively.

The initial collapse fragility function is defined to have a median collapse capacity of 2.19 g and dispersion of 0.43. Median collapse capacity is estimated within ±40% with 90% confidence.

At IM_1, prior information leads to a distribution of $Beta(\alpha_1 = 1.75, \beta_1 = 17.42)$. Using the Bayesian approach along with the observed data of 6 collapses out of 30 analyses, this distribution is updated to $Beta(\hat{\alpha}_1 = 7.75, \hat{\beta}_1 = 41.42)$.

Similarly, at IM_2, prior information leads to a distribution of $Beta(\alpha_2 = 2.55, \beta_2 = 3.34)$. Using Bayesian approach along with the observed data of 13 collapses out of 30 analyses, this distribution is updated to $Beta(\hat{\alpha}_2 = 15.55, \hat{\beta}_2 = 20.34)$.

Using the maximum likelihood estimation method, where the posterior distributions at the two IM levels are used as probability distributions defining the likelihood function, the final collapse fragility curve is obtained to have a median collapse capacity of 2.22 g and dispersion of 0.7.

We see that Bayesian method increases the dispersion of the initial fragility function by 64%. It also increases the median collapse capacity of the initial fragility function by 1.5%.

3.5. Discussions

To benchmark the collapse response of the case study structure, we conduct an extensive collapse response analysis of the structure incorporating ground motion and modeling uncertainties. We select 200 ground motions consistent with conditional spectra at each IM level. We use five IM levels correspond to probabilities of exceedance of 5% in 50 years, 2% in 50 years, 1% in 50 years, 1% in 100 years and 1% in 200 years. 200

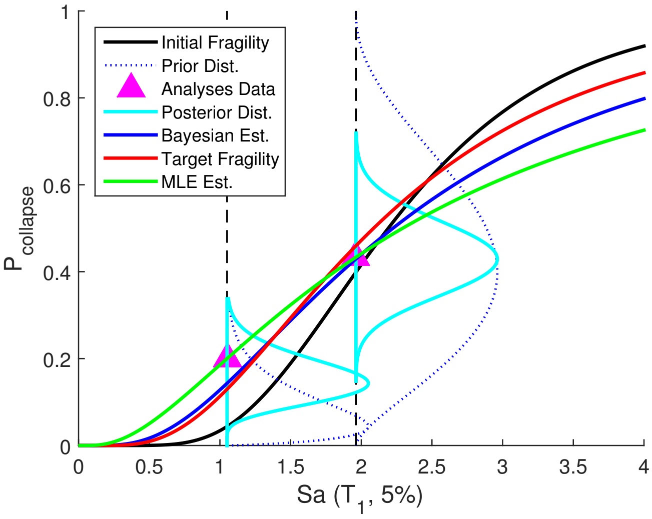


Figure 4: Collapse fragility functions obtained using different methods

model realization are obtained using Latin Hypercube sampling. Each ground motion is matched with a model realization, and in total 1000 nonlinear time history analyses are conducted. Using a maximum likelihood estimator, we obtain the collapse fragility curve of the structure having a median of 2.09 g and a dispersion of 0.61. Figure 4 shows the collapse fragility functions obtained using different approaches. The red line shows the result from 1000 analyses, and is indicated as the target fragility function. The fragility curve obtained using the Bayesian approach is shown in blue, whereas the estimate obtained by applying maximum likelihood estimation on observed structural analyses data only (MLE) is in green. Table 1 summarizes the results obtained using the aforementioned approaches in terms of different collapse metrics. Listed in Table 1 are the collapse fragility parameters θ and β along with λ_c, which is obtained using the site hazard curve, and the probability of collapse in 50 years, which is obtained using a Poisson assumption. Collapse risk estimates obtained using the median model parameters for the case study structure are also given in Table 1.

Table 1 shows that the Bayesian method shifts the median collapse capacity away from the target value. Although the initial estimate overestimates the median collapse capacity by 5%, updating the initial curve increases this difference to 6.6%. We

Table 1: Collapse risk estimates

Estimator	θ (g)	β	$\lambda_c(10^{-5})$	$P_{col_{50years}}$
Initial	2.19	0.43	10.61	0.0053
Bayesian	2.22	0.7	31.76	0.0158
MLE	2.29	0.93	71.99	0.0354
Target	2.09	0.61	25.37	0.0126
Median	2.17	0.49	13.9	0.0069

also see that dispersion of the target fragility function is overestimated by 15.6% by the Bayesian approach.

Although median collapse capacities and dispersions of the target and the Bayesian estimate differ, Figure 4 shows that the lower portion of the fragility function, up until 25%, is well-constrained by the Bayesian method. Table 1 shows that the Bayesian method provides a good estimate of λ_c. This is because of the good match in the lower portion of the fragility function, since the lower tail of the fragility function is an important contributor to λ_c.

The Bayesian method starts with an initial estimate of λ_c which underestimates the target λ_c by 58%. After applying by the method, the final estimate of λ_c differs from the target response by 25%. This difference is 184% using an MLE approach. It is also observed that by neglecting model uncertainty and using median model parameters, λ_c is underestimated by 45%.

4. CONCLUSIONS

This study presents a Bayesian method for collapse risk assessment of structures. The method uses prior information on collapse response of structures and augments this information with data from a small number of structural analyses. The approach enables propagating ground motion variability and model uncertainty through efficient sampling of model realizations. An illustration of the method is provided through a collapse risk assessment of a 4-story frame structure. The Bayesian method is observed to significantly improve statistical efficiency of collapse risk predictions compared to alternative methods, and provide considerable reduction in computational demand for probabilistic collapse risk assessment of structures.

5. ACKNOWLEDGEMENTS

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