

# A FORM-based analysis of lifeline networks using a multivariate seismic intensity model

M. Miller, J. Baker

*Stanford University, Stanford, CA, USA*

H.-W. Lim, J. Song

*University of Illinois Urbana-Champaign, Urbana, IL, USA*

N. Jayaram

*Risk Management Solutions Inc., CA, USA*

**ABSTRACT:** Governmental organizations, private firms, and others in seismically active regions are interested in how reliable lifeline networks will be in the event of an earthquake. Assessing risk in these networks is more complicated than assessing risk for individual sites using traditional Probabilistic Seismic Hazard Analysis (PSHA) because of interdependencies among different system links, as well as correlations among ground motion intensities across a region. The focus of this paper is three-fold: (1) to construct a multivariate probability distribution of seismic intensities at many locations in an example network, by analyzing simulated earthquake data, (2) to develop a framework based on the First Order Reliability Method (FORM) to calculate network flow capacity after a disaster, and (3) to illustrate the importance of more accurately describing joint distributions of ground motion intensities when computing lifeline network reliability. This proposed approach provides probability distributions of network flow capacity given an earthquake, and also quantifies the importance of each lifeline component, which would allow system administrators to identify the most critical links in their lifeline systems and prioritize network upgrades. The example results indicate that neglecting the correlation of the ground motions can result in gross overestimation or underestimation of the probability of achieving a given system flow.

## 1 INTRODUCTION

Officials deciding how to mitigate risk to their lifeline networks, whether through maintenance, retrofitting, new construction, or other policies, face uncertainty in network demands as well as capacity. Assessing risk for a network is yet more complex than that of a single site, because of interactions among network components. For example, there is no simple closed-form equation that can be evaluated to find network performance, such as maximum flow capacity, even if the ground motion intensities at each location are known. A further complication is that the demands due to ground motions are correlated among a region, and similar bridge construction methods or codes can lead to a correlation among structural capacities too, as described by Lee & Kiremidjian (2007) among others.

A common choice for assessing network performance is Monte Carlo Simulation (MCS), such as used by Crowley & Bommer (2006), and Jayaram & Baker (2010). However, this method does not directly determine which components or uncertainties have the most impact on the overall failure probability, nor the most likely failure scenario.

This paper proposes the use of the First-Order Reliability Method (FORM) to determine probabili-

ties of exceeding certain levels of performance loss in a sample San Francisco Bay Area (USA) transportation network. FORM overcomes the limitations of MCS described above because it naturally provides a “design point,” i.e. most probable set of values of the random variables that causes a failure (Der Kiureghian 2005). However, the error in FORM increases with the nonlinearity of the problem and the number of random variables. Furthermore, FORM traditionally uses a closed-form limit state expression. The proposed approach overcomes some obstacles traditionally prohibiting researchers from using FORM for network reliability analyses.

First, the network is focused to 38 main links in the SF Bay Area. Second, careful mathematics enable the demand and capacity uncertainty to be captured by only two random variables per location, which is less than that required by some other analysis approaches. The selected random variables are a) the natural logarithm of the spectral intensity demand,  $\ln S_a$ , and b) a parameter that captures the combined effect of uncertainty in both the damage state and the flow capacity for a given demand,  $\varepsilon_T$ . These two variables enable the calculation of traffic flow across each link. Using only the random variables  $\ln S_a$  and  $\varepsilon_T$ , total maximum network flow capacity can be estimated. The formulation pre-

sented is also carefully selected to create a smooth limit state function, which is critical for the gradient-based algorithm used by FORM to find the design point. Third, the analysis captures the complex network performance by linking to an external subroutine that efficiently calculates the network flow at each iteration, while finding the design point. This subroutine takes the place of an explicit limit state function.

In addition, this paper presents a multivariate seismic intensity model, which is crucial because FORM traditionally requires that the random variables can be completely parameterized by a standard probability distribution. The seismic intensity in this paper is modeled as:

$$\ln(S_{a_{ij}}) = \ln(\bar{S}_{a_{ij}}) + \sigma_{ij}\varepsilon_{ij} + \tau_j\eta_j \quad (1)$$

where  $S_{a_{ij}}$  is the spectral acceleration at a period of 1s at site  $i$  during earthquake  $j$ ,  $\bar{S}_{a_{ij}}$  is the median predicted spectral acceleration that is a function of period, magnitude, distance, shear wave velocity, and other local conditions, and the other terms comprise residual terms as described by Jayaram & Baker (2009).

While researchers such as Jayaram & Baker (2008) have shown close matching of the residual terms to a normal distribution, the term  $\ln\bar{S}_{a_{ij}}$ , in contrast, is a nonlinear function of many variables, including magnitude and distance to the fault. There is no clear reason why the composite  $\ln S_a$  values will follow any particular distribution, however results below indicate that a normal distribution predicts the log spectral intensity values well. The following sections describe how a 38-dimensional probability model can then be created to describe the joint distribution of ground shaking intensity at each component location. This result, combined with the previously described methods, allows for a risk assessment of this sample network under a probabilistic scenario of future earthquakes.

## 2 DESCRIPTION OF MULTIVARIATE SEISMIC INTENSITY MODEL

### 2.1 Simulation of ground motion fields

Simulations of  $S_a(1s)$  at all locations of interest were obtained using the following MCS procedure (Jayaram & Baker 2010). First, simulations of earthquakes of varying magnitudes on the active faults in the region are produced using appropriate magnitude–recurrence relationships (the Gutenberg–Richter relationship and the characteristic earthquake model). The Boore & Atkinson (2008) ground-motion model is then used to obtain the ground-motion intensity medians and variances at the sites of interest for each of the simulated earthquakes. Finally, realizations of ground motion inten-

sities are obtained by combining the median intensities with values of inter-event residuals and spatially-correlated intra-event residuals from MCS. The spatial correlation is computed using the model proposed by Jayaram & Baker (2009). The result is a set of tens of thousands of simulated ground motion fields that aim to capture all uncertainty in earthquake occurrence, size and resulting ground motion intensity. Because high intensities are generally more interesting than low intensities, the simulations were produced using importance sampling, so each realization has a weight that reflects the difference between the target and sampling distributions. The simulations are not directly usable for FORM analysis but can be used to calibrate the needed multivariate probability distribution, as described in the following section.

### 2.2 Creation of multivariate seismic intensity model

The model creation is broken into three parts: 1) selecting a candidate distribution, 2) fitting the distribution using the method of moments, and 3) testing the distribution with other candidate distributions in order to pick a final model.

#### 2.2.1 Distribution selection and fitting

Candidate distributions for  $S_a$  at a given site include the Gumbel extreme value distribution, the exponential distribution, and the lognormal distribution. The sample mean and variance are calculated from 14,750 Monte Carlo simulations, for the purposes of estimating distribution parameters. This calculation requires incorporating the importance sampling weight assigned to each simulation. For example, the mean is estimated as follows:

$$\mu_{\ln S_a} = \left( \sum_{i=1}^{14,750} w_i \cdot \ln S_{a_i} \right) / \left( \sum_{i=1}^{14,750} w_i \right) \quad (2)$$

where  $\mu_{\ln S_a}$  = mean of the natural logarithm of spectral acceleration intensity;  $w_i$  = importance sampling weight of the simulation  $i$ ; and  $\ln S_{a_i}$  = natural logarithm of spectral acceleration intensity of simulation  $i$ .

The results indicate that the exponential distribution is a poor fit to both the empirical CDF and PDF curves. In contrast, the Gumbel distribution and the normal distribution visually match the empirical CDF and PDF plots.

The Kolmogorov-Smirnov test (KS-test) is considered to provide an appropriate metric to determine which of two candidate distribution CDF curves had the least difference from the empirical CDF (Boes et al. 1974). The results indicate some sudden jumps in the empirical CDF at low  $\ln S_a$  due to some large importance sampling weights for low intensity high frequency events. After smoothing to reduce the impact of this artifact, the KS Statistic for one dimen-

sion suggests the normal distribution is the best candidate distribution for  $\ln S_a$  values.

### 2.2.2 Correlation coefficients

Correlation between  $\ln S_a$  values at each pair of bridge locations is computed using a sample correlation coefficient that accounts for the importance sampling weights. Since the marginal distributions appear to be normally distributed, we make the minor assumption that the  $\ln S_a$  values are multivariate normal, and thus correlation coefficients are sufficient to completely describe the joint distribution.

### 2.2.3 Results of distribution fitting

The fitted distributions are tested to examine how closely the chosen bivariate joint distributions fit the empirical results.

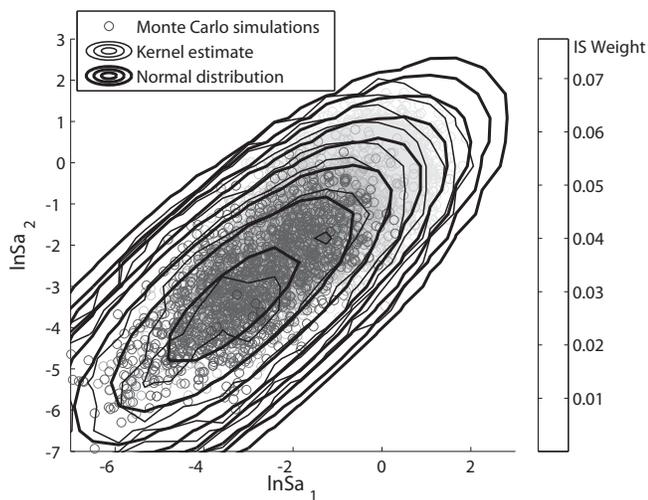


Figure 1. Simulated  $\ln S_{a1}$  vs.  $\ln S_{a2}$  data (in dots), with shading indicating importance sampling weights. The contours of the Kernel estimate PDF are shown in thin lines, and contours of the estimated bivariate normal distribution are shown in thick lines.

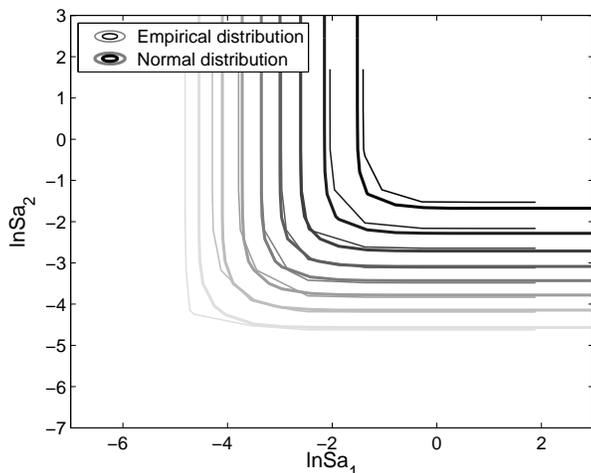


Figure 2. Cumulative distribution function contours of the distributions shown in Figure 1. Kernel estimated contours are in thin lines and Normal distribution contours are in thick lines.

Figure 1 shows  $\ln S_a$  values at two different locations, i.e.  $\ln S_{a1}$  vs.  $\ln S_{a2}$ , for individual earthquakes (one circle per simulation). The color gradient indicates the importance sampling weight of each realization, which is important to consider when visualizing the data. The contour lines further help visualize the empirical results. As Figure 1 shows, the candidate distribution relatively accurately defines the empirical results, including skew and correlation (tightness of the simulation results band). Figure 2 shows the close matching of the bivariate Normal CDF curves with the empirical ones.

## 3 NETWORK CHARACTERISTICS

### 3.1 Network description

This paper uses an aggregated transportation network of the San Francisco Bay Area (USA). The data is from Stergiou & Kiremidjian (2006) and includes network topology, bridge flow capacities, and the classification of each bridge into one of 28 HAZUS bridge types (based on abutment type, number of spans, etc.). This paper analyzes flow between San Jose and Oakland, CA, indicated by asterisks in Figure 3.

### 3.2 Network clustering

Although the total Metropolitan Transportation Commission network contains 30,000+ links and 10,000+ nodes, this study uses an aggregated network topology with 38 total road segments, or links, each with bidirectional flow. While FORM can incorporate more than 100 random variables in the authors' experience, which is more than some other analytical methods, all 30,000 links would be too many random variables for a viable FORM analysis.

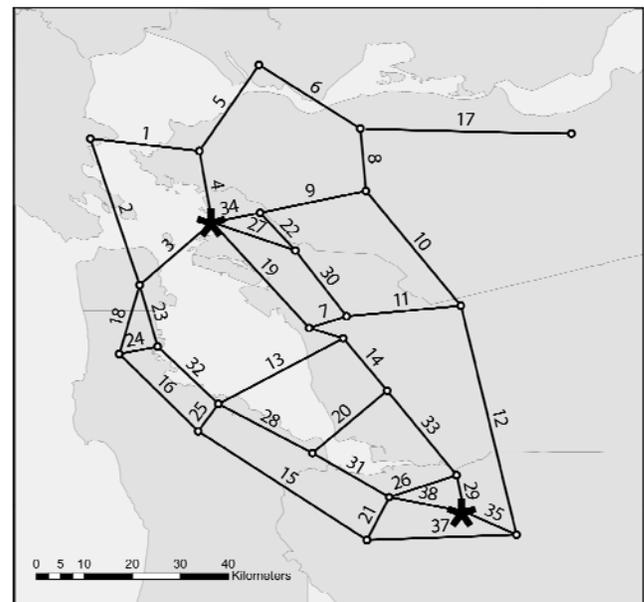


Figure 3. Aggregated SF Bay Area (USA) transportation network map used for the analysis.

The aggregated network topology, as shown in Figure 3, follows precedent (e.g., Wakabayashi and Kameda 1992), but includes extra links that capture the redundancy in the major North-South highways.

There are 1,036 bridges in the detailed network, and each of the 38 simplified links contains numerous bridges. This study uses one bridge per link to represent all the bridges on the link and thus reduce the complexity of the FORM analysis. The selection procedure is as follows:

1. Determine bridges on each link
2. Use the bridge classification and HAZUS data to find the bridge with the lowest mean capacity
3. In case of ties, choose the bridge with the highest mean demand from the model of ground motion intensities

This procedure defines the network used for the study. Improving the analysis granularity by increasing the number of links would lead to better estimation of significant links, however the results are not sensitive to randomly choosing a different bridge along a given link when using the same link topology.

#### 4 FORM-BASED ANALYSIS

Performance metrics for transportation networks can be divided into three broad categories: travel delay cost, maximum flow capacity and connectivity (Chang 2010). In particular, maximum flow capacity is defined as the largest feasible flow between a pair of nodes, i.e. the source and sink. It is a function of network topology and capacity of each link, as described by Ahuja et al. (1993). Maximum flow capacity is typically considered as the essential metric in evaluating the system serviceability when the damage state is specifically determined (Fenves & Law 1979). Thus, researchers, such as Lee et al. (2011), commonly use maximum flow capacity when evaluating the capacity of a transportation network in an evacuation scenario. Typically, maximum flow capacity is defined in the unit of vehicles per hour, i.e. veh/hr.

The main goal of the FORM-based analysis in this study is to compute the probability that maximum flow capacity between two cities in the San Francisco Bay Area is less than a certain threshold level. Thus, the problem formulation can be expressed as  $P(T_{a \leftrightarrow b} \leq t)$  where  $T_{a \leftrightarrow b}$  is the maximum flow capacity between cities a and b. For FORM analysis of a transportation network, the processes are divided into those at the component (link) and network levels. The random variables should capture uncertainties in both.

#### 4.1 Component Analysis: bridge traffic flow and random variables

Traffic flow capacity at bridges is influenced by many factors including ground motion intensities, bridge capacities and the relationship between flow reduction and damage states. These values have inherent uncertainties that impact traffic flow capacity in bridges and consequently, the overall network performance. Thus, the flow capacity at each bridge should be formulated with random variables that capture the uncertainties. These random variables and their link to calculating traffic flow capacity will now be explained.

In this paper, the uncertainties in the seismic demand and capacity of bridges are considered at each bridge location. The uncertain spectral acceleration intensity,  $S_a$ , can be used as one class of random variables in FORM.

Next, the random variable for uncertainty in the seismic capacity of the structure should be determined. A first idea might be to assign a random variable to each damage state for the bridges. However this formulation poses two hurdles for analysis using FORM. First, with a random variable for each damage state for each bridge, the calculation error in FORM will increase as the number of random variables increases because FORM is a method of linear approximation. Second, FORM typically implements a gradient-based constraint optimization such as Hasofer-Lind Rackwitz-Fiessler (HL-RF) algorithm. However, gradient-based optimization fails if the failure surface exhibits characteristics of step functions due to discrete damage states. Thus, this analysis develops a method to minimize the number of random variables and make the gradient-based FORM feasible while still capturing the uncertainty in the bridge capacity.

The fragility of bridges is defined as the conditional probability of being in or exceeding a particular damage state,  $ds_i$ , given the spectral acceleration  $S_a$ :

$$P(\text{exceeds } ds_i | S_a = s_a) = \Phi \left[ \frac{\ln s_a - \ln \bar{S}_{a, ds_i}}{\beta_{ds_i}} \right] \quad (3)$$

where,  $\bar{S}_{a, ds_i}$  is the median value of spectral acceleration at which the bridge reaches the threshold of the damage state,  $ds_i$ ; and  $\beta_{ds_i}$  is the logarithmic standard deviation of the spectral acceleration of the damage state.

HAZUS tabulates the reduced traffic flow capacity in bridges according to damage states and elapsed days. This analysis uses the results with a 3 day-long restoration period, i.e. functional flow capacities of 100%, 100%, 60%, 5% and 2% for the damage states none ( $ds_1$ ), slight ( $ds_2$ ), moderate ( $ds_3$ ), extensive ( $ds_4$ ) and complete ( $ds_5$ ) respectively (HAZUS 2008).

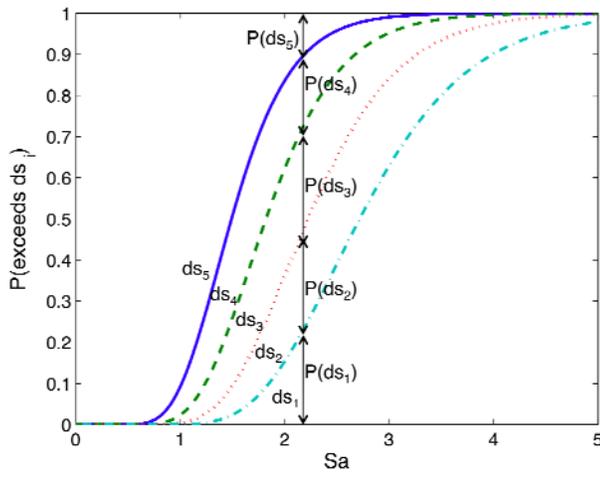


Figure 4. A representative bridge fragility curve and the corresponding definitions of the damage states.

The probability of each damage state is computed by use of the fragilities as shown in Figure 4. Given the five damage state and flow capacity definitions by HAZUS, conditional mean flow capacity  $\mu_{T|S_a}$ , and conditional variance  $\sigma_{T|S_a}^2$  given  $S_a = s_a$  is obtained as

$$\mu_{T|S_a}(s_a) = \sum_{i=1}^{n_{ds}} T_i \cdot P(ds_i | S_a = s_a) \quad (4)$$

$$\sigma_{T|S_a}^2(s_a) = \sum_{i=1}^{n_{ds}} T_i^2 \cdot P(ds_i | S_a = s_a) - [\mu_{T|S_a}(s_a)]^2 \quad (5)$$

where  $T_i$  is the flow capacity in the damage state  $ds_i$ . The traffic capacity  $T$  can be expressed as a function of two random variables  $S_a$  and  $\varepsilon_T \sim N(0, \sigma_T)$ , assuming the normality of  $T$  given  $S_a = s_a$ , as detailed in Eq. (6).

$$T = \mu_{T|S_a}(S_a) + \frac{\sigma_{T|S_a}(S_a)}{\sigma_T} \cdot \varepsilon_T \quad (6)$$

where  $\sigma_T$  is the maximum value of the conditional standard deviation function  $\sigma_{T|S_a}(s_a)$ . Note that the mean and variance of  $T$  given  $S_a = s_a$  in Eq. (6) match the conditional mean and variance obtained by Eq. (4) and Eq. (5). For the particular example in this study, the conditional variance function in Eq. (5) was fitted by a simple function of the conditional mean as follows for a convenient implementation:

$$\sigma_{T|S_a}^2(s_a) \cong \alpha \cdot \mu_{T|S_a}(s_a) \cdot [1 - \mu_{T|S_a}(s_a)] \quad (7)$$

where  $\alpha$  is the scaling constant identified during the fitting process.

It is important to note that the bridge traffic capacity  $T$  is capped between 0 and 100%, which is the domain that is physically feasible. This capping does not affect the FORM process given a reasonable starting point, since FORM iterates towards the design point that is within the physically feasible domain. However, MCS is affected by the extreme re-

gions, in which the high variance in this example would cause values below 0 and above 100% without capping.

The approach outlined above describes the uncertainty in the traffic capacity propagated from the uncertainty in the seismic capacity (or damage state) by use of one random variable  $\varepsilon_T \sim N(0, \sigma_T)$  for each bridge. Note that this paper does not consider the uncertainty in the functional traffic flow capacity for a known damage state and instead uses the deterministic values detailed in HAZUS (2002).

In summary,  $S_a$  and  $\varepsilon_T$  capture the traffic flow capacity  $T$  at each bridge in the network given the previously described modeling assumptions.

## 4.2 Network Analysis: FORM as implemented in FERUM

FORM analysis in this study is run through the Finite Element Reliability Using Matlab (FERUM) program within the MATLAB® shell (Der Kiureghian et al. 2006). Instead of explicitly defining the evaluation function, FERUM is modified to call an external algorithm to calculate the maximum flow capacity for a given set of values of the random variables. Specifically, a MATLAB® version of Boost Graph Library is used (Gleich 2008).

The modified FERUM calculates not only the loss exceedance probabilities for the network as a whole, but also other measures to support decision-making such as importance measures for each random variable and sensitivity values for distribution and limit state parameters.

## 5 RESULTS AND DISCUSSION

This section presents computed probabilities of network flow given a seismic event, and illustrates the importance of characterizing correlations in ground shaking intensities at multiple locations across a network.

### 5.1 Lifeline risk assessment results

The analysis results, as displayed in Figure 5, show that in almost all cases the network will be at or very near full capacity, which is 7,600 veh/hr in this example. This is because the model includes a full distribution of future earthquakes, and most earthquakes cause very low intensity  $S_a$  values.

The number of FORM iterations before convergence varies depending on the starting point, but for the starting point used in this analysis, the average number of iterations is around 5. The total time averages about 6 seconds using a personal computer with a 2.2 GHz processor and 2GB SDRAM.

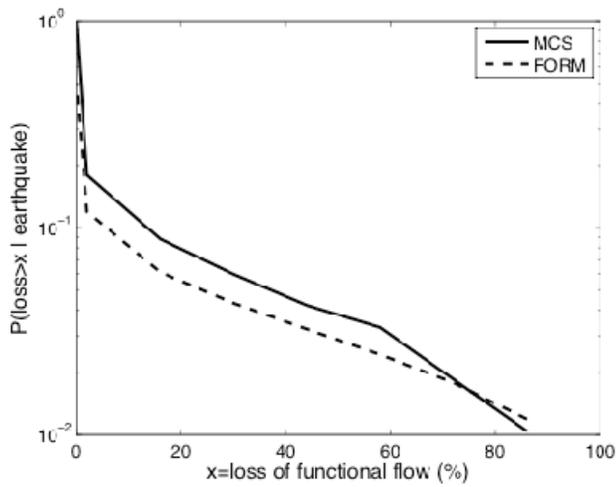


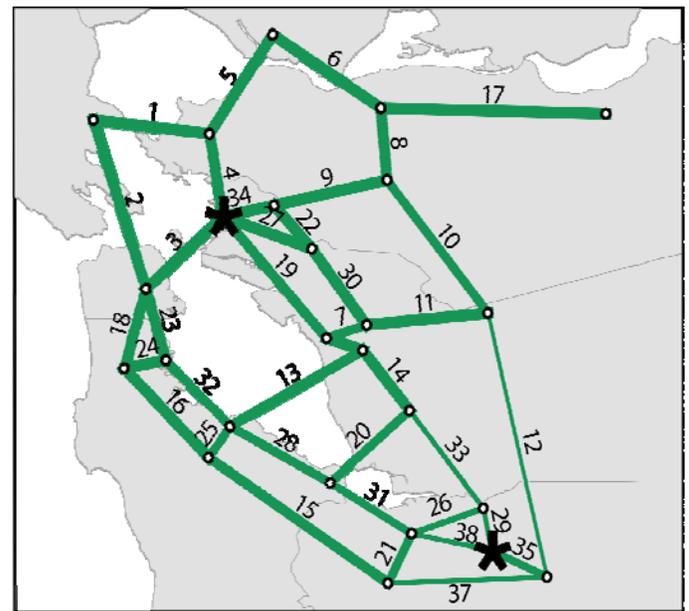
Figure 5.  $P(\text{loss} > x \mid \text{earthquake})$  where  $x$  is the loss of maximum network flow between the start and end nodes.

Verification of the analysis results with MCS is necessary for multiple reasons, including concerns that FORM might find a local minimum instead of the true design point. Figure 5 shows the similarity of the MCS and FORM results. One potential source of discrepancy is the problem formulation, where flow capacity is capped between 0% and 100%. However, due to high variance, it is possible for a value to be above 100%. This case is not reached in these FORM iterations since FORM narrows in on the design point and does not explore the extreme domains. MCS, by contrast, likely does sample from extremes where this capping occurs. The formulation is a subject of continuing study, and it is hoped that the issue will be eliminated or confirmed to not affect computed results. The second reason, which likely has the largest impact, is that FORM implies the linearization of the limit state surface at the design point. The analysis shows that the surface has significant nonlinearity. The impact of nonlinearity will be thoroughly investigated in the future research by using the Second Order Reliability Method (SORM).

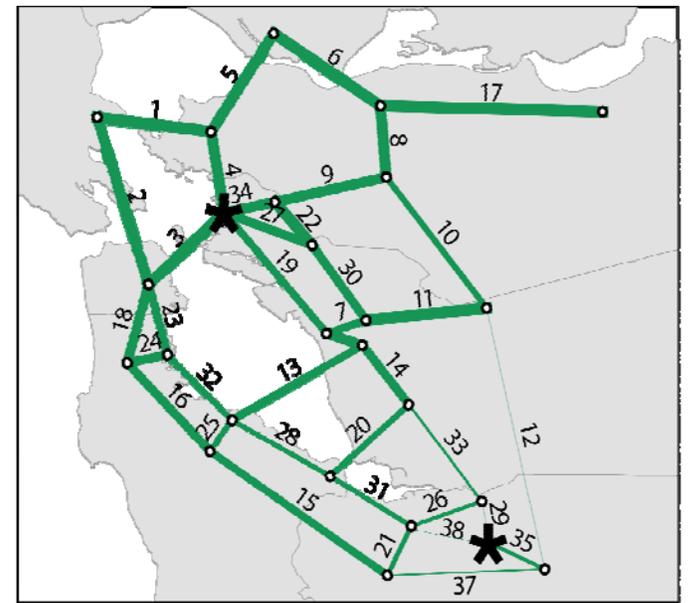
The MCS results took much longer than FORM to converge, particularly for rare events. For example, the MCS results for a 60% loss in flow ( $t=3,000$  veh/hr) were obtained in about 120 seconds.

### 5.2 Network flow at different design points

FORM, in contrast to MCS, naturally provides the design point, i.e. the most likely failure case, in addition to the probability of being in the failure domain. Figure 6 shows link flows associated with design points for two levels of network disruption, and indicate that the design points correspond to reduced link flows near the end node.



(a)



(b)

Figure 6. Most probable flows at the design point are indicated by the thickness of the line for each link for a) 74% loss of maximum network flow, i.e.  $t=2,000$  veh/hr, and b) 21% loss of maximum network flow, i.e.  $t=6,000$  veh/hr.

As will be shown in the next section, the links with the reduced flow capacity are similar to the ones with the highest importance factors. Furthermore, as might be expected, the design point for lower loss of network flow shows fewer links experiencing reductions in flow capacity.

### 5.3 Impact of correlations

To determine the impact of a more accurate description of the high-dimensional probability distributions, the analysis is repeated for the case where marginal distributions of  $\ln S_a$  at each location are unmodified, but correlations between all pairs of  $\ln S_a$  are assumed to be zero. Figure 7 shows the dramatic difference in the computed network per-

formance if correlation in ground motion intensities is neglected. Disregarding correlation is not a “conservative” approximation, which might be preferred by officials determining if a network meets a certain maximum level of acceptable risk. The rate of exceeding a given flow disruption, particularly for large disruptions, diverges dramatically.

The difference is not unexpected since the ground motion intensities contain both “primary” correlation from the common-source effects in  $\ln \bar{S}_a$  and “secondary” correlation from the residual terms. As Park et al. (2007) found for residual correlations only, ignoring total correlation leads to overestimating the probability of exceedance at low loss levels and underestimating the probability at higher ones.

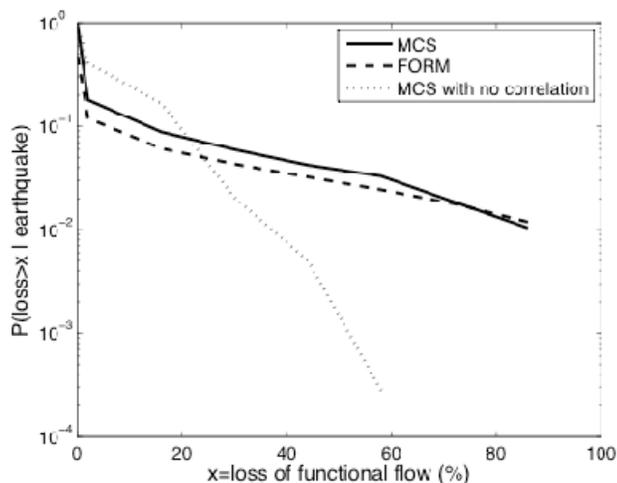


Figure 7.  $P(\text{loss} > x \mid \text{earthquake})$  where  $x$  is the relative to the case with more network flow loss of maximum network flow between the start and end nodes.

#### 5.4 Sensitivity and importance factors

In addition to the most probable flows, FORM, in contrast to MCS, enables calculating sensitivity and importance factors. For example, the gamma importance factor vector (see Der Kiureghian 2005) quantifies relative contribution of each random variable to the variability of the limit-state function with statistical dependence between random variables fully considered. In this example, the gamma vector helps highlight the bridges that most significantly impact the probability of being in the failure domain. As shown in Figure 8, random variables associated with four links near to the end node have relative high importance factors. At  $t=2,000$  veh/hr, for example, these seismic demands at 4 links have importance factors between 0.32 and 0.66 while the next highest value is negligible. The factors for  $\varepsilon_T$  are between  $-0.32$  and  $-0.15$  for the four links discussed above and near 0 for other links. By comparing importance factor results for various node pairs, a similar analysis could highlight bridges most significant for flow capacity levels in this SF Bay Area network regardless of origin and destination.

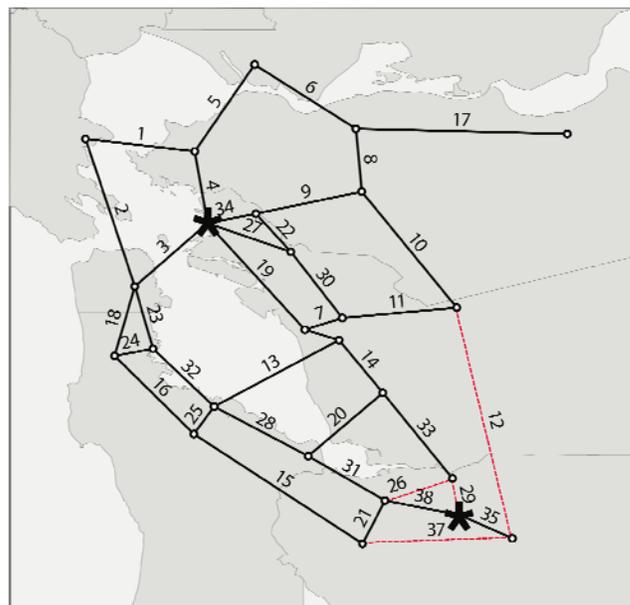


Figure 8. The dotted links correspond to those with highest importance vector values, at 74% loss of maximum network flow.

## 6 CONCLUDING REMARKS

### 6.1 Future work

This study focuses exclusively on bridge damage due to ground shaking, and does not consider reduction in flow due to other damage, such as landslides or displacements across fault crossings. This is partially to reduce the numerical complexity for FORM analysis, and partially because the results are intended to illustrate the potential of this approach rather than provide an exact answer. The aggregated network also eliminates many damage mechanisms, and obscures the effect of drivers using local roads to detour around damaged stretches of highway. Further analysis could incorporate a multi-scale approach to more accurately capture the effects of highway failures and rerouting onto local roads. The model could also be further improved by parameterizing the effect of correlation of the structural capacity of the bridges, and research is currently in progress on this topic. Finally, further research will explore flow behavior under scenarios that include the simultaneous loading from many origin and destination pairs.

### 6.2 Summary

The seismic intensity model developed in this paper results from an analysis of thousands of simulated earthquakes in the San Francisco Bay Area generated by MCS as described by Jayaram & Baker (2010). The Monte Carlo data are particularly well suited for this study because of the relative dearth of recordings of high-intensity earthquakes as well as the complexity of analytically characterizing ground

motions across a whole region. These simulations are used to study joint distributions of spectral accelerations at pairs of locations in the region. Analytic probability distributions are fit to the simulations, which allow for use of the data in closed-form reliability methods.

The First Order Reliability Method transforms points to normal space to iterate to the most likely set of values of the random variables that would cause failure, which in this paper is loss of functional flow exceeding a minimum percentage loss. The probability of failure is computed from this “design point”. Since FORM requires a linearization of the limit state, error tends to increase with increasing random variables, which is why this paper uses 38 links to represent the transportation network. Another limitation is that the demand, capacity, and performance metric must all be expressed analytically. This paper presents a methodology to express each in closed form, given various modeling assumptions including lognormal distributions and the  $\epsilon_T \sim N(0, \sigma_T)$  flow formulation. While MCS, in contrast, is less restrictive in the form of modeling parameters, it does not provide importance factors and design points as efficiently as FORM does.

This paper provides an example calculation of maximum traffic flow capacity between select pairs of cities in the San Francisco Bay Area to estimate the effect of correlated hazards on the seismic reliability of an infrastructure network and to demonstrate the benefits and limitations of FORM for network reliability analyses. A maximum flow capacity algorithm is integrated into the analysis to predict probability distributions of traffic flow capacity given an earthquake. The design point in FORM allows for identifying important uncertainties and network components. This information helps support the decision of officials determining if and where to improve their lifeline networks.

## 7 ACKNOWLEDGEMENTS

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