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UNCERTAINTY ESTIMATION IN SEISMIC COLLAPSE ASSESSMENT OF MODERN REINFORCED CONCRETE MOMENT FRAME BUILDINGS

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ABSTRACT

Robust characterization of seismic performance requires the quantification and propagation of various sources of uncertainties throughout the analysis and assessment. This paper presents recent research to characterize and evaluate the effect of structural modeling uncertainties, along with ground motion and hazard uncertainties, on the collapse safety of buildings. The focus of the study is on modern reinforced concrete frame buildings, but the proposed methods and findings are generally applicable to other structural materials and systems. Uncertainties and correlations in structural component model parameters are evaluated by applying random effects regression models to a database of tests on reinforced concrete beam-columns. These analyses show that model parameters within structural components tend to be uncorrelated, whereas the data suggest that there are strong correlations between like parameters of different components within a building. The influence of modeling uncertainties on the collapse behavior of the systems is evaluated using several alternative methods, including Monte Carlo simulations, First-Order Second-Moment, response surface and artificial neural network methods. Collapse risk estimates are used to illustrate the relative merits of each method, depending on the size and other characteristics of the problem. Simulation-based methods are explored in terms of the computational effort involved in uncertainty propagation in the presence of high dimensional random variables for collapse safety assessments. The results emphasize the sensitivity of collapse response to modeling uncertainties and the challenges of balancing of computational efficiency and robust uncertainty characterization.

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Robust characterization of seismic performance requires the quantification and propagation of various sources of uncertainties throughout the analysis and assessment. This paper presents recent research to characterize and evaluate the effect of structural modeling uncertainties, along with ground motion and hazard uncertainties, on the collapse safety of buildings. The focus of the study is on modern reinforced concrete frame buildings, but the proposed methods and findings are generally applicable to other structural materials and systems. Uncertainties and correlations in structural component model parameters are evaluated by applying random effects regression models to a database of tests on reinforced concrete beam-columns. These analyses show that model parameters within structural components tend to be uncorrelated, whereas the data suggest that there are strong correlations between like parameters of different components within a building. The influence of modeling uncertainties on the collapse behavior of the systems is evaluated using several alternative methods, including Monte Carlo simulations, First-Order Second-Moment, response surface and artificial neural network methods. Collapse risk estimates are used to illustrate the relative merits of each method, depending on the size and other characteristics of the problem. Simulation-based methods are explored in terms of the computational effort involved in uncertainty propagation in the presence of high dimensional random variables for collapse safety assessments. The results emphasize the sensitivity of collapse response to modeling uncertainties and the challenges of balancing of computational efficiency and robust uncertainty characterization.

Introduction

There are various sources of uncertainties that should be considered in seismic performance assessment. One important source of uncertainty is ground motion intensities and it is addressed by a site-specific hazard curve. Hazard curves provide a probabilistic representation of ground motion intensities and this enables to propagate the effects of uncertainty in ground motions on the response of a structure through the performance-based earthquake engineering framework. Another important source of uncertainty is structural modeling and analysis. Structural response simulations are affected by the choice of structural idealizations. Furthermore, analysis model parameters that define structural idealizations are subject to uncertainty. Modeling uncertainty becomes more pronounced for collapse response simulations than for elastic or mildly nonlinear

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simulations, due to the relatively limited knowledge of model parameters and behavior associated with collapse. Collapse response simulations require idealizations that are capable of simulating nonlinear deformations demands and various sources of degradation both at component and structure levels, and phenomenological concentrated plasticity models are better candidates for modeling collapse behavior [1]. However, model parameters that define concentrated hinge/spring models are generally calibrated by empirical relationships that relate model parameters to physical engineering parameters. This constitutes a major source of uncertainty to collapse response simulations.

Previous research on the effects of modeling parameters on collapse response predictions highlights the importance of analysis model parameters defining component ductility capacity, post-yield characteristics and hysteretic energy dissipation capacity on collapse capacity predictions ([2], [3]). Furthermore, propagation of uncertainties related to modeling has been studied by several researchers ([2], [3], [4], [5], [6]). Although researchers utilize different methods to characterize the uncertainty, the results in general emphasize the importance of modeling uncertainties in collapse response predictions. Because of the computational demand involved in uncertainty propagation methods and difficulties in modeling, judgment based factors to adjust the uncertainty in collapse fragility functions are proposed in ATC-58 [7] and FEMA-P695 [8].

In this study, we evaluate the effects of modeling uncertainty on collapse response assessment of a modern reinforced concrete frame building. Multiple reliability methods are explored for use in propagating uncertainty. The focus of the study is to evaluate the accuracy of these methods in characterizing the overall uncertainty, and their computational demand. The challenges in balancing computational efficiency and robust uncertainty characterization for collapse response predictions are discussed.

Overview of Case Study

The techniques for capturing modeling uncertainty are demonstrated through a case study of a 4-story reinforced concrete moment frame building that has been used in previous studies of collapse [3]. It is designed according to the seismic provisions of [9], and evaluated by nonlinear analysis using OpenSees. Frame elements are modeled as elastic members having nonlinear rotational springs at their ends with a concentrated plasticity hinge model. P-Delta effects are taken into account by using a leaning column carrying gravity loads. The first-mode period of the structure is 0.94 s. Details of the building design and analysis model can be found in [3].

Uncertainty in Collapse Response Predictions

Ground Motion Variability

Selection and scaling of ground motions to be used in dynamic response history analysis is important since structural response depends highly on the ground motion characteristics. Several researchers have developed methods to address this issue ([10], [11]). Using generic ground

motion sets for response history analysis provides an alternative when there is no specific site of interest. In this study, we use the generic far-field ground motion set of FEMA-P695 [8], consisting of 22 record pairs from extreme events.

Structural Analysis Techniques and Structural Idealizations

Incremental dynamic analysis (IDA) is a popular nonlinear dynamic analysis procedure for collapse response assessments. It involves the scaling of ground motions to a range of ground motion intensity levels until the structure displays dynamic instability. The same ground motions are used over the full-range of considered intensities. The counted fractions of ground motions causing collapse at each intensity level are used to calibrate a collapse fragility curves relating probability of collapse to a predefined ground motion intensity measure (IM). In this study, the IM considered is spectral acceleration at the first mode period of the structure, $S_a(T_1)$.

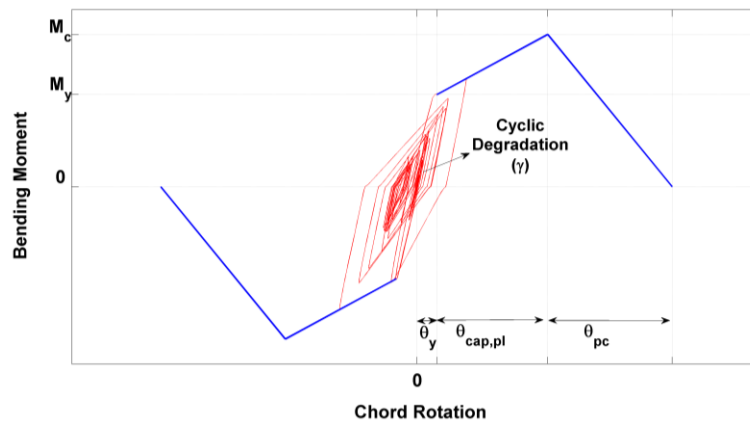


Figure 1. Backbone curve for concentrated plasticity hinge model

Component structural idealizations differ in the way they model plasticity along the member and through the cross-section. Phenomenological models are well suited for collapse response simulations since they can capture highly nonlinear degradation effects that are difficult to reproduce with more fundamental fiber or continuum models. In this study, the Ibarra-Medina-Krawinkler concentrated plasticity model [12] is used to model component level response. The plastic hinge model is defined by a trilinear backbone curve shown in Fig. 1. The six parameters defining the backbone curve and hysteretic behavior of a component are treated as random variables. These parameters are flexural strength (M_y), ratio of maximum moment and yield moment capacity (M_c/M_y), effective initial stiffness which is defined by the secant stiffness to 40% of yield force (EI_{stf40}/EI_g), plastic rotation capacity ($\theta_{cap,pl}$), post-capping rotation capacity (θ_{pc}) and energy dissipation capacity for cyclic stiffness and strength deterioration (γ). The variability in the modeling parameters is represented by the following: logarithmic standard deviations for θ_{pc} , γ , $\theta_{cap,pl}$ and M_c/M_y are equal to 0.86, 0.64, 0.63 and 0.13, respectively from [13], and logarithmic standard deviations for EI_{stf40}/EI_g and M_y are equal to 0.43 and 0.3 as calculated in this study. Equivalent viscous damping and elastic footing rotational stiffness are also treated as random having logarithmic standard deviations of 0.6 and 0.3 [3], respectively.

Correlations of Model Parameters

Uncertainty propagation methods require the characterization of correlations among the analysis model parameters, which are treated as random variables. Assessment of correlations of model parameters involves the quantification of correlations both within a single structural component and between different structural components of the frame. A one-way random effects model [14] is used to quantify the two types of correlations. The model is applied to the component calibration database of Haselton et al. [13], which consists of 255 tests of rectangular RC columns from 42 different test groups (i.e., series of tests run at independent laboratories). Test groups are represented by the random effects in the model. It is assumed that tests done within a test group represent different components in a structure and the correlations among different tests within a group can be used to make inference about the correlations between components in a structure. Based on this assumption, the random effects model enables the quantification of correlations between components in a structure. Details of the statistical analyses conducted for quantification of correlation coefficients are omitted for brevity, but the resulting correlation coefficients are summarized in Table 1. Within a component, it is observed that correlations between model parameters are rather small, with the largest being 0.4 corresponding to the correlation between M_c/M_y and M_y . Between components, like parameters are observed to have large correlation coefficients in the range of 0.6 to 0.9. Equivalent viscous damping and elastic footing rotational stiffness are assumed to be uncorrelated from the model parameters that define component backbone curve and hysteretic behavior.

Table 1. Correlations of analysis model parameters within and between components

		COMPONENT 1						COMPONENT 2					
		$\theta_{cap,p1}$	θ_{pc1}	EI_{stf1}	M_{y1}	M_c/M_{y1}	γ_1	$\theta_{cap,p2}$	θ_{pc2}	EI_{stf2}	M_{y2}	M_c/M_{y2}	γ_2
COMPONENT 1	$\theta_{cap,p1}$	1	0.3	0	0.1	0.3	0.1	0.6	0.3	0	0.1	0.2	0
	θ_{pc1}		1	0.1	0.1	0.1	0.3		0.8	0.1	0.1	0.1	0.3
	EI_{stf1}			1	0.1	0	0			0.9	0.1	0.1	0
	M_{y1}				1	0.4	0.1				0.9	0.4	0.1
	M_c/M_{y1}		(sym.)			1	0.2		(sym.)			0.8	0.1
	γ_1						1						0.6
COMPONENT 2	$\theta_{cap,p2}$							1	0.3	0	0.1	0.3	0.1
	θ_{pc2}								1	0.1	0.1	0.1	0.3
	EI_{stf2}									1	0.1	0	0
	M_{y2}										1	0.4	0.1
	M_c/M_{y2}											1	0.2
	γ_2												1

Uncertainty Propagation in Collapse Response Assessment

Probabilistic collapse response modeling is challenging since failure modes are correlated with loading and resistance variables. This fact, combined with the high dimensionality of uncertain variables, makes the exact solution impossible in general. Structural reliability methods, ranging

from approximate methods such as First-Order Second-Moment (FOSM) to Monte Carlo simulation (MCS) methods, enable uncertainty propagation for collapse response assessment.

Sensitivity analysis can be used to infer the relative importance of individual random variables, and this information can be further used for reducing the dimensionality of the problem by omitting unimportant variables. Several researchers have conducted sensitivity analyses for quantifying model parameter uncertainty on collapse response assessment with component hinge models ([2], [3], [4]). Although individual perturbations of parameters provide insight in terms of significance in behavior, they are not able to capture interactions between model parameters. Fig. 2 shows an intermediate result from the sensitivity study on the case study structure. Median collapse capacity is plotted with respect to two representative random variables for illustrative purposes. Perturbations of individual parameters at $\pm\sqrt{3}$ logarithmic standard deviations are displayed together with joint perturbations of two parameters at ± 1 logarithmic standard deviations to capture interaction effects. A quadratic surface fitted through these points is also shown. Different collapse modes are obtained for different values of random variables. This emphasizes the nonlinear relationship between random variable values and collapse capacities, and points to the need for realistic characterization of correlations between random variables.

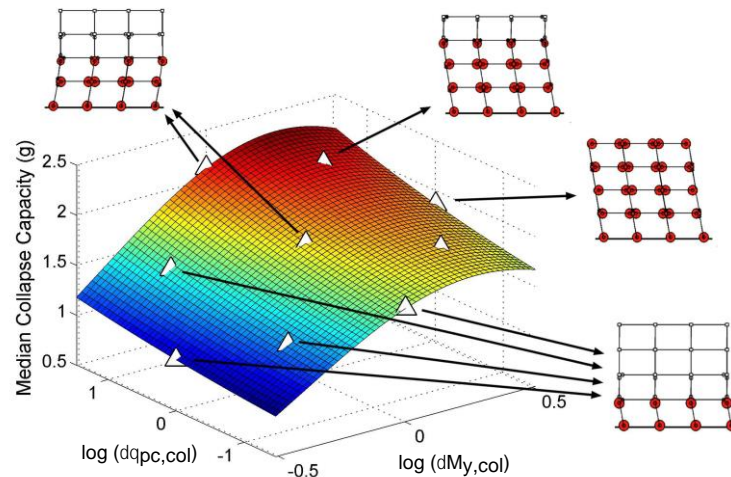


Figure 2. Nonlinear relationship between median collapse capacity and perturbations of strength and post-capping deformation capacity of columns. Collapse modes corresponding to different realizations of random variables are displayed.

The current practice for structural response assessment is to conduct structural analysis with median (or expected) values of model parameters. Several researchers have shown that collapse capacity estimates obtained using this approach underestimate true collapse risk ([3], [4]). In this study, collapse risk of the case study structure is also estimated using median values of model parameters. The capability of using median values for model parameters is investigated in terms of handling the variability in the properties and the response characteristics of structural components.

For uncertainty propagation, simulation-based methods are easy to implement and converge to the exact solution as the sample size increases. In this study, MCS are conducted to provide a benchmark collapse capacity estimate for the case study structure. 3608 random realizations of the model parameters are obtained using the joint probability distribution of the random variables. The parameters defining component hinges are assumed to be perfectly correlated within beam components as well as within column components (meaning that six parameters define the variations for all beams and six define for all columns). When combined with the damping and footing stiffness variables, 14 total random variables are used in this study. Correlation coefficients given in Table 1 are used to define the correlation within the parameters defining a component and correlation between beam and column component parameters.

FOSM is an approximate uncertainty propagation method, and an alternative to MCS, which employs a linear limit state function to establish the relationship between random variables and collapse capacity [15]. Response Surface (RS) methods offer an advancement of FOSM by employing quadratic functions to model the same relationship, which enables capturing nonlinearity of the problem to some degree ([4], [16]). Orthogonal design (OD), in which each variable has two levels, is used for FOSM. RS is also calibrated with OD to make a fair comparison among the methods requiring similar computational demand. For FOSM and RP, each model realization obtained using OD is analyzed with all ground motions in the suite.

In this study, we present an artificial neural network (ANN) based collapse response assessment. ANNs are commonly used for function approximations. The most common ANNs are multi-layer feed forward ANNs. In this study, multi-layer feed forward ANNs are trained with back propagation [17]. The architecture of the network consists of five hidden layers and two output layers in addition to the input layers. The output layers consist of the ground motion intensities corresponding to probabilities of collapse of 10% and 25%. Two output layers are used since two points are capable of defining a lognormal fragility curve. Representing fragility curves by two points is motivated by the work of Eads et al. [18], who propose selecting two IMs that have significant contributions to the collapse risk. The selection of the two points in this study is somewhat arbitrary and our future work will concentrate on refining this selection. However, based on the finding of [18] that the lower tail of the collapse fragility curve governs collapse risk, we use collapse probabilities that are less than 50%. For calibrating ANN, 18 model realizations obtained using Latin hypercube sampling (LHS) are used in training. For testing and validation, six model realizations that are obtained using randomly sampling.

The lognormal collapse fragility functions obtained using the aforementioned methods are provided in Fig. 3.a. It is observed that the fragility curve obtained using median model parameters lies far from the fragility functions that incorporate model uncertainty and underestimates the collapse potential of the structure. Among the uncertainty propagation methods considered, close estimation of the MCS curve is provided by ANN predictions. FOSM and RS in general have considerable discrepancy from MCS. Estimation by RS is comparable to MCS at the important lower tail of the curve. It gives good estimations of collapse probability until 10%.

Collapse fragility curves are integrated with hazard curves for sites in Los Angeles, CA and Memphis, TN. Seismic hazard curves for these sites are given in Fig. 3.b. These two sites are selected because of the different hazard characteristics. The site at Los Angeles (LA) is located at a high seismicity region and the hazard curve is steeper compared to the hazard curve of Memphis. Mean annual frequencies of collapse (λ_c) at these sites are obtained using Eq. 1.

$$\lambda_c = \int_0^{\infty} P(C|im) \left| \frac{d\lambda_{IM}(im)}{d(im)} \right| d(im) \quad (1)$$

where $\left| \frac{d\lambda_{IM}(im)}{d(im)} \right|$ is the slope of the hazard curve and $P(C|im)$ is the probability of collapse at a given im . The plot showing the product of $P(C|im)$ and slope of the hazard curve with respect to IM is called a collapse risk deaggregation curve [18]. These curves provide insight on the ground motion intensities that contribute most to the collapse risk at the site of interest. Collapse risk deaggregation curves obtained using different fragility curves are provided in Figs. 3.c and 3.d for the sites at LA and Memphis, respectively.

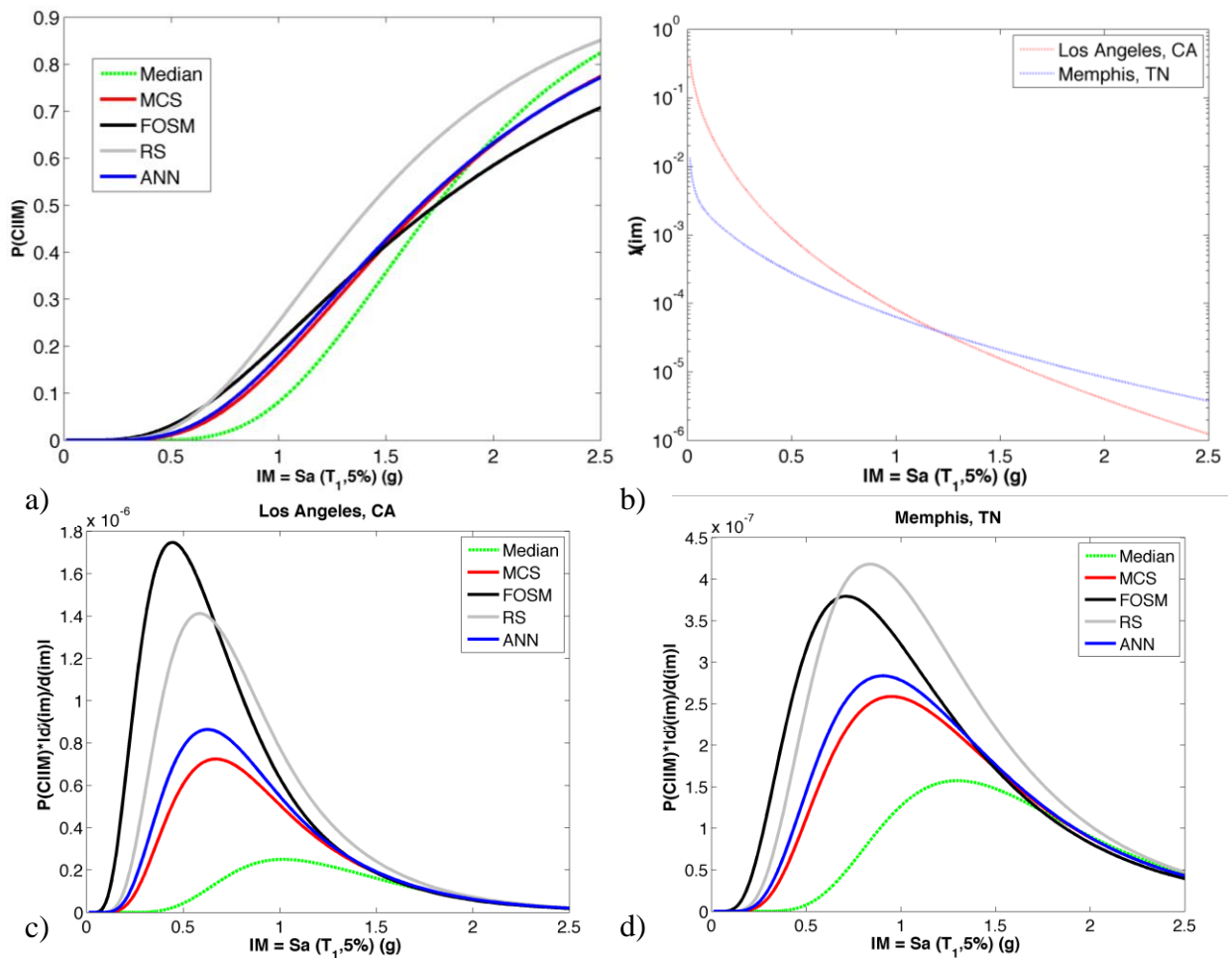


Figure 3. a) Collapse fragility curves obtained using different uncertainty propagation methods. b) Seismic hazard curves. Collapse risk deaggregation curves for c) Los Angeles, CA and d) Memphis, TN

The site at LA has a steeper hazard curve and thus the collapse probabilities at each IM are multiplied with larger slopes. This results in higher collapse risk deaggregation values and higher λ_c in comparison to the site at Memphis. It is also observed that the mode of the MCS based curve occurs at 0.65 g and 0.95 g for LA and Memphis sites, respectively. We see that for LA site, close estimations of the MCS based collapse deaggregation curve are provided by ANN, followed by RS. As mentioned before, these two methods yield good estimations at the lower tail of the collapse fragility curve. The largest contribution to the collapse risk at Memphis occurs at around 0.95 g. At this intensity level, collapse fragility curve estimations are similar for ANN and MCS. Therefore, they yield comparable estimations for collapse risk at this site.

Table 2 summarizes the number of structural models used and estimations for median collapse capacity, dispersion (σ_{ln}) and λ_c at LA and Memphis sites using the uncertainty propagation methods considered in this study. Collapse fragility function is best estimated by ANN. Although ANN provides the best estimate for the collapse fragility curve, it overestimates collapse risk by 15% and 9% for LA and Memphis, respectively. It is noted that RS calibrated with central composite design provides an advancement to OD, however, for the given dimensionality of uncertain variables, it requires as many structural analyses as MCS.

Table 2. Collapse risk estimations obtained using different uncertainty propagation methods

	Median Model	MCS	FOSM	RS	ANN
# of IDA	44	3608	1276	1276	1056
Median Sa(T ₁) (g)	1.73	1.68	1.73	1.43	1.66
σ_{ln}	0.39	0.53	0.67	0.54	0.55
$\lambda_{c,LA}$ ($\times 10^{-5}$)	2.80	6.70	13.10	11.60	7.70
$\lambda_{c,Memphis}$ ($\times 10^{-5}$)	2.20	3.40	4.60	5.10	3.70

Simulation-based methods enable practical treatment of uncertainties due to record-to-record variability and modeling uncertainty for collapse safety assessment. One significant advantage of these methods is that their computational demand associated does not increase significantly as the number of random variables increases. In this study, we next explore the computational effort involved in uncertainty propagation using simulation-based methods in the presence of 170 random variables. Component hinge model parameters for 16 columns and 12 beams for the 4-story reinforced concrete moment frame building are now treated as random and non-perfectly-correlated, in addition to the equivalent viscous damping and elastic footing rotational stiffness. Table 1 is used to define the correlation of the parameters defining the 28 components. Table 3 shows the collapse risk estimates obtained using MCS with 170 random variables. Table 2's estimates of collapse risk were obtained with the assumption of perfect correlation among beam and column hinges. By treating beam and column hinges as random variables, in which correlation among components are defined using partial correlation coefficients, the dispersion of collapse fragility curve decreases, whereas median collapse

capacity stays unchanged. Therefore, realistic estimates of correlations lead to smaller λ_c and thus smaller collapse risk.

Among simulation-based methods, LHS provides an efficient alternative to MCS. Given the joint probability distribution of uncertain variables, LHS ensures that the samples are distributed evenly in accordance with the given variability of the random variables. For more details about this method and its application to collapse capacity estimations readers are referred to [5] and [6]. It is noted that stochastic optimization using simulated annealing is applied to preserve the correlation structure among the random variables. For treating record-to-record variability and modeling uncertainty, every record in the ground motion suite is matched with a single simulated structural model realization that is obtained using LHS. To study the variability in the collapse response simulations, we repeat the matching of ground motion records and structural model realizations 100 times. The right half of Table 3 shows the means and coefficients of variation of the resulting 100 estimates of median collapse capacity, dispersion, $\lambda_{c,LA}$ and $\lambda_{c,Memphis}$. Also displayed are the results for matching the records in the ground motion suite with 2 and 4 model realizations, which correspond to in total 88 and 176 collapse response analyses, respectively. It is observed from Table 3 that with 44 incremental dynamic analyses, on average, one gets close estimations of collapse fragility curve. The results are observed to be stable with coefficients of variation for median collapse capacity and dispersion less than 10%. For λ_c , however, higher coefficients of variations are obtained. With increasing number of analyses, the variation in the results decreases.

Table 3. Collapse risk estimates using simulation-based methods with 170 random variables

# of IDA (using MCS)	Median Sa(T ₁) (g)	σ_{ln}	$\lambda_{c,LA}$ ($\times 10^{-5}$)	$\lambda_{c,Memphis}$ ($\times 10^{-5}$)				
3608	1.68	0.48	4.98E-5	2.96E-5				
# of IDA (using LHS)	Mean Estimate among 100 estimates for				Coefficient of variation among 100 estimates in			
	Median Sa(T ₁) (g)	σ_{ln}	$\lambda_{c,LA}$ ($\times 10^{-5}$)	$\lambda_{c,Memphis}$ ($\times 10^{-5}$)	Median Sa(T ₁) (g)	σ_{ln}	$\lambda_{c,LA}$	$\lambda_{c,Memphis}$
44	1.66	0.49	5.67E-5	3.16E-5	0.02	0.08	0.25	0.13
88	1.66	0.49	5.57E-5	3.14E-5	0.02	0.06	0.19	0.09
176	1.66	0.48	5.34E-5	3.09E-5	0.01	0.04	0.13	0.07

Conclusions

We have presented collapse assessment of a 4-story concrete frame structure incorporating uncertainties related to ground motion and structural modeling. Correlations of component model parameters are quantified and incorporated uncertainty propagation. MCS, LHS, FOSM, ANN and RS reliability methods are used to incorporate uncertainty in collapse predictions. The ANN approach provided good estimates of the collapse fragility curve for the case study structure. Although RS yielded a less accurate estimate of the overall fragility curve, it closely estimated the lower tail of the fragility curve, which is important for collapse risk predictions.

For uncertainty propagation studies, treatment of collapse analyses with a large number of random variables (~20 or more) is challenging and requires dimension reduction techniques in

order to use approximate methods such as FOSM, RS and ANN. In contrast, simulation-based methods (MCS, LHS) are robust and scalable to high dimensionality. While simulation-based methods have high computational demands due to repeating simulations, smart sampling techniques can be employed to reduce the computational demands while handling cases with high dimensionality. The example presented in this paper illustrates the use of LHS for collapse response assessment, where the computational demand is reduced significantly and a close estimation of the collapse fragility curve is obtained. In this case, estimates of median collapse capacity and dispersion are stable with coefficient of variation less than 0.1. Our future work will explore the dimensionality problem for uncertainty propagation in collapse response assessment, concentrating on dimension reduction techniques for approximate methods and importance sampling for simulation methods.

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