Stochastic model for earthquake ground motions using wavelet packets

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Abstract

For performance-based design, non-linear dynamic structural analysis for various types of input ground motions is required. Since the number of ground motion recordings is limited for such analysis, synthetic ground motions can be used as additional input. Here a stochastic ground motion model with time-frequency nonstationarity is developed using wavelet packets.

The wavelet packet transform can decompose time-series data into wavelet packets with constant resolution in the time and frequency axes, and it can also reconstruct a time-series data from wavelet packets. This allows time series to be modeled intuitively in the time-frequency domain using the wavelet packets.

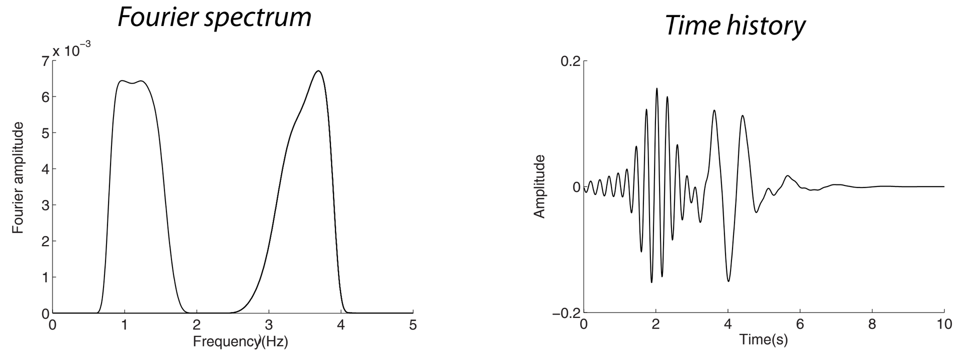
Ground motions siumulated using the proposed approach reasonably match ground motion recordings in several respects, including elastic and inelastic spectral acceleration, duration, bandwidth, cumulative energy, and time-frequency nonstationarity. In addition, the median and log standard deviation of spectral accelerations from the simulated ground motions match those of empirical ground motion prediction models. These results suggest that the synthetic ground motions generated by our model can be used as realistic inputs for non-linear dynamic structural analyses.

Wavelet packet transform

The wavelet packet transform is an extended version of the discrete wavelet transform. By controlling wavelet packets at varying scales and locations, the time-frequency characteristics of the time series can be determined.

The following figures show the relationship between a time series its Fourier spectrum, and its wavelet packets.

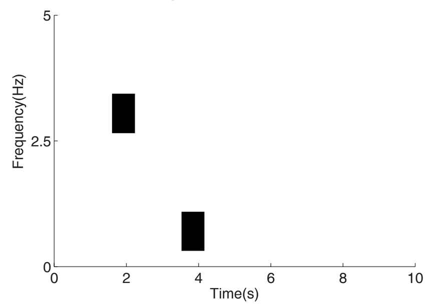
Fourier spectrum *Time history*



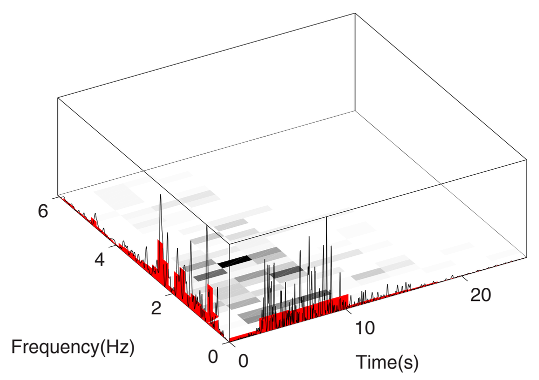
$$f(t)=\sum_{j=0}^{2^{p}-1}\sum_{k=1}^{2^{n-p}} W_j^p(j,k)\cdot\psi_{j,k}^p(t)$$

- $f(t)$: Time history
- $W_j^p(j,k)$: Wavelet packet coefficients
- $\psi_{j,k}^p(t)$: Scaling function
- j : Frequency order (frequency axis)
- k : Scale factor (time axis)
- p : Depth (band width of each level j)
- 2^n : Total number of data

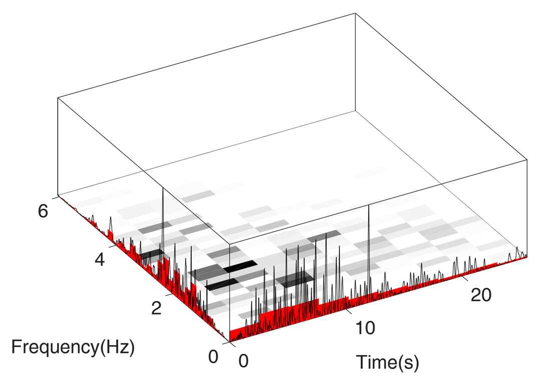
Wavelet packets distribution



Wavelet packets of ground motion recordings from the Northridge Earthquake



Example near field recording
Northridge - 17645 Saticoy St
Rhyp=18km, Vs30=281m/s



Example far field recording
Santa Barbara - UCSB Goleta
Rhyp=123km, Vs30=339m/s

Stochastic model for ground motions

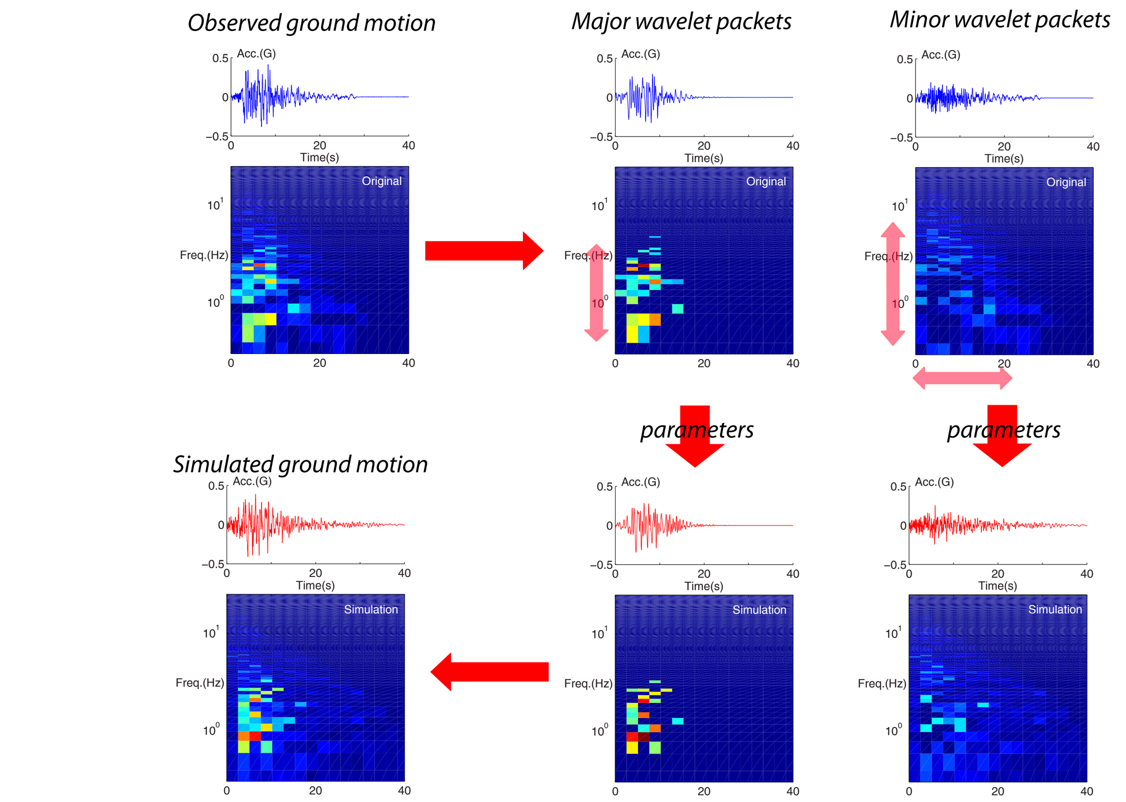
In our model, time series are defined by the combination of two types of wavelet packet groups: the major group (which contains the largest coefficients representing 70% of the total ground motion energy) and the minor group (which contains the rest). To determine the amplitudes of the wavelet packets, the following 12 parameters are required for three independent probability density functions.

for both wavelet packet groups, we define the following parameters
- E(t) - mean time
- S(t) - standard deviation of time (related to duration)
- E(f) - mean frequency (related to dominant frequency)
- S(f) - standard deviation of frequency (related to bandwidth)
- R(t,f) - correlation between time and frequency (related to frequency evolution in time)

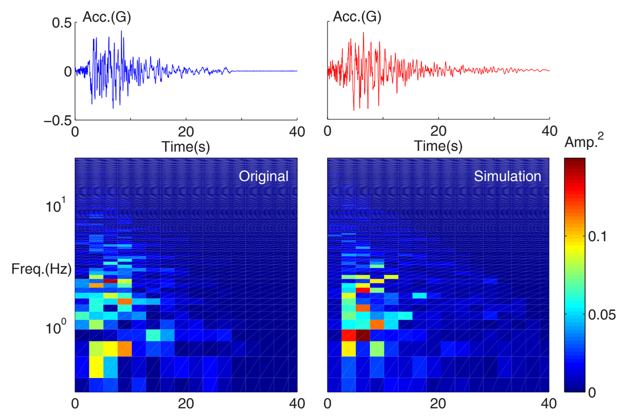
for the major wavelet packets group, we define the following
- E(a) - mean amplitude

Finally, we specify the total energy in the time series
- Energy - total energy

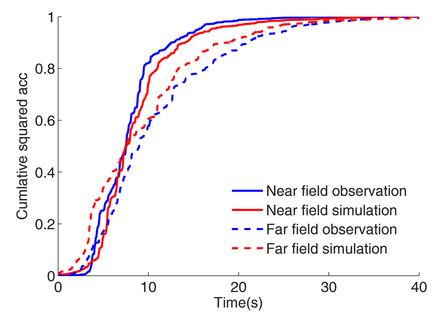
How to simulate ground motions



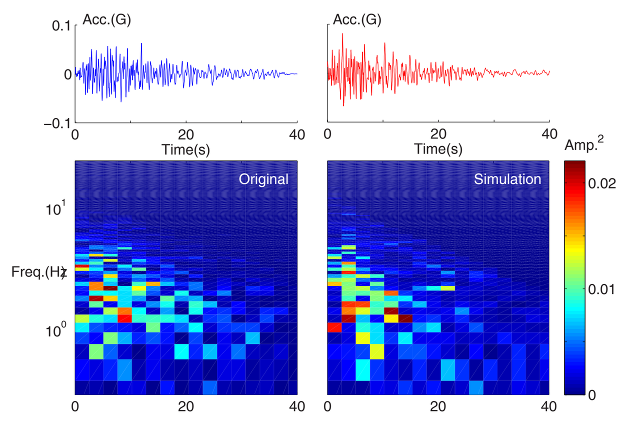
Simulated ground motions



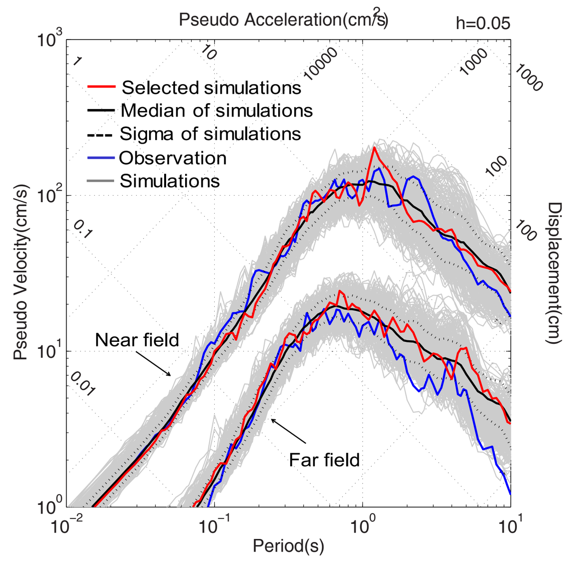
Near field station
Northridge - 17645 Saticoy St [Rhyp=18km, Vs30=281m/s]



Cumulative Squared Acceleration



Far field station
Santa Barbara - UCSB Goleta [Rhyp=123km, Vs30=339m/s]



Spectral Acceleration

Regression analysis to predict model parameters

The 12 parameters are predicted by the regression analysis calibrated using 1408 strong ground motion recordings from 25 earthquakes. The predictors of our model are moment magnitude (Mw), rupture distance (Rrup), hypocentral distance (Rhyp) , and Vs30. The following equation predicts the Total Energy parameter (Y). Other model parameters have similar equations.

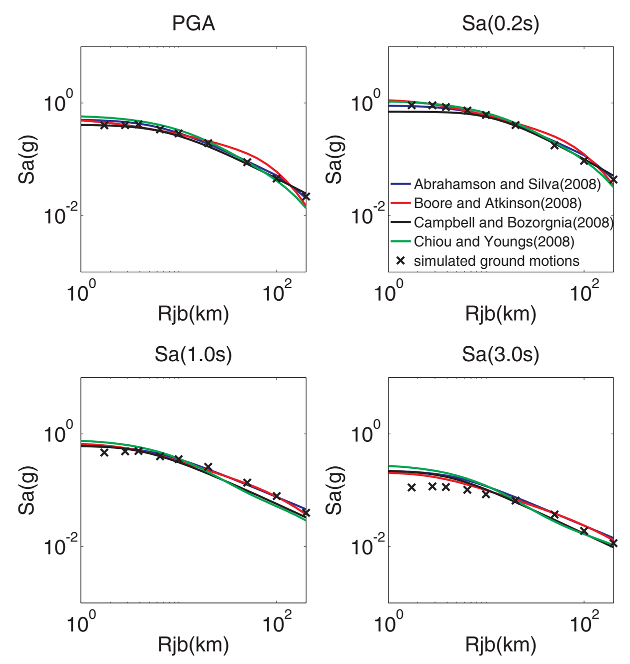
$$\ln(Y)=a+b_1M_W+b_2\ln(M_W)+c_2\ln(R)+d\ln(V_{S30})+\eta_i+\varepsilon_{ij}$$

$$R=\sqrt{R_{rup}^2+h^2}\qquad R_{dif}=R_{hyp}-R_{rup}$$

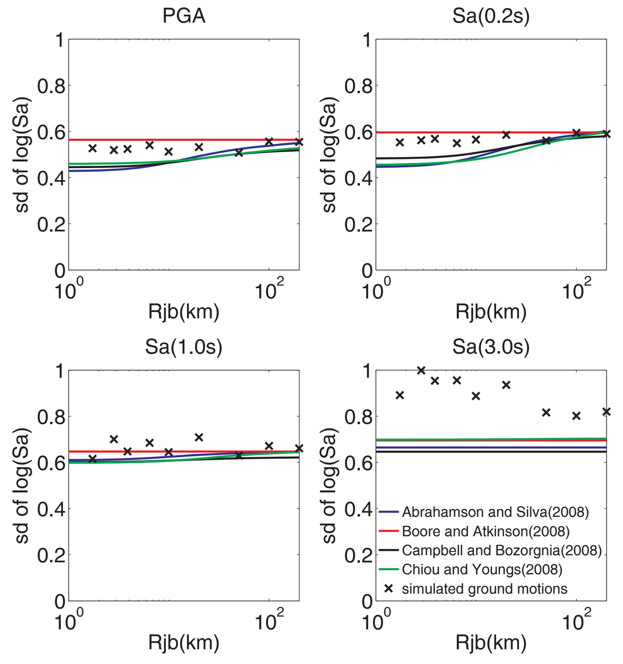
η is the interevent residual and ε is the intraevent residual, and other variables are determined by regression analysis. η and ε values for pairs of model parameters are correlated, and this correlation is tracked using a correlation matrix that is then used for simulations .

As seen below, characteristics of the simulated ground motions obtained using these regression models reasonably match the characteristics of recorded ground motions as predicted by empirical ground motion prediction ("attenuation") models.

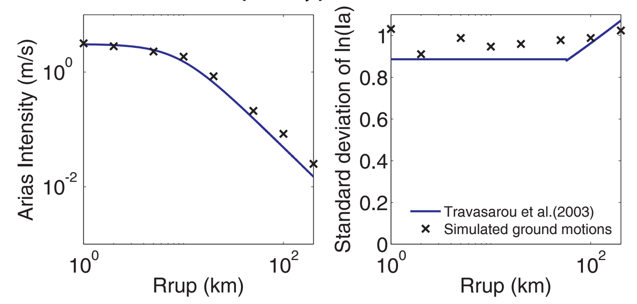
Spectral Acceleration (Sa) vs Rjb
Mw=7, VS30=270m/s



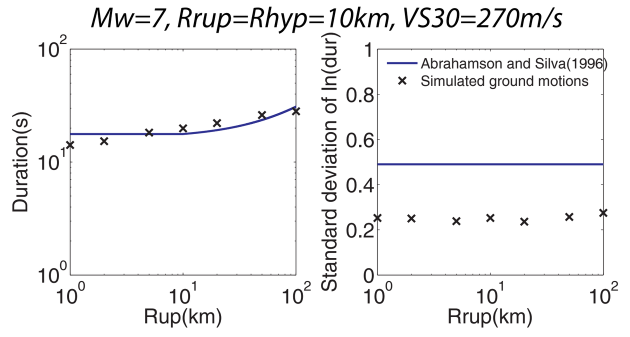
Standard deviation of log(Sa) vs Rjb
Mw=7, VS30=270m/s



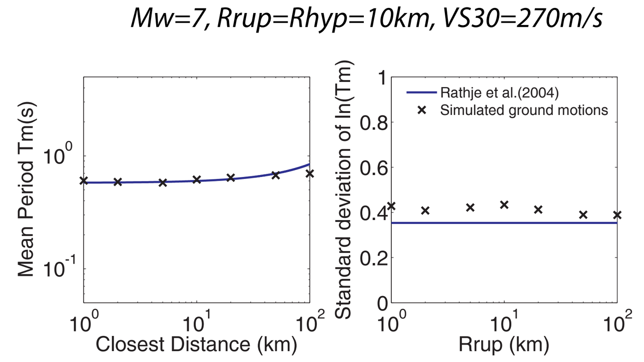
Arias Intensity vs Rrup
Mw=7, Rrup=Rhyp=10km, VS30=270m/s



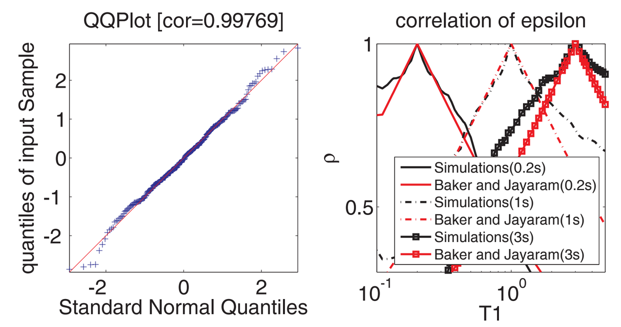
5-95% duration vs Rrup
Mw=7, Rrup=Rhyp=10km, VS30=270m/s



mean Period(Tm) vs Rrup
Mw=7, Rrup=Rhyp=10km, VS30=270m/s

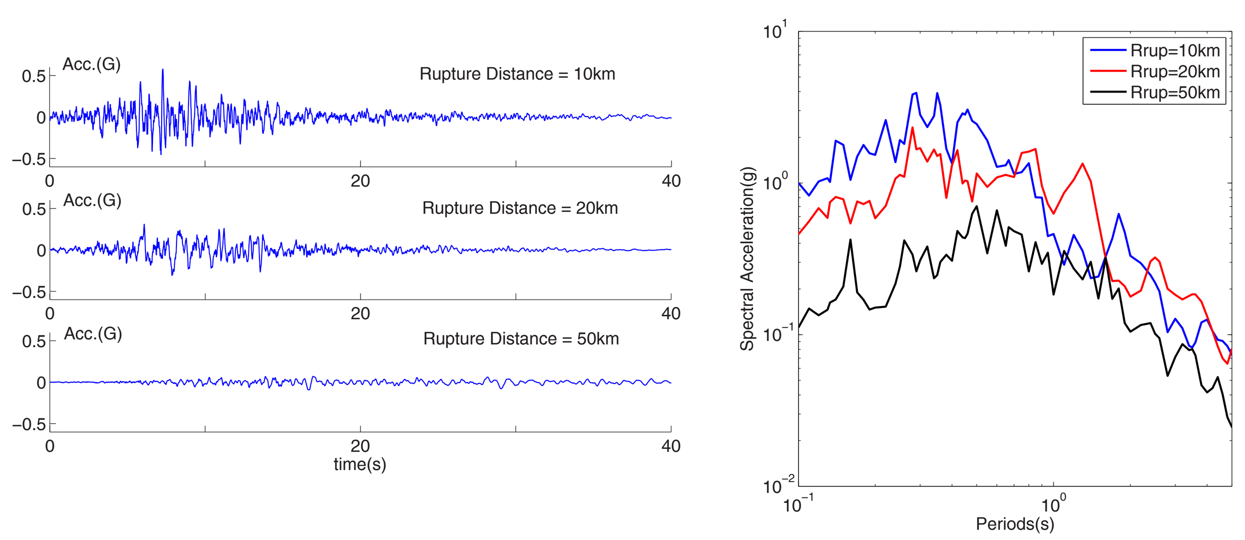


characteristics of epsilon
Mw=7, Rrup=Rhyp=10km, VS30=270m/s

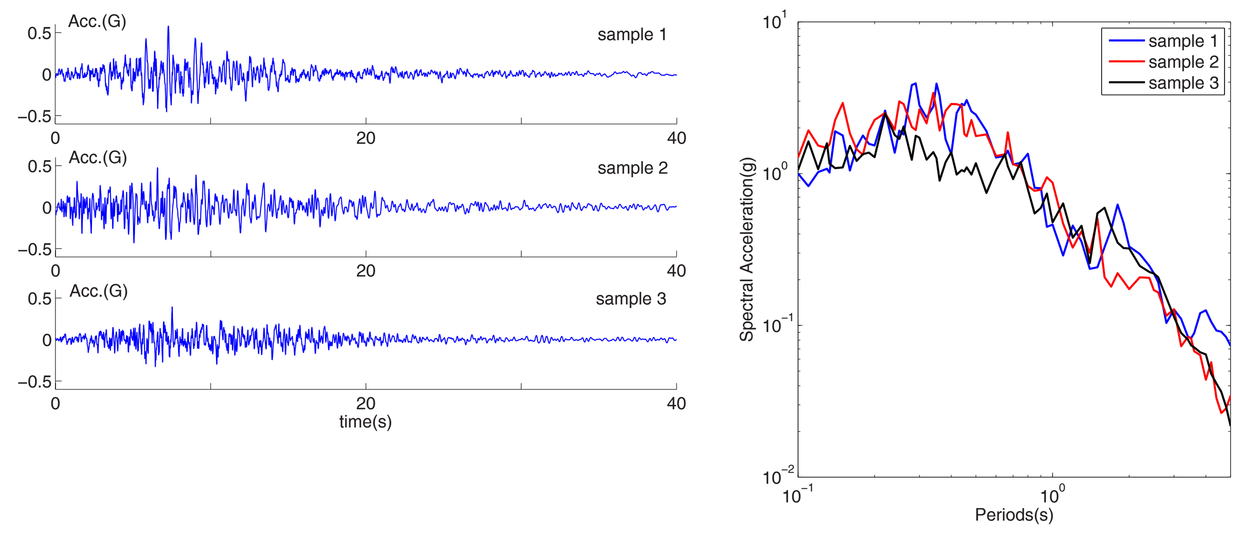


The standard deviation of log spectral accelerations in long period is a little larger than that of the attenuation models. This is because the constant time-frequency resolution of the wavelet packets makes it difficult to control the long-period energy of the simulations. An appealing solution would be to use this technique to produce high-frequency simulations, and combine them with physics-based low-frequency simulations.

Simulated ground motions
Mw=7, Vs30=270m/s



Simulated ground motions
Mw=7, Rrup=10km, Vs30=270m/s



Summary of results to date

- **A stochastic ground motion model with time-frequency nonstationarity has been developed. A wavelet packet based model is used to allow easy control of time-frequency nonstationarity**
- **The model is empirically calibrated and captures important characteristics of observed ground motion recordings. Simulations reasonably match the empirical ground motion prediction models for spectral accelerations, Arias Intensity, duration, mean period and epsilon.**
- **Simulations have parameters that vary with earthquake magnitude, distance and local site conditions. This differs from some stochastic simulations which reproduce properties of only a recorded seed ground motion.**
- **The regression and wavelet packet transform calculations are extremely computationally inexpensive, so simulating large numbers of ground motions for engineering use is easy. A desktop PC can produce 1000 simulations in one hour.**
- **The high freqency portion of these stochastic simulations could be combined with low frequency physics-based simulations to produce broadband simulated ground motions.**