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Correlation of ground motion intensity parameters used for predicting structural and geotechnical response

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ABSTRACT: By combining probabilistic descriptions of ground motion intensity with predictions of structural or geo-technical response as a function of that intensity, it is possible to compute the seismic reliability of engineering systems. This approach has been used for assessment of structural reliability considering seismically-induced collapse, as well as geotechnical reliability considering liquefaction failures. But reliability assessments that simultaneously consider both structural and geotechnical failures are currently not possible using this approach, because structural and geotechnical responses are generally predicted using different ground motion intensity parameters, and the tools are not yet available for determining a probabilistic characterization of the joint occurrence of these parameters. This paper develops models for the stochastic dependence between observed values of elastic response spectral values, peak ground acceleration, and Arias Intensity. By combining these correlation models with existing ground motion prediction equations, it is possible to characterize the joint distribution of the various ground motion intensity parameters needed to predict structural and geotechnical failures. The correlation coefficients of interest are calculated empirically from a large set of recorded and processed strong ground motions, and analytic predictive equations are fitted to the results. Once the correlation coefficients have been determined, a simple example calculation is performed to demonstrate the use of the result, and to illustrate the importance of considering this correlation when performing a seismic reliability analysis that considers both structural and geotechnical failures.

1 INTRODUCTION

By combining probabilistic descriptions of ground motion intensity with predictions of structural or geotechnical response as a function of that intensity, it is possible to compute the seismic reliability of engineering systems. This approach has been used for assessment of structural reliability (e.g., Bazzurro and Cornell 1994; Cornell et al. 2002) as well as geotechnical reliability considering liquefaction failures (Kramer et al. 2006). But reliability assessments that attempt to simultaneously consider both structural and geotechnical failures are currently not possible using this approach, because structural and geotechnical responses are generally predicted using different ground motion intensity parameters, and the tools are not available for determining a probabilistic characterization of the joint occurrence of these parameters.

This paper develops the correlation coefficient models necessary to achieve the goal of considering both structural and liquefaction failures simultaneously. Structural response (and structural failure) is often predicted using elastic spectral acceleration (*Sa*) at the first-mode period of the structure (e.g., Pinto et al. 2004). Liquefaction failure, on the other hand, is typically predicted using peak ground acceleration (*PGA*) (e.g., Cetin et al. 2004; Youd et al. 2001) or Arias Intensity (I_A) (e.g., Kayen and Mitchell 1997). Arias Intensity has also been seen to be a useful predictor of damage due to other failure mechanisms such as slope instability (Travasarou et al. 2003).

To evaluate system reliability considering both structural and geotechnical failures, it is necessary to obtain knowledge about the simultaneous occurrence of several of these ground motion parameters at once. This calculation can be performed using vector-valued probabilistic seismic hazard analysis (VPSHA) (Bazzurro and Cornell 2002). The calculation is a direct extension of traditional PSHA, using the same information about the magnitudes, locations, and recurrence rates of earthquakes and the same ground motion prediction (attenuation) models. The only additional requirement is knowledge of the joint distribution of the ground motion intensity values for a given magnitude and distance. Logarithmic *Sa,PGA* and I_A values have been observed to be well-represented by the normal distribution, so the mild assumption that pairs of values are well-represented by the joint normal distribution is probably a reasonable one (but has not been investigated as yet to the author's knowledge). Under this assumption, only correlation coefficients between ground motion intensity values are needed to define the joint distribution and proceed with VPSHA. This paper will provide predictive models for these correlation coefficients, furthering the development of the vector-valued probabilistic seismic hazard analysis. The models are developed by calculating empirical correlation coefficients from a large set of strong ground motions, and using the results to fit a predictive model.

Once a vector-valued PSHA has been performed, engineers can use this new information to perform more comprehensive reliability assessments. A simple example calculation is performed here to demonstrate the use of the results, and to illustrate the importance of considering this correlation when performing a seismic reliability analysis that considers both structural and geotechnical failures.

2 ANALYSIS PROCEDURE

The needed correlation coefficients are obtained by first selecting a large set of recorded ground motions. Ground motion intensity parameters are then computed for each ground motion, along with predicted values for these parameters provided by ground motion prediction models. Correlations among the prediction residuals of the ground motion intensity parameters are then computed, and simple analytic equations are fit to provide a simple means of calculating the required correlation coefficients. Details are given in the following sections.

2.1 *Record selection*

The PEER Next Generation Attenuation ground motion database (2005) was used to compute the empirical results shown in this study. From this database, all records meeting the following criteria were selected:

1. The recording site had an average shear wave velocity in the top 30 meters of between 180 and 1500 m/s (NEHRP classification class B-D), and Bray and Rodriguez-Marek SGS classification B-D (Rodríguez-Marek et al. 2001).
2. The recording was made in the free field or the first story of a structure.
3. Both horizontal components of the ground motion had high-pass filter corner frequencies less than 0.2 hertz and low-pass filter corner frequencies greater than 18 hertz.
4. The earthquake magnitude was greater than 5.5.
5. The closest distance from the earthquake rupture area to the site was less than 100 km.

These criteria resulted in 1106 ground motions being selected from the ground motion library. Of these ground motions, 589 came from the 1999 Chi-Chi, Taiwan earthquake and aftershocks. These were removed because of concerns that they might overly influence the results due to any unusual features of that single event. After removing these records, 517 ground motions remained for the analyses performed below.

2.2 *Computation of ground motion intensity parameters*

The ground motion intensity parameters of interest here consist of spectral acceleration, peak ground acceleration and Arias Intensity. Here (pseudo) spectral acceleration is computed, which is equal to the peak displacement of an elastic oscillator with a given period, T, multiplied by the square of the oscillator's natural frequency, $2\pi/T$ (Chopra 2001). Here oscillators with damping equal to 5% of critical damping are used, and a range of periods between 0.05 and 5 seconds are considered. Note that the ground motions have two horizontal components, so a geometric mean spectral acceleration is computed here using the following equation

$$Sa_{gm}(T) = \sqrt{Sa_x^{\,2}(T) \cdot Sa_y^{\,2}(T)} \tag{1}$$

where $Sa_{gm}(T)$ is the geometric mean spectral acceleration and $Sa_x(T)$ and $Sa_y(T)$ are the spectral acceleration values of the x and y components of the ground motion, respectively (Baker and Cornell 2006b).

Arias Intensity is defined using the following equation (Arias 1970)

$$I_{A,x} = \int_0^\infty a_x(t)^2 \,\mathrm{d}t \tag{2}$$

where $I_{A,x}$ is the Arias Intensity of the x component of the ground motion and $a_x(t)$ is the acceleration of the x component of the ground motion at time t. To compute a single measure of Arias Intensity for a two component ground motion, the quantity $I_A = (I_{A,x} + I_{A,y})/2$ is used.

The final ground motion parameter of interest is peak ground acceleration. It is defined as the maximum absolute value of acceleration observed at any instance in the ground motion. A geometric mean *PGA* is computed here, in a manner analogous to that of equation 1.

All of these intensity parameters are well represented by lognormal distributions, conditional upon earthquake magnitude, distance, and other parameters. Mean values and standard deviations for the possible values that the logarithms of these parameters may take in a given earthquake scenario are given by ground motion prediction ("attenuation") models. These models provide a predictive equation of the form

$$\ln IM = f(M, R, \theta) + \sigma(M)\varepsilon \qquad (3)$$

where IM is the intensity measure of interest, such as PGA or spectral acceleration at a given period. Because these variables are observed to have lognormal distributions, predictive equations are generally formulated for the logarithm of the variable, which is then normally distributed. The term $f(M, R, \theta)$ is the mean prediction of the logarithm of the IM, as a function of earthquake magnitude (M), distance (R) and other paramters (θ). The term $\sigma(M)$ is the standard deviation of the ground motion intensity, and is sometimes a function of earthquake magnitude. Finally, the random variable ε accounts for the variability in observed intensities of ground motions with a given set of predictive parameters. Because the other terms in equation 3 have already accounted for the means and standard deviations of logarithmic intensity, the ε terms have means of zero and unit standard deviations.

To evaluate equation 3, the prediction model of Abrahamson and Silva (1997) is used for spectral acceleration values, the model of Travasarou et al. (2003) is used for I_A and the model of Boore et al. (1997) is used for PGA. Note that the Boore et al. model also provides predictions for spectral acceleration, but it was not used here because is does not cover as large a range of periods as the Abrahamson and Silva model. These prediction models define the intensity of two-component ground motions in the same manner as was done in this section, so comparisons between predicted and observed values are valid.

Once these models are used to compute the means and standard deviations of intensity for the magnitude and distance associated with a given ground motion, one can compute a normalized residual that indicates the number of standard deviations away from the mean prediction a given observation is, using the following equation

$$\varepsilon_i = \frac{\ln x_i - f(m_i, r_i, \theta_i)}{\sigma(m_i)} \qquad (4)$$

where x_i is the observed ground motion intensity (defined using one of the above intensity parameters) for ground motion i, and m_i, r_i and θ_i are the ground motion's associated magnitude, distance, and other predictor parameteres, respectively. The term $f(m_i, r_i, \theta_i)$ is the predicted mean value of the logarithm of that intensity (given the appropriate value of magnitude, distance, etc.) and $\sigma(m_i)$ is the predicted standard deviation of the log intensity. These "epsilons" represent the record-to-record aleatory variability that is not explained by the predictive equations. This variability is explicitly considered in probabilistic assessments such as probabilistic seismic hazard analysis (PSHA). A given ground motion will have a different ε value for each ground motion intensity measure considered, and it is the correlation among these different ε values that must be considered if seismic reliability analysis is to be performed considering multiple intensity measure parameters (Bazzurro and Cornell 2002).

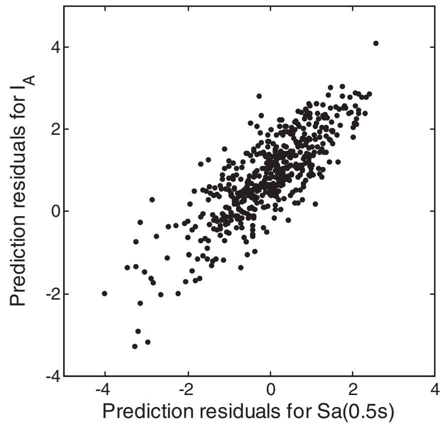

Figure 1. Scatter plot of prediction residuals used to compute the correlation coefficient between I_A and Sa(0.5s).

3 CORRELATION RESULTS

Using the large database of ε values computed using the approach of the previous section, empirical correlation coefficients are computed. The maximum likelihood estimator, sometimes referred to as the Pearson product-moment correlation coefficient, is used here (Neter et al. 1996).

Equation 3 can be used to show that, for a given ground motion with fixed magnitude and distance, the correlation coefficient between two ground motion intensity parameters is equivalent to the correlation coefficient between the ε values for those parameters. Thus, the ε values computed in the previous section are used to compute these correlation coefficients. An example of the data used to compute these correlation coefficients is shown in Figure 1.

The first result obtained is the correlation between two spectral acceleration values at different periods. These correlations have been investigated by others (Abrahamson et al. 2003; Baker and Cornell 2006a; Inoue and Cornell 1990), so this result serves as a validation that the dataset and calculation approach is reasonable. The correlation coefficient depends upon the two periods of interest, so an efficient way to present results for many period combinations is to plot contours of correlation coefficients. In Figure 2. In Figure 3, the predictions of Baker and Cornell (2006a)

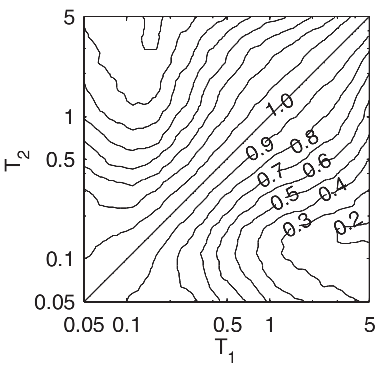

Figure 2. Contours of empirical correlation coefficients between $Sa(T_1)$ and $Sa(T_2)$, as a function of T_1 and T_2.

are shown; the predictive equation from that paper is repeated here for reference

$$\rho_{Sa(T_1),Sa(T_2)} = 1 - \cos\left(\frac{\pi}{2} - \left(\begin{array}{l} 0.359 \\ + 0.163 I_{T_{\min}<0.189} \ln \dfrac{T_{\min}}{0.189} \end{array}\right) \ln \frac{T_{\max}}{T_{\min}}\right) \quad (5)$$

where $T_{\min}$ and $T_{\max}$ are used to denote the larger and smaller of T_1 and T_2, respectively, and $I_{Tmin<0.189}$ is an indicator function equal to 1 if $T_{min} < 0.189$ and equal to 0 otherwise. The agreement between the empirical and predictive equation is good, especially considering that the prediction of equation 5 was developed using only half the number of ground motions that are used in this study. This suggests that the predictive equation of equation 5 can not be significantly improved using the new Next Generation Attenuation ground motion database (2005) used here. The Boore et al. (1997) prediction model was also used to compute residuals and correlation coefficients of spectral acceleration values, and the results were seen to agree closely with those shown in Figure 2. This suggests that the predicted correlations are reasonable for use with ground motion prediction models other than the specific model used to develop the prediction.

The correlations between Arias Intensity and spectral acceleration are considered next. The results are shown in Figure 4. It is seen that short- and moderate-period (i.e., 0.05 to 0.4s) spectral acceleration values have high correlations of about 0.8, while longer period spectral acceleration values are less correlated with I_A. A simple piecewise linear functional approximation to this data is given in the following equation

$$\rho_{I_A,Sa(T)} = \begin{cases} 0.344 - 0.152 \cdot \ln(T) & \text{if } 0.05 \le T < 0.11 \\ 0.971 + 0.131 \cdot \ln(T) & \text{if } 0.11 \le T < 0.4 \\ 0.697 - 0.166 \cdot \ln(T) & \text{if } 0.4 \le T \le 5 \end{cases} \quad (6)$$

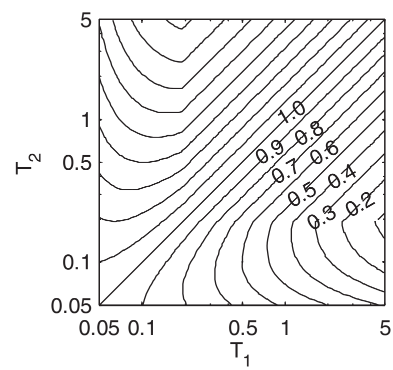

Figure 3. Predicted contours of correlation coefficients between $Sa(T_1)$ and $Sa(T_2)$ from Baker and Cornell (2006a).

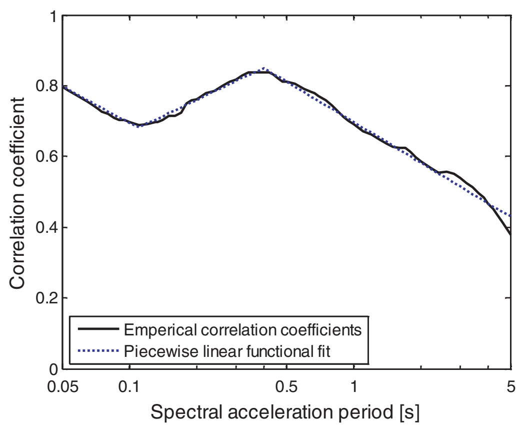

Figure 4. Correlation coefficients between Arias Intensity and $Sa(T)$, as a function of T.

To quantify the level of uncertainty in these correlations due to the finite sample size for estimation, the bootstrap was utilized (Efron and Tibshirani 1993). With this approach, sets of 517 ground motions are selected *with replacement* from the original 517 motions, and correlation coefficients are then computed with these resampled datasets. These results indicated that the level of variability in the correlation estimates is relatively low, with standard deviations of less than 0.05 at all periods.

Correlation results for peak ground acceleration versus spectral acceleration are shown in Figure 5. The general trend in this figure is similar to that in Figure 4. The following piecewise linear equation provides a good fit to the observed values

$$\rho_{PGA,Sa(T)} = \begin{cases} 0.500 - 0.127 \cdot \ln(T) & \text{if } 0.05 \le T < 0.11 \\ 0.968 + 0.085 \cdot \ln(T) & \text{if } 0.11 \le T < 0.25 \\ 0.568 - 0.204 \cdot \ln(T) & \text{if } 0.4 \le T \le 5 \end{cases} \quad (7)$$

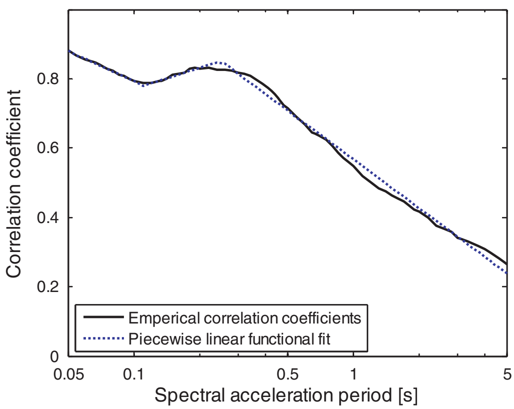

Figure 5. Correlation coefficients between PGA and $Sa(T)$, as a function of T.

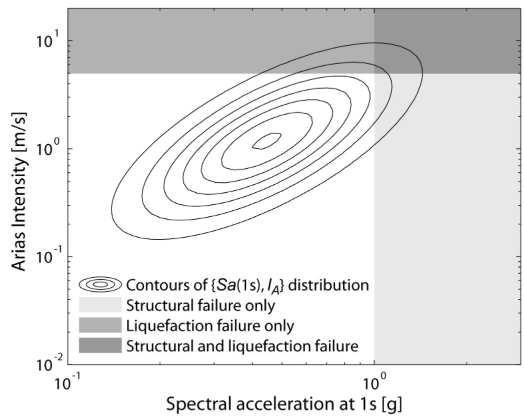

Figure 6. Contours of the joint probability density function for Arias Intensity and $Sa(1\text{s})$, and the assumed failure regions for the example structure, when correlation of ground motion parameters is accounted for.

Finally, the correlation between PGA and I_A was computed to be 0.82. It is not clear that this correlation is currently needed for any common reliability assessments, but it is reported here for completeness.

Equations 6 and 7 were fit manually rather than using a statistical criterion such as least-squares, but the close agreement of these functions with the empirical results suggests that statistically fitted functions would be nearly identical. Equations 6 and 7 are provided to facilitate calculations in software applications; without these analytic functions it would be necessary to provide large tables in order to look up correlation coefficients at a specified period.

The correlations of spectral acceleration values have previously been found to be independent of parameters such as earthquake magnitude or distance (Baker and Cornell 2005, Appendix B). It has been assumed that this independence holds for these ground motion intensity parameters as well, without further investigation.

4 EXAMPLE CALCULATION

To demonstrate the importance of the results provided here, a simple example calculation is performed. Consider a hypothetical site located 10 km from a fault that has produced a magnitude 7 strike-slip earthquake. The site has NEHRP classification C and Bray and Rodriguez-Marek SGS classification C. Of interest is the potential failure of a building with first-mode period of 1 second, due to structural collapse or liquefaction of the underlying soil.

Based on ground motion prediction equation for Arias Intensity (Travasarou et al. 2003), we find that Arias Intensity is lognormally distributed with a median of 1.17 m/s and logarithmic standard deviation of 1.06. Spectral acceleration at one second likewise has a lognormal distribution with a median of 0.45 g and a logarithmic standard deviation of 0.59 (Abrahamson and Silva 1997).

From equation 6, we see that the two parameters have a correlation coefficient of 0.70. With this information, it is possible to fully specify the joint normal distribution for the logarithms of Arias Intensity and $Sa(1\text{s})$. Contours of this joint distribution are shown in Figure 6. To illustrate the effect of this correlation, contours are shown in Figure 6. To illustrate the effect of this correlation, contours are shown in Figure 7 for the same distribution but with zero correlation.

To simplify the reliability computation, it is assumed here that a structural failure will occur if $Sa(1\text{s}) > 1$ g and a liquefaction failure will occur if $I_A > 5$ m/s. In real applications, these failure criteria would not be deterministic; structural failures could be predicted using a fragility function (e.g., Pinto et al. 2004), and liquefaction failures could be predicted using a liquefaction failure model (e.g., Cetin et al. 2004). The simplified criteria used nonetheless serve to illustrate the need for consideration of ground motion correlation. The failure regions associated with these two failure modes are shaded in Figure 6 and and Figure 7.

The probability of a structural failure can be computed as

$$\begin{aligned} P(\text{structural failure}) &= 1-\Phi\left(\frac{\ln Sa_{failure}-\mu_{\ln Sa}}{\sigma_{\ln Sa}}\right) \\ &= 1-\Phi\left(\frac{\ln 1-\ln(0.45)}{0.59}\right) \\ &= 0.085 \end{aligned} \tag{8}$$

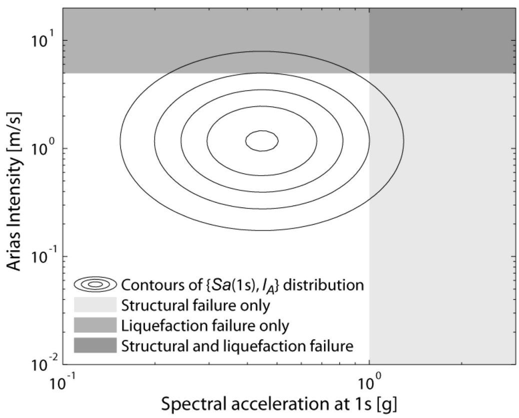

Figure 7. Contours of the joint probability density function for Arias Intensity and $Sa(1\text{s})$, and the assumed failure regions for the example structure, when correlation of ground motion parameters is *not* accounted for.

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

Similarly, the probability of liquefaction can be computed to be 0.087. Because these are not mutually exclusive events, however, it is not valid to simply add these two probabilities to find the total probability of failure. Instead, the joint distribution can be integrated over the shaded regions shown in Figure 6. This integration gives a total failure probability of 0.13. In the incorrect case of Figure 7, where correlation is assumed to be zero, the individual probabilities of structural and liquefaction failure remain the same, but the total failure probability becomes 0.17. This illustrates the effect of considering correlation between these ground motion parameters when simultaneously considering both structural and geotechnical failures.

This example considers only a single magnitude and distance event, however, while real sites may be potentially subjected to a wide range of events coming from a variety of nearby earthquake sources. Probabilistic seismic hazard analysis can be used to consider all of these potential events, along with their corresponding rates of occurrence (Kramer 1996; McGuire 2004). The standard, widely used, PSHA approach provides rates of exceedance for only a single "scalar" ground motion intensity parameter such as spectral acceleration. What is needed here is a vector-valued PSHA result. The mathematical approach has been developed by Bazzurro and Cornell (2002), and is directly applicable to the present problem as well. The output of this procedure is a set of rates of jointly exceeding, for example, both $Sa = x$ and $PGA = y$. The result could then replace the joint distributions used in the previous section when reliability calculations are performed. Results of this type for vectors of spectral acceleration parameters have been obtained and used by others, and future work will focus on obtaining analogous vector-valued PSHA results for spectral acceleration and PGA or I_A. This approach is currently being implemented for use in the the seismic reliability framework of the MERCI project (Bayraktarli et al. 2006).

5 CONCLUSIONS

Models for correlation of ground motion intensity parameters have been presented that are necessary for computing seismic reliability of structures vulnerable to both structural and geotechnical failures. The models characterize the probabilistic dependence among spectral acceleration ($Sa(T)$), peak ground acceleration (PGA) and Arias Intensity (I_A). Results have previously been reported for correlation of spectral response values, but this is believed to be the first time that results incorporating peak ground acceleration and Arias Intensity have been considered. The dependence between ground motion intensity parameters was characterized in the form of correlation coefficients of standardized residuals ("epsilons") from ground motion prediction models. The correlations were determined empirically be computing epsilon values from 517 recorded ground motions and fitting predictive equations to the results in order to allow easy calculation of correlation coefficients in other applications.

The following approximate results may provide useful "rule-of-thumb" guidelines: I_A and $Sa(T)$ have correlation coefficients of approximately 0.8 when $T < 0.4$s, and the correlation decreases to 0.4 by a period of 5s; PGA and $Sa(T)$ have correlation coefficients between 0.8 and 0.9 in the period range $0.05 < T < 0.25$s, and the correlation decreases to 0.25 by a period of 5s. Correlations between $Sa(T_1)$ and $Sa(T_2)$ were computed and were seen to agree closely with the predictive model developed by Baker and Cornell (2006a) (who used a smaller set of ground motions than was used here).

A simple example calculation was presented to demonstrate the need for these correlation results in seismic reliability calculations. In the idealized situation considered above, the probability of failure of a system subjected to a scenario earthquake was computed to be 0.21, compared to a probability of 0.25 that would incorrectly be computed if one neglected to account for correlation in the ground motion intensity parameters. Implementation of these correlations in vector-valued probabilistic seismic hazard analysis will be straightforward, and should facilitate the more widespread dissemination of these results. Note that the example calculation considered only correlation in the ground motion intensity parameters. The interaction between failure mechanisms was not addressed in this paper, although it might be significant in some cases due to, e.g., soil-structure interaction.

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