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Probabilistic structural response assessment using vector-valued intensity measures

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SUMMARY

Methods for using scalar and vector ground motion intensity parameters to estimate the probabilistic relationship between ground motion intensity and structural response are described and compared. Options include using regression analysis on structural analysis results from a set of unscaled (or uniformly scaled) ground motions, or fitting a probability distribution to the analysis results from scaled ground motions analysed using incremental dynamic analysis and related methods. Past methods for using scalar ground motion intensity are reviewed, and methods for utilizing improved vector-valued intensity measures (IMs) are proposed. 'Hybrid' estimation methods that obtain the benefit of vector-valued IMs using simplified techniques such as careful record selection are also discussed. The results are then combined with models for ground motion occurrence obtained from probabilistic seismic hazard analysis to compute seismic reliability, and the results obtained from the various methods are compared. In general, a tradeoff must be made between the accuracy of the functional relationship between ground motion intensity and structural response versus the number of structural analyses needed for estimation. Copyright © 2007 John Wiley & Sons, Ltd.

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INTRODUCTION

Relationships between ground motion intensity and structural response can be combined with ground motion hazard models to compute the seismic reliability of structures (e.g. [1–3]). With this approach, the link between ground motion hazard and structural response is often made

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using scalar intensity measures (IMs) such as spectral acceleration at the fundamental period of the structure, but recently researchers have considered vector-valued IMs consisting of multiple parameters. Probabilistic predictions of structural response as a function of the IM, obtained using statistical analysis of nonlinear dynamic analysis results using a set of ground motions, are required with this approach. A variety of methods for obtaining this prediction using scalar or vector IMs are described here, and their relative advantages and disadvantages are discussed. Previously developed methods for using scalar IMs are reviewed, and several new methods for use with vector-valued IMs are presented and discussed. It is seen that the preferred approach depends on the sample size used for estimation (i.e. the number of nonlinear dynamic analyses performed), the level of nonlinearity in the structure, whether the IM is scalar valued or vector valued, and the validity of several potential functional form approximations. Note that non-IM-based methods for computing seismic reliability have also been proposed [4–6], but the focus here is on a detailed study of IM-based approaches.

Following the terminology conventions of the Pacific Earthquake Engineering Research (PEER) center, the structural response parameter is termed as an Engineering Demand Parameter (EDP) in the discussion that follows. The focus of this paper is on methods for estimating the probability distribution of EDP for a given IM (this conditional distribution is denoted as EDP|IM). The estimate of EDP|IM can then be combined with a ground motion hazard curve to compute the mean annual rate of exceeding an EDP level y, $\lambda_{\mathrm{EDP}}(y)$, using the equation

$$\lambda_{\mathrm{EDP}}(y) = \int_{\mathrm{IM}} G_{\mathrm{EDP|IM}}(y|\mathrm{im}) \left| \frac{\mathrm{d}\lambda_{\mathrm{IM}}(\mathrm{im})}{\mathrm{dim}} \right| \mathrm{dim} \tag{1}$$

where $G_{\mathrm{EDP|IM}}(y|\mathrm{im})$ is the complementary cumulative distribution function (CCDF) of EDP|IM (i.e. the probability that EDP$>y$, given that IM$=$im) obtained using the estimation methods described in this paper. The term $\lambda_{\mathrm{IM}}(\mathrm{im})$ is the mean annual rate of exceeding the IM level im, obtained using Probabilistic Seismic Hazard Analysis, a procedure that is well documented elsewhere (e.g. [7, 8]). The rate of exceeding a given EDP level, $\lambda_{\mathrm{EDP}}(y)$, is the seismic reliability measure of interest here (e.g. if EDP$=y$ is a response level associated with collapse, then $\lambda_{\mathrm{EDP}}(y)$ is the annual rate of collapse of the structure). This equation can also be generalized to incorporate vector-valued IMs, as will be seen in the following section.

The needed relationship between EDP and IM is modelled using several methods in the following sections. Both scalar and vector IMs are used, and the results are compared to determine whether the various estimates of structural response and seismic reliability are equivalent. Dynamic analysis results from an example structure will be used to illustrate the various scalar- and vector-IM-based methods. The structure is a 1960s era reinforced-concrete moment-frame building located on a NEHRP category S_D site near Los Angeles, which has been used as a testbed for a variety of PEER research activities [9]. A nonlinear, stiffness and strength degrading 2D model of the transverse frame is used [10]. This frame of the building has an elastic first-mode period of 0.8 s. So spectral acceleration at the first-mode period of the building, $\mathrm{Sa}(T_1)$, is used to demonstrate the scalar IM approaches; the parameter ε, described briefly below, is combined with $\mathrm{Sa}(T_1)$ to create an example vector IM. The EDP considered here is the maximum interstorey drift ratio observed in any storey of a building. These specific parameters are used only for illustration: the proposed procedures are also applicable for other structures, EDPs, and IMs.

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MOTIVATION FOR USING VECTOR-VALUED INTENSITY MEASURES

Equation (1) can potentially be improved by increasing the number of parameters in the IM so that it more completely describes the properties of ground motions. The benefit can be explained using the following argument, which has also been made by others [11, 12]. Consider an EDP that is potentially dependent upon two ground motion parameters: IM_1 and IM_2. The rate of exceeding a specified value of EDP, y, can be computed using knowledge of the conditional distribution of EDP given IM_1 and IM_2, along with knowledge of the joint rates of occurrence of the various levels of IM_1 and IM_2. This generalization of Equation (1) can be written as

$$\lambda_{\mathrm{EDP}}(y)=\int_{\mathrm{IM}_1}\int_{\mathrm{IM}_2} G_{\mathrm{EDP}|\mathbf{IM}}(y|\mathrm{im}_1,\mathrm{im}_2)\, f_{\mathrm{IM}_2|\mathrm{IM}_1}(\mathrm{im}_2|\mathrm{im}_1)\left|\frac{\mathrm{d}\lambda_{\mathrm{IM}}(\mathrm{im}_1)}{\mathrm{dim}_1}\right| \mathrm{dim}_2\mathrm{dim}_1 \qquad (2)$$

where $G_{\mathrm{EDP}|\mathrm{IM}_1,\mathrm{IM}_2}(y|\mathrm{im}_1,\mathrm{im}_2)$ denotes the probability that EDP is greater than y, given an earthquake ground motion with intensity such that $\mathrm{IM}_1=\mathrm{im}_1$ and $\mathrm{IM}_2=\mathrm{im}_2$. The term $f_{\mathrm{IM}_2|\mathrm{IM}_1}(\mathrm{im}_2|\mathrm{im}_1)$ denotes the conditional probability density function of IM_2 given IM_1, and $\lambda_{\mathrm{IM}_1}(\mathrm{im}_1)$ is the annual rate of IM_1 exceeding im_1 at the site being considered. The ground motion hazard ($f_{\mathrm{IM}_2|\mathrm{IM}_1}(\mathrm{im}_2|\mathrm{im}_1)$ and $\lambda_{\mathrm{IM}_1}(\mathrm{im}_1)$) can be obtained either through vector probabilistic seismic hazard analysis [12] or, in the case of $\mathrm{Sa}(T_1)$ and ε used below, with standard probabilistic seismic hazard analysis (PSHA) and disaggregation.

If the ground motion parameter IM_2 is ignored, and only IM_1 is used for predicting structural response, then the distribution of EDP will not depend (explicitly) on IM_2. In the case where IM_2 actually does not affect structural response (given IM_1), $G_{\mathrm{EDP}|\mathrm{IM}_1,\mathrm{IM}_2}(y|\mathrm{im}_1,\mathrm{im}_2)$ is equal to $G_{\mathrm{EDP}|\mathrm{IM}_1}(y|\mathrm{im}_1)$, and Equations (1) and (2) are equivalent. This should be intuitive: if the ground motion parameter IM_2 has no effect on structural response, then a calculation which considers IM_2 (Equation (2)) should not produce a different answer than a calculation which does not (Equation (1)).

If the parameter IM_2 *does* affect response, however, then more care is needed in the reliability assessment. If one uses only IM_1 to estimate structural response in this case, then the estimate of EDP given IM_1 depends implicitly upon the distribution of IM_2 values of the ground motion record set used for analysis. This can be seen by expanding Equation (1) using the total probability theorem [13] to explicitly note that EDP is a function of both IM_1 and IM_2:

$$\lambda_{\mathrm{EDP}}(y)=\int_{\mathrm{IM}_1}\int_{\mathrm{IM}_2} G_{\mathrm{EDP}|\mathrm{IM}_1,\mathrm{IM}_2}(y|\mathrm{im}_1,\mathrm{im}_2)\, \tilde{f}_{\mathrm{IM}_2|\mathrm{IM}_1}(\mathrm{im}_2|\mathrm{im}_1)\left|\frac{\mathrm{d}\lambda_{\mathrm{IM}}(\mathrm{im}_1)}{\mathrm{dim}_1}\right| \mathrm{dim}_2\mathrm{dim}_1 \qquad (3)$$

This is similar to Equation (2), but with one important difference. The conditional distribution of IM_2 given IM_1 occurring at the site, $f_{\mathrm{IM}_2|\mathrm{IM}_1}(\mathrm{im}_2|\mathrm{im}_1)$, has been replaced by the conditional distribution of IM_2 given IM_1 *within the record set used for analysis*, which is denoted as $\tilde{f}_{\mathrm{IM}_2|\mathrm{IM}_1}(\mathrm{im}_2|\mathrm{im}_1)$.

By comparing Equation (2) to Equation (1) (and its equivalent expanded form in Equation (3)), one can see that there are two ways to obtain the answer of Equation (2): use a vector IM, or ensure that the distribution of IM_2 given IM_1 in the record set ($\tilde{f}_{\mathrm{IM}_2|\mathrm{IM}_1}(\mathrm{im}_2|\mathrm{im}_1)$) is equal to the target distribution obtained from hazard analysis ($f_{\mathrm{IM}_2|\mathrm{IM}_1}(\mathrm{im}_2|\mathrm{im}_1)$). Methods of implementing these two approaches will be considered in detail below.

A further benefit provided by vector-valued IMs is increased estimation efficiency. If by considering IM_2 one can further explain a ground motion's effect on a structure, then the remaining

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unexplained statistical variability in EDP will be reduced. This means that fewer nonlinear dynamic analyses will be needed to characterize the relationship between structural response and the IM. An IM that results in small variability of EDP given IM is termed 'efficient' by Luco and Cornell [14]. Several researchers have found that vector-valued IMs can achieve significant gains in efficiency, so this approach can reduce the number of dynamic analyses that must be performed to assess a structure's performance [12, 15–17]. The potential of vector-valued IMs to improve estimation efficiency and more accurately predict seismic reliability provides the motivation for using vector IMs to predict EDP in this paper.

EDP ESTIMATION FROM A STATISTICAL INFERENCE PERSPECTIVE

The field of statistical inference is concerned with estimating the properties of a random variable (in this case, EDP given an IM level) from a finite sample of data. Two classes of statistical inference approaches will be considered here. Using a parametric approach, one assumes that the random variable EDP has some probability distribution (e.g. lognormal) that is defined by a few parameters. One then estimates these parameters to define the distribution [18]. Generally, the choice of a distribution is made based on past experience, and statistical tests exist to identify when a data set is not well represented by the specified distribution. Another class of models, which use a non-parametric approach, do not require assumptions about the distribution of the data [19]. Non-parametric models have the advantage of being robust when the data do not fit a specified parametric distribution, but generally require more data for estimation in cases where the data *do* fit a parametric distribution.

Complicating the EDP estimation problem relative to simple statistical inference situations is the fact that the distribution of EDP is a function of IM (i.e. structural response tends to be larger when the intensity of ground motion is greater). For some applications, the distribution of EDP is needed over a continuous range of IMs, or at least a somewhat finely discretized set. In other applications, the distribution of EDP may be needed for only a single IM level. The choice of an estimation method used will depend on the range of IM levels over which the distribution estimate is needed. These concepts will help to identify the positive and negative aspects of the various estimation methods described in the following sections.

ESTIMATION METHODS USING A SCALAR INTENSITY MEASURE

EDP estimation using scalar IMs has been addressed elsewhere [10], but the potential approaches are briefly reviewed here to facilitate later comparisons with similar approaches using vector IMs.

Regress on response data from unscaled ground motions

With this method, the nonlinear dynamic analysis of a structure is performed using a set of unscaled ground motion records (or records scaled by a constant factor). The records' IM values and associated EDP values obtained from nonlinear dynamic analysis are sometimes referred to as a 'cloud,' because they form a rough ellipse when plotted, as seen in Figure 1(a). Regression can be used with this cloud of data to compute the conditional mean and standard deviation of

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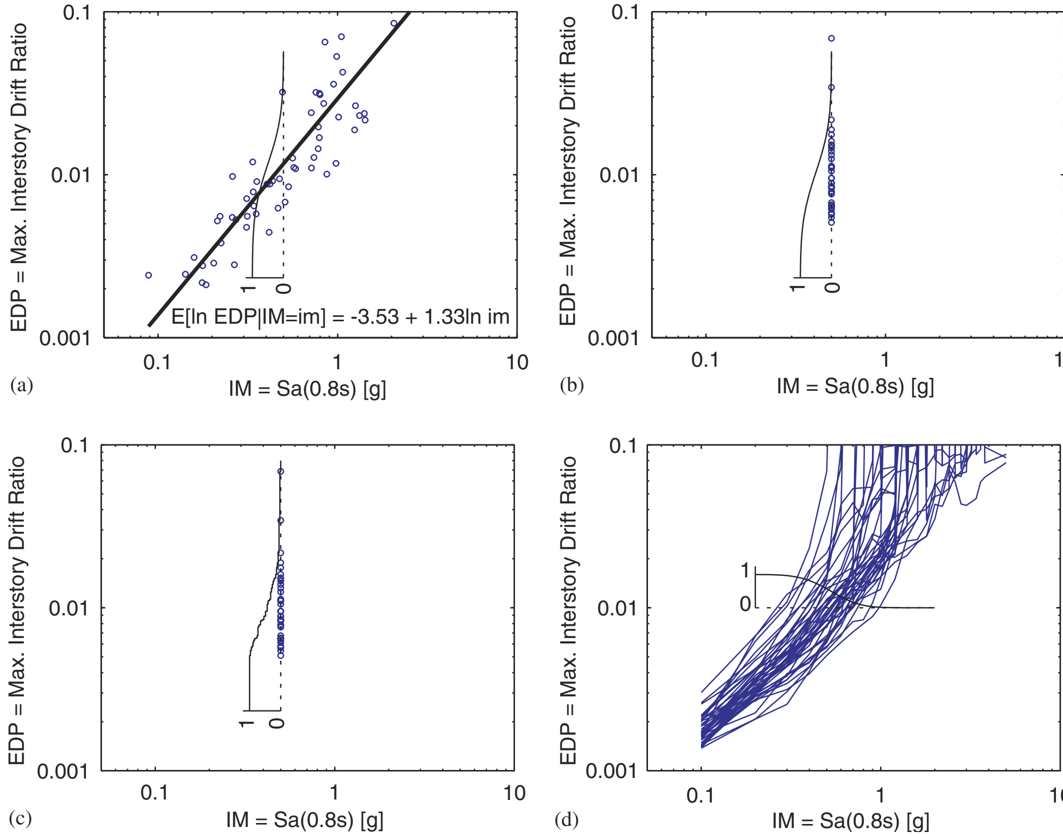

Figure 1. Illustration of methods for estimating the conditional distribution of $\ln \mathrm{EDP}|\mathrm{IM}$ at $\mathrm{Sa}(0.8\,\mathrm{s}) = 0.5g$: (a) a cloud of $\ln \mathrm{EDP}|\mathrm{IM}$ data, the conditional mean value from linear regression, and a Gaussian CCDF fitted to the mean and standard deviation from the regression; (b) a stripe of $\ln \mathrm{EDP}$ data and a Gaussian CCDF based on the sample mean and standard deviation; (c) a stripe of $\ln \mathrm{EDP}$ data and its empirical CCDF; and (d) incremental dynamic analysis curves, and a Gaussian CDF of $\ln \mathrm{IM_{Cap}}$ obtained from the sample mean and standard deviation of the first exceedances of maximum interstorey drift ratio $= 0.01$.

EDP given IM. A linear relationship between the logarithms of the two variables often provides a reasonable estimate of the mean value of $\ln \mathrm{EDP}$ over a small range, yielding the model

$$E[\ln \mathrm{EDP}|\mathrm{IM} = \mathrm{im}] = \hat{\beta}_0 + \hat{\beta}_1 \ln \mathrm{im} \tag{4}$$

where $\hat{\beta}_0$ and $\hat{\beta}_1$ are constant coefficients to be estimated from linear regression [20]. Using this mean prediction, regression residuals are defined as

$$e_i = \ln \mathrm{EDP}_i - \ln \widehat{\mathrm{EDP}}_i \tag{5}$$

where $\ln \mathrm{EDP}_i$ is the natural logarithm of the EDP associated with record i, and $\ln \widehat{\mathrm{EDP}}_i$ is the prediction from Equation (4) based on the record's IM value. The e's by definition have a mean of

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zero, and if they are assumed to have a constant variance for all IM, then their standard deviation can be estimated as

$$\hat{\sigma}_e = \frac{1}{n-2}\sqrt{\sum_i^n (\ln \mathrm{EDP}_i - \ln \hat{\mathrm{EDP}}_i)^2} \tag{6}$$

where n is the number of records. If $\ln \mathrm{EDP}|\mathrm{IM}$ is further assumed to have a Gaussian distribution, then the estimated conditional probability of exceeding an EDP level y given $\mathrm{IM}=\mathrm{im}$ is

$$G_{\mathrm{EDP}|\mathrm{IM}}(y|\mathrm{im}) = 1 - \Phi\left(\frac{\ln y - (\hat{\beta}_0 + \hat{\beta}_1 \ln \mathrm{im})}{\hat{\sigma}_e}\right) \tag{7}$$

where $G_{\mathrm{EDP}|\mathrm{IM}}(y|\mathrm{im})$ is the CCDF of EDP given IM and $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard Gaussian distribution. Example results are shown in Figure 1(a) for an Sa(0.8 s) level of $0.5g$, using regression on structural response results obtained from 60 ground motions that have been scaled by a factor of two (record details are available in [21, Tables A.1 and A.2]).

This method has more restrictions on the form of the conditional distributions than any other method to follow. In particular, it requires the relationship between EDP and IM to be linear with constant variance after transformations (logarithms are typically taken of EDP and IM to obtain an approximately linear relationship and constant variance). These restrictions may be appropriate only over a limited range of IM levels, but they have the benefit of reducing the computational expense of the estimation (few data are needed because there are few parameters to estimate). This method has the additional advantage of being the only one associated with a closed-form analytic solution for Equation (1) [22]. For these reasons, it is the preferred approach in many applications [10, 14, 23–25].

The above assumptions can be relaxed if they are not believed to be valid, although a closed-form solution to Equation (1) will no longer be available. A linear relationship may not be reasonable for the entire IM range of interest, and the variance of EDP may not be constant. To address these problems, the regression should be limited to the IM range where the assumed form is reasonable, or piecewise linear relationships may be fit (e.g. [26]). Structural collapses are not considered in the basic model, but can be incorporated by adding a supplemental probability-of-collapse estimation as a function of IM [27]. With these modifications, the resulting assessment then lies somewhere between the basic cloud method and the less restrictive methods described below, both in terms of the required number of analyses and the number of parametric assumptions made.

Scale records to the target IM level and fit a parametric distribution to response results

Rather than using regression analysis with ground motions having a range of IM levels, one can instead scale the motions so that each has the IM level of interest, and then directly estimate the distribution of EDP from the resulting structural responses. To estimate a distribution from this data, one can estimate the mean and standard deviation of the responses and use these values to fit a distribution. For example, one can fit a normal distribution to $\ln \mathrm{EDP}$ values, giving a CCDF defined as

$$G_{\mathrm{EDP}|\mathrm{IM}}(y|\mathrm{im}) = 1 - \Phi\left(\frac{\ln y - \hat{\mu}}{\hat{\sigma}}\right) \tag{8}$$

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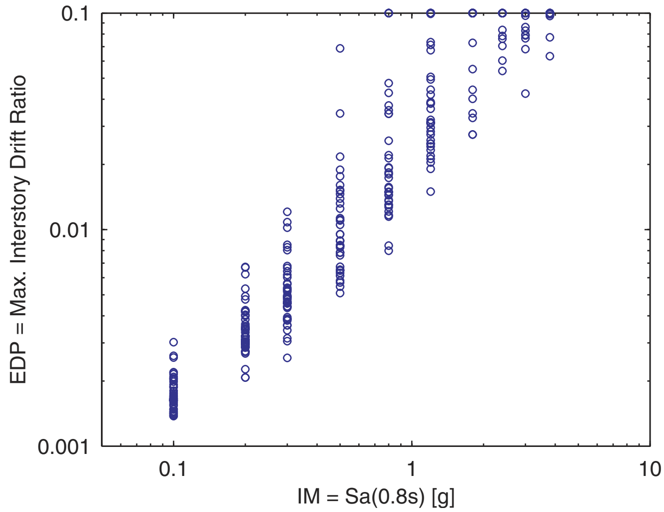

Figure 2. Multiple stripes of data used to re-estimate distributions at varying IM levels. Records that cause maximum interstorey drift ratios of larger than 0.1 are displayed with interstorey drift ratios of 0.1.

where $\hat{\mu}$ and $\hat{\sigma}$ are the mean and standard deviation, respectively, of the $\ln \text{EDP}$ values from a set of ground motions scaled so that $\text{IM} = \text{im}$. Forty records and a fitted CCDF are shown in Figure 1(b); because the data lie on a straight line, this method is sometimes referred to as the 'stripe' method. Note that the 40 records used here and below are a subset of the 60 used for the cloud method. The number of records was reduced because the use of 60 records at multiple IM levels would increase computational expense without significantly improving prediction accuracy. By repeating this procedure for several IM values and interpolating if needed, the EDP distribution can be obtained as a function of IM. Multiple stripes of data are shown in Figure 2 (using a suite of 40 ground motions scaled to 10 spectral acceleration levels between $0.1g$ and $4g$). It is visually apparent that the standard deviation of $\ln \text{EDP}$ is not constant over the range of IM considered here. It also appears that the mean value of $\ln \text{EDP}$ is not a linear function of $\ln \text{IM}$. This indicates that the most basic cloud approach would need to be generalized in order to accurately model responses over the entire IM range shown here.

This method can easily account for records that cause collapse of the structure. Here collapse is defined to have occurred if the dynamic analysis algorithm fails to converge or if the drift ratio at any storey exceeds 10%, although other criteria can be easily adopted. This is done by first estimating the probability of collapse (denoted C) as the fraction of records in a stripe that cause collapse, and then fitting a parametric distribution to the non-collapse ($\overline{C}$) responses. The lognormal CCDF is given by

$$G_{\text{EDP}|\text{IM}}(y|\text{im}) = 1 - P(\overline{C}|\text{IM} = \text{im})\Phi\left(\frac{\ln y - \hat{\mu}}{\hat{\sigma}}\right) \tag{9}$$

where $\hat{\mu}$ and $\hat{\sigma}$ are the sample mean and standard deviation, respectively, of the non-collapse responses, and $P(\overline{C}|\text{IM} = \text{im})$ is the counted fraction of records at the $\text{IM} = \text{im}$ stripe that do not cause collapse. When estimating the probability of collapse at multiple IM levels, the probability

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can be counted at each IM = im stripe, or a parametric function can be fit over a range of IM levels [27].

This approach is especially useful if response at only a single IM level is of interest (e.g. in code methods concerned with response at a target ground motion intensity level). The method potentially requires more structural analyses than the cloud method, however, if response estimates are needed at many IM levels. Four hundred dynamic analyses were used to produce Figure 2 for research purposes, although in practice this number could be reduced significantly. Questions regarding the validity of record scaling also arise when this method is used. While scaled ground motions differ from the naturally observed ground motions, empirical studies of peak displacements in frame structures suggest that if the records are selected carefully then the introduction of bias can be avoided [28–30].

Scale records to the target IM level and compute an empirical distribution for response

With this non-parametric method, a stripe of data is obtained in the same manner as in the previous section, but the fitted parametric distribution is replaced by an empirical CCDF [19]. The probability of exceeding an EDP level y is estimated by simply counting the fraction of records that cause a response larger than y:

$$G_{\mathrm{EDP|IM}}(y|\mathrm{im}) = \frac{\text{number of responses} > \text{y}}{\text{total number of records}} \tag{10}$$

An empirical CCDF is superimposed on a stripe in Figure 1(c). With this approach, no assumptions are needed regarding distributions or functional relationships between EDP and IM. The eliminated assumptions have a cost, however: more data are needed to characterize the conditional distributions. Empirical distributions can also have difficulties precisely estimating the probability of exceeding extreme values [31], which are often of concern for reliability analysis.

Fit a distribution for IM capacity to IDA results

With this method, the distribution of EDP|IM is not estimated directly. Rather, the results from Incremental Dynamic Analysis (IDA) [32] are used to determine the probability that the IM level of a ground motion is less than im, given that the ground motion caused a level of response EDP $= y$ [1, 33, 34]. This can be expressed as

$$F_{\mathrm{IM_{Cap}|EDP}}(\mathrm{im}|y) = P(\mathrm{IM_{Cap}} < \mathrm{im}|\mathrm{EDP} = y) \tag{11}$$

where $\mathrm{IM_{Cap}}$ is a random variable representing the distribution of IM values that result in an EDP level y occurring in the structure (i.e. the 'capacity' of the structure to resist a given EDP level, defined in terms of the IM of the ground motion). Rather than using Equation (1) to compute the structural response hazard, Equation (11) can be used with the following equation:

$$\lambda_{\mathrm{EDP}}(y) = \int_{\mathrm{IM}} F_{\mathrm{IM_{Cap}|EDP}}(\mathrm{im}|y) \left| \frac{\mathrm{d}\lambda_{\mathrm{IM}}(\mathrm{im})}{\mathrm{dim}} \right| \mathrm{dim} \tag{12}$$

An example of IDA results and a fitted distribution for a maximum interstorey drift ratio of 0.01 is shown in Figure 1(d). The IDAs in this figure were obtained by interpolating the stripes

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from Figure 2, but with wise selection of analysis points it is possible to reduce the number of needed analyses [32]. If one is only interested in a single structural response level y, such as that associated with a critical performance level, then Equation (11) need only be evaluated at a single EDP value, further reducing the number of dynamic analyses needed. This method also does not require a separate treatment of collapse responses, unlike previous methods.

A drawback of this method is that it will likely still require more analyses than a cloud analysis (although it will provide more accuracy than the cloud method if used over a large range of IMs). The need to have continuous IDAs also means that one must use the same records for analysis at all IM levels, rather than re-selecting different records at increasing IM levels in order to reflect the differing causal earthquake events [35]. One should be aware that the IM value causing $\mathrm{EDP} = y$ may not be unique, because some IDA traces are not always monotonically increasing in some ranges of IM, as seen in Figure 1(d). This can be addressed by defining $\mathrm{IM}_{\mathrm{Cap}}$ as the smallest value of IM such that $\mathrm{EDP} = y$.

Comparison of results from alternative methods

The results from Figure 1 can be compared in several ways. The estimated mean and standard deviation of $\ln \mathrm{EDP}|\mathrm{IM}$ from stripe and cloud methods are shown in Figure 3; the values at each stripe are interpolated linearly to obtain continuous estimates. Only the most basic cloud method is used, without piecewise linear fits or probability of collapse estimates (to evaluate collapses the cloud record set would need to be scaled up uniformly, as none of the records in this example caused a collapse). Because a large number of dynamic analyses were performed for the stripes in this example, the stripe estimate can be reasonably treated as the true answer. For spectral acceleration values of approximately $1g$, the stripe method reveals an increase in mean EDP values as the structure reaches large levels of nonlinearity. These results are for only the non-collapsing cases, where here collapse is assumed to occur after exceedance of a 10% maximum interstorey drift ratio. Thus, the stripes

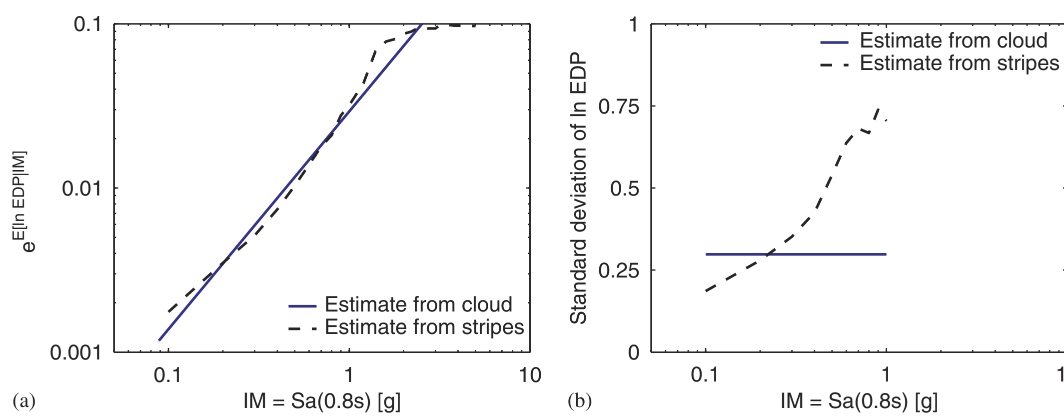

Figure 3. (a) Mean and (b) standard deviation of $\ln \mathrm{EDP}|\mathrm{IM}$ for non-collapsing records, estimated using the cloud and stripe methods. For the stripe method, the values between stripes are determined using linear interpolation. Only the IM levels where less than 50% of records cause collapse are displayed because at levels with higher probability of collapse, the statistics estimated using only non-collapse responses are less meaningful.

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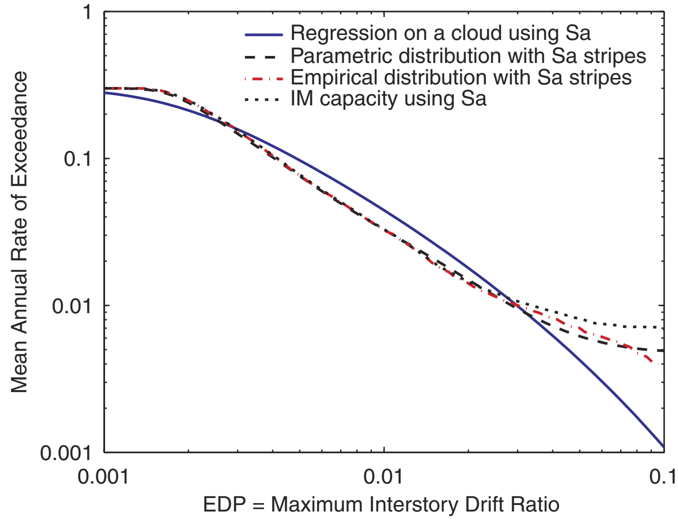

Figure 4. Comparison of drift hazard results for the example structure using the four scalar-IM-based estimation methods.

produce a mean estimate that peaks at about 10% for the extremely high ground motion levels (greater than $2g$).

A comparison of the estimated log standard deviations of EDP is shown in Figure 3(b). The simple cloud method provides a constant estimated standard deviation of $\ln \mathrm{EDP}$ for all spectral acceleration levels, but the stripe estimates suggest that log standard deviation is increasing with increasing IM level. The shortcomings of the cloud method identified here could be addressed using the generalizations discussed earlier, or by performing the regression over a more narrow range of IM than was used here, so that the assumptions are more likely to be reasonable.

To evaluate the effect of the complete distributional estimates for all spectral acceleration levels, the probabilistic estimates of EDP|IM can be combined with an IM hazard curve to compute an EDP hazard curve, per Equation (1) (and Equation (12) for the capacity formulation). The IM hazard is computed for the location in Van Nuys, California, where the example structure is located. Using each of the above methods to estimate EDP|IM, the EDP hazard curve is computed and displayed in Figure 4. Because of the large number of analyses used, the two stripe methods and the capacity method capture the distribution of EDP|IM well, and the resulting drift hazard curves show reasonable agreement. The stripe methods and the capacity method should in general produce equivalent results, given sufficient data for estimation, lognormality of the conditional distributions, and monotonicity of the IDA traces. The convergence of estimates for large sample sizes is termed *consistency* in statistical inference. Connections and comparisons between these methods have also been made by others [10, 11, 32]. For this example, the cloud method agrees somewhat with the other methods, but functional form constraints limit its accuracy. With a greater number of analyses and generalizations to remove the basic assumptions, however, the cloud method could be made to agree more closely with the other methods.

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ESTIMATION METHODS USING A VECTOR-VALUED INTENSITY MEASURE

Vector-valued IMs have several advantages for estimating seismic reliability, as discussed above. But comparison of methods for predicting response as a function of the vector IM has received little or no attention to date. Modifications of the scalar IM methods will be described here, and their relative merits considered. In the text below, a vector of IM parameters will be denoted **IM**, and individual parameters will be denoted IM_1, IM_2, etc. Several of the methods assume that one of the parameters, IM_1, is the dominant predictor of structural response, and treat it differently than the others (e.g. by scaling to match IM_1 and using regression to account for the effects of IM_2, IM_3, etc.).

To illustrate the vector-**IM**-based approaches, an IM consisting of Sa(0.8 s) and a parameter denoted ε will be used to predict response of the same example structure used earlier. The parameter ε, defined as a measure of the difference between the spectral acceleration of a record and the mean of a ground motion prediction equation at the given period, has been found to be an effective predictor of structural response because it is an implicit measure of spectral shape [35, 36]. More specifically, ε (measured at a period T) tends to indicate whether $\mathrm{Sa}(T)$ is in a peak or a valley of the response spectrum. Records with positive ε values tend to have smaller responses than records with negative ε values, given that they have the same $\mathrm{Sa}(T_1)$ values. For this example IM, $\mathrm{Sa}(T_1)$ is treated as the dominant parameter, with ε providing supplementary information. This **IM** is only used to illustrate application of the EDP estimation approaches, without further considering its merits relative to other **IM**s.

Multiple linear regression with a cloud of ground motions

The cloud method can be easily adapted to vector-valued **IM**s through the use of multiple linear regression, which provides a well-developed theory regarding model selection, confidence intervals for regression coefficients, etc. [20]. A linear functional form (after variable transformations) is again used to model the relationship between the mean value of EDP and IMs. For example,

$$E[\ln \mathrm{EDP}|\mathrm{IM}_1=\mathrm{im}_1, \mathrm{IM}_2=\mathrm{im}_2]=\hat{\beta}_0+\hat{\beta}_1 \ln \mathrm{im}_1+\hat{\beta}_2 \ln \mathrm{im}_2 \tag{13}$$

where $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$ are estimated coefficients obtained using multiple linear regression. An example prediction is shown in Figure 5(a). Assuming that $\ln \mathrm{EDP}$ is normally distributed, then the probability of exceeding an EDP level y given $\mathbf{IM}=(\mathrm{im}_1, \mathrm{im}_2)$ is

$$G_{\mathrm{EDP}|\mathbf{IM}}(y|\mathbf{im})=1-\Phi\left(\frac{\ln y-(\hat{\beta}_0+\hat{\beta}_1 \ln \mathrm{im}_1+\hat{\beta}_2 \ln \mathrm{im}_2)}{\hat{\sigma}_e}\right) \tag{14}$$

where $\hat{\sigma}_e$ is estimated from the observed prediction errors, as in Equation (6), but now using Equation (13) to generate the predictions.

This method has the advantage of easily accommodating many **IM** parameters by simply adding additional terms to Equation (13) (e.g. $\beta_3 \ln \mathrm{IM}_3$). It also requires many fewer parameters to be estimated than with the other methods, typically allowing estimates to be obtained from fewer analyses. The model can also be generalized, as in the scalar case, to incorporate features such as non-constant variance or records that cause collapse. Because of these appealing features, this was the analysis method of choice for several investigations using vector-valued IMs [11, 17, 37].

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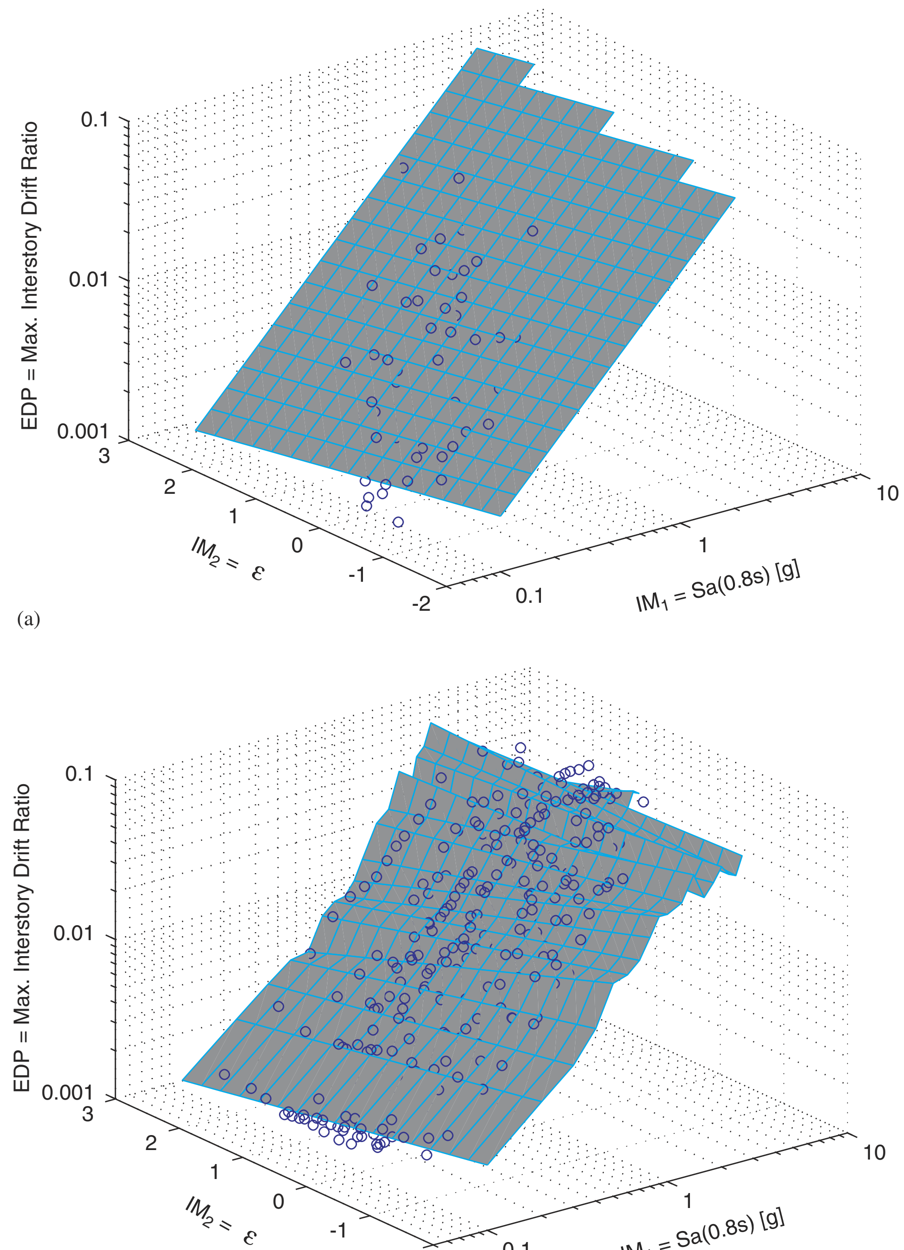

Figure 5. Estimates of mean $\ln \mathrm{EDP}$ as a function of $\mathrm{Sa}(0.8\,\mathrm{s})$ and ε: (a) estimate obtained using linear regression on a cloud of data and (b) estimates obtained using linear regression on ε at a series of $\mathrm{Sa}(0.8\,\mathrm{s})$ stripes.

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One difficulty with this method is that when the parameters IM_1 and IM_2 are highly correlated, it is difficult to separate their effects and estimate with confidence the coefficients $\hat{\beta}_1$ and $\hat{\beta}_2$ in Equation (13). This condition is referred to as *collinearity*. A somewhat related problem is that extrapolation becomes much more common when using regression in higher dimensions. Although marginally one might have predictor variables that span the **IM** range of interest, their joint distribution likely covers a lesser range than expected. This can be seen in Figure 5(a): although some data take values in the range $2<\varepsilon<3$, and some data take values in the range $0.1g<\mathrm{Sa}<0.3g$, there are no data with both $2<\varepsilon<3$ and $0.1g<\mathrm{Sa}<0.3g$. These extrapolations become more frequent, and harder to detect, as the number of dimensions increases.

A final challenge with this method involves modeling interactions between **IM** parameters. For example, if IM_1 is spectral acceleration at the first-mode period of the structure and IM_2 is spectral acceleration at a larger period (to account for nonlinear response), then IM_2 may have a small effect on response at low IM_1 levels when the structure is linear and a large effect on response at large IM_1 levels when the structure is very nonlinear. This would imply that the β_2 coefficient in Equation (13) should be non-constant. Additional interaction terms (e.g. $\beta_3 \ln \mathrm{IM}_1\mathrm{IM}_2$) can help, but it can be difficult to identify and incorporate an appropriate functional form. Thus, while this method has several appealing features, it should be verified that assumptions made regarding the functional form are appropriate for a given application.

Scale records to the target IM_1 *and regress on additional parameters*

With this method, records are scaled to the primary IM parameter before performing structural analysis, and regression analysis is used on the resulting response data to determine the effect of the remaining IM parameters. Logistic regression is used to compute the probability of collapse, and linear regression is used to model the non-collapse responses.

Rather than estimating the probability of collapse as simply the fraction of records at an IM_1 stripe that cause collapse, IM_2 is used to predict the probability of collapse using logistic regression [20]. With this procedure each record has a value of IM_2, which is used as the predictor variable. Using the indicator variable C to designate occurrence of collapse ($C=1$ if the record causes collapse and 0 otherwise), the following functional form is fitted

$$P(C|\mathrm{IM}_1=\mathrm{im}_1, \mathrm{IM}_2=\mathrm{im}_2)=\frac{\mathrm{e}^{\hat{\beta}_0+\hat{\beta}_1 \mathrm{im}_2}}{1+\mathrm{e}^{\hat{\beta}_0+\hat{\beta}_1 \mathrm{im}_2}} \tag{15}$$

where $\hat{\beta}_0$ and $\hat{\beta}_1$ are coefficients estimated from regression on a record set that has been scaled to $\mathrm{IM}_1=\mathrm{im}_1$. By repeating this regression for multiple IM_1 levels, one can obtain the probability of collapse as a function of both IM_1 and IM_2.

For the remaining non-collapse data at an IM_1 stripe, the relationship between IM_2 and the mean value of EDP is modelled by a linear function (after transformations). For example,

$$E[\ln \mathrm{EDP}|\mathrm{IM}_1=\mathrm{im}_1, \mathrm{IM}_2=\mathrm{im}_2]=\hat{\beta}_2+\hat{\beta}_3 \ln \mathrm{im}_2 \tag{16}$$

where $\hat{\beta}_2$ and $\hat{\beta}_3$ are constant coefficients estimated using linear regression, and the standard deviation of the regression residuals, $\hat{\sigma}_e$, is again estimated from the observed prediction errors. In contrast to the cloud method, these coefficients and the residual standard deviation are re-estimated at every IM_1 stripe. Assuming a Gaussian distribution for the residuals, the probability that EDP

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exceeds y, given $\mathrm{IM}_1 = \mathrm{im}_1$, $\mathrm{IM}_2 = \mathrm{im}_2$, and no collapse ($\overline{C}$) is

$$G_{\mathrm{EDP}|\mathbf{IM}}(y|\mathrm{im}_1, \mathrm{im}_2, \overline{C}) = 1 - \Phi\left(\frac{\ln y - (\hat{\beta}_2 + \hat{\beta}_3 \ln \mathrm{im}_2)}{\hat{\sigma}_e}\right) \tag{17}$$

By repeating this scaling and regression for multiple IM_1 levels, one can obtain the distribution of non-collapse responses as a function of both IM_1 and IM_2. The mean value of non-collapse responses as a function of Sa(0.8 s) and ε is shown in Figure 5(b).

The collapse and non-collapse cases are combined using the Total Probability Theorem to compute the conditional probability that EDP exceeds y

$$G_{\mathrm{EDP}|\mathbf{IM}}(y|\mathrm{im}_1, \mathrm{im}_2) = \hat{P}(C) + (1 - \hat{P}(C))\left(1 - \Phi\left(\frac{\ln y - (\hat{\beta}_2 + \hat{\beta}_3 \mathrm{im}_2)}{\hat{\sigma}_e}\right)\right) \tag{18}$$

where

$$\hat{P}(C) = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 \mathrm{im}_2}}{1 + e^{\hat{\beta}_0 + \hat{\beta}_1 \mathrm{im}_2}}$$

This approach can be used with IM vectors of more than two parameters by adding additional predictor variables to Equations (15) and (16).

This method has several desirable attributes. Scaling to stripes is already used for scalar IMs, so the record processing is not new. In fact, the current scalar parametric stripe case is merely a special case of this method, with the β_1 and β_3 coefficients omitted from Equation (18). Only two additional fitted parameters per stripe are used for estimation by moving to a vector, and the need for these extra parameters should be offset by the increased efficiency (i.e. decreased $\hat{\sigma}_e$) obtained from the vector. Collinearity and extrapolation problems that arise with the cloud method are reduced with this approach, and interactions between IM_1 and IM_2 are captured automatically because regression coefficients are re-estimated at each IM_1 stripe. This approach will, however, require a greater number of structural analyses than the cloud method.

Fit a conditional distribution for IM_1 *capacity to IDA results*

Here, the IM capacity method presented earlier is generalized to incorporate a vector **IM**. Although only a single **IM** parameter (IM_1) is scaled during IDA, it is still possible to determine the effects of other **IM** parameters. Here, IDA is first performed by varying IM_1 until the EDP value of interest is obtained. This provides the distribution of $\mathrm{IM}_{1\mathrm{Cap}}$ values as in the scalar case. Then IDA curves can be plotted along with IM_2, as shown in Figure 6(a). The point where each IDA curve first reaches the EDP level of interest (max interstorey drift ratio $= 0.1$ for this example) defines a set of **IM** capacity values. These points are plotted in Figure 6(b) for exceedance of EDP $= 0.1$ maximum interstorey drift ratio (corresponding to the tips of the IDA traces in Figure 6(a)). It is apparent in Figure 6(b) that in this example IM_2 can explain part of the variation of IM_1 capacity (i.e. the $\mathrm{IM}_{1\mathrm{Cap}}$ values tend to be larger for positive values of IM_2). In Figure 6, the conditional distribution of $\ln \mathrm{IM}_{1\mathrm{Cap}}$ appears to be linearly dependent upon IM_2, so linear regression can be used to find the conditional mean of $\ln \mathrm{IM}_{1\mathrm{Cap}}$ given IM_2:

$$E[\ln \mathrm{IM}_{1\mathrm{Cap}}|\mathrm{IM}_2 = \mathrm{im}_2] = \hat{\beta}_0 + \hat{\beta}_1 \mathrm{im}_2 \tag{19}$$

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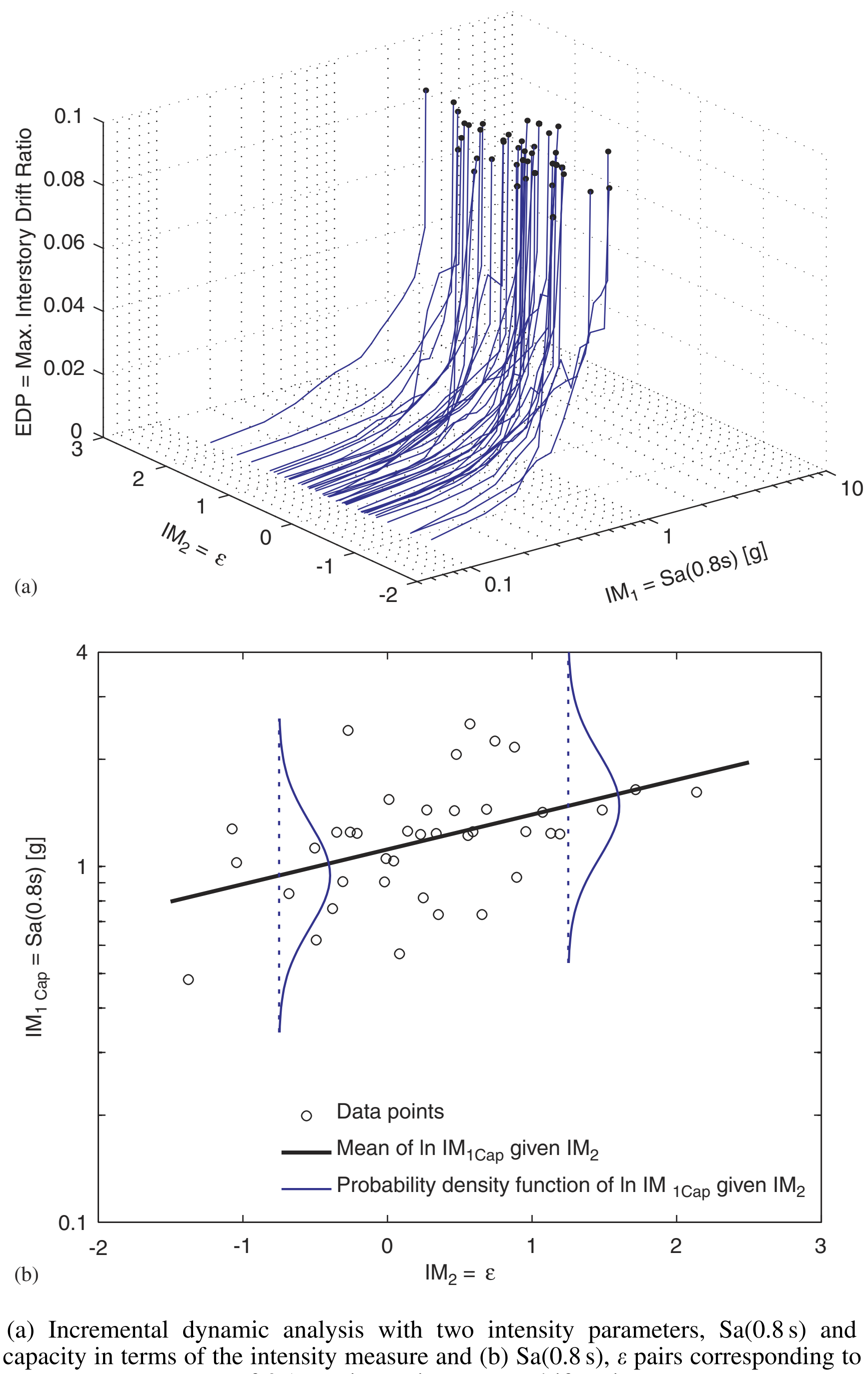

Figure 6. (a) Incremental dynamic analysis with two intensity parameters, Sa(0.8 s) and ε, used to determine capacity in terms of the intensity measure and (b) Sa(0.8 s), ε pairs corresponding to occurrence of 0.1 maximum interstorey drift ratio.

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where $\hat{\beta}_0$ and $\hat{\beta}_1$ are estimated from linear regression using, e.g. the data points from Figure 6(b). The conditional standard deviation of $\ln \mathrm{IM}_{1\mathrm{Cap}}$ given IM_2, denoted $\hat{\sigma}_{\mathrm{Cap}}$, can be estimated by computing the standard deviation of the regression residuals. If the conditional distribution of $\ln \mathrm{IM}_{1\mathrm{Cap}}$ is assumed to be Gaussian, then it can be computed as

$$P(\mathrm{IM}_{1\mathrm{Cap}} < \mathrm{im}_1 | \mathrm{EDP} = y, \mathrm{IM}_2 = \mathrm{im}_2) = \Phi\left(\frac{\ln \mathrm{im}_1 - \hat{\beta}_0 + \hat{\beta}_1 \mathrm{im}_2}{\hat{\sigma}_{\mathrm{Cap}}}\right) \tag{20}$$

The probability density function of this distribution is superimposed on Figure 6(b) for two different values of IM_2. Using Equation (20), the drift hazard can then be computed as

$$\lambda_{\mathrm{EDP}}(y) = \int_{\mathrm{im}_1} \int_{\mathrm{im}_2} P(\mathrm{IM}_{1\mathrm{Cap}} < \mathrm{im}_1 | \mathrm{EDP} = y, \mathrm{IM}_2 = \mathrm{im}_2) f_{\mathrm{IM}_2|\mathrm{IM}_1}(\mathrm{im}_2|\mathrm{im}_1) \times \left|\frac{\mathrm{d}\lambda_{\mathrm{IM}}(\mathrm{im}_1)}{\mathrm{dim}_1}\right| \mathrm{dim}_2\, \mathrm{dim}_1 \tag{21}$$

where the $\mathrm{IM}_{1\mathrm{Cap}}$ points were defined as the IM_1 level associated with occurrence of the EDP level y. This procedure can be extended to larger vectors by adding additional IM parameters to Equations (19)–(21). This approach has previously been used by Vamvatsikos and Cornell [15].

The number of analyses needed for this method will generally be greater than with the cloud method but less than with the stripes method. Interactions among the **IM** parameters are captured because the conditional capacity distribution (as a function of im_2) is re-estimated at each EDP level. Wise selection of IDA analysis points and interpolation is slightly more complicated than the procedure needed for the stripes method.

Scale records to specified IM levels and calculate an empirical distribution

With this method, records are scaled to target IM_1 levels and then bins of records having a specified range of IM_2 values are selected. Within each bin, an empirical CCDF is computed and used as $G_{\mathrm{EDP}|\mathbf{IM}}(y|\mathrm{im}_1, \mathrm{im}_2)$. This computation is the same as the scalar-IM empirical CCDF approach earlier, except that with the scalar IM, no distinction is made between records with differing IM_2 values. It can be shown that this method is equivalent to fitting a scalar-IM empirical CCDF after using the re-weighting procedure that will be discussed later. It is simpler to present this method in the context of re-weighting, so discussion will be reserved for that section.

Process records to match target values of all IM parameters

It may sometimes be possible to process a suite of records to match all parameters specified in the **IM** vector, while leaving other record properties random. The conditional distribution of EDP could then be estimated directly from the modified records. If the same suite of records is used for a range of **IM** values by repeatedly re-processing the records, then this can be thought of as a generalization of IDA, where there are now two or more scaling parameters rather than the single parameter used in standard IDA.

For example, when the **IM** consists of spectral acceleration plus a measure of spectral shape (e.g. [16]), one would scale the record to match the spectral acceleration value, and then use a spectrum-compatibilization scheme to match the spectral shape [38, 39]. Rather than completely

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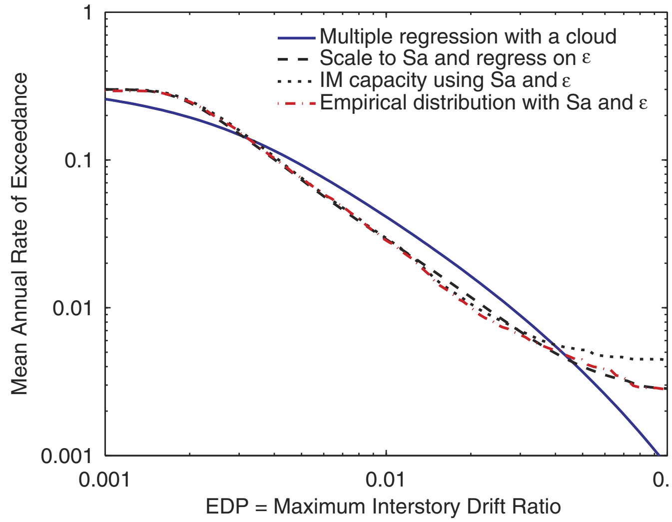

Figure 7. Comparison of drift hazard results using a vector-valued intensity measures consisting of $\mathrm{Sa}(T_1)$ and ε.

smoothing the spectrum, the spectrum roughness should be retained and only the general shape of the spectrum should be modified, in accordance with the target spectral shape parameter. This approach is sometimes referred to as a 'one-pass' compatibilization, because it requires only a single iteration of spectrum modification as opposed to the multiple iterations required to create a smooth spectrum [40]. This approach has been used to create ground motions representative of Eastern U.S. earthquakes from Western U.S. recordings [41], but it has not yet been used with vector **IM**s as described here.

Comparison of results from alternative methods

Mean values of EDP given Sa(0.8 s) and ε were shown for the cloud and stripe methods in Figure 5 and seen to match reasonably well, at least for values of Sa(0.8 s) less than 1 or $2g$. The EDP predictions can also be combined with the **IM** hazard using Equation (2) to predict the mean annual rate of exceeding various EDP levels. A comparison of this result for each of the methods is shown in Figure 7. As with the scalar IM approaches, the results are in good agreement except that the cloud method differs somewhat because the linear functional form is not appropriate over the large range of fitted IM values. Note that the curves have shifted down slightly with respect to the corresponding scalar IM curves from Figure 4. This suggests that the example vector IM has provided more information about structural response and thus affected the estimates of structural reliability, as examined in detail elsewhere [36].

HYBRID ESTIMATION METHODS

Although vector-valued IMs facilitate improved estimation of EDP distributions, they are somewhat more complicated to use than scalar IMs. Several methods are described here that can achieve the

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gains of a vector-valued IM while simplifying the analysis procedure. All of these methods mimic in some way the conditional distribution of $IM_2|IM_1$ that is expected at the site in the ground motion data set used for analysis. In other words, in the context of the vector-IM discussion above, the goal is to ensure that $\tilde{f}_{IM_2|IM_1}(im_2|im_1)$ in Equation (3) is equal to $f_{IM_2|IM_1}(im_2|im_1)$ in Equation (2), so that a scalar IM calculation produces the same result as the vector IM calculation.

Fit a distribution to a stripe of data after re-weighting to match a target distribution of $IM_2|IM_1$

Rather than explicitly predicting response as a function of IM_2, one could select ground motions with a range of IM_2values and then re-weight the data, after scaling to IM_1, so that the re-weighted data set has the proper distribution of $IM_2|IM_1$ at each IM_1 level [10, 11]. After discretizing IM_2 into a set of 'bins,' the weight for a record with an IM_2 value in bin j is

$$\text{weight}_j = \frac{f_{IM_2|IM_1}(im_{2,j}|im_1)}{n_j} \tag{22}$$

where $f_{IM_2|IM_1}(im_{2,j}|im_1)$ is the target probability that $IM_2 = im_2$ (in bin j), given $IM_1 = im_1$ (obtained from vector-valued PSHA, or disaggregation if $IM_2 = \varepsilon$), and n_j is the number of records in the record-set with IM_2 values falling in bin j. The weighted record-set will have a probability distribution equal to the target $f_{IM_2|IM_1}(im_2|im_1)$. The scalar stripe methods can then be applied to this weighted data set, and the procedure works as before.

It is necessary to have at least one record in every IM_2 bin considered, to avoid having a denominator of zero in Equation (22). Ensuring that there are records in each IM_2 bin requires either careful record selection or large bin sizes, but large bin sizes may mask the effect of the underlying IM parameter. In addition, some records may be assigned weights close to zero, meaning that data are essentially discarded for the purposes of estimation; this is undesirable, given the expense of obtaining response data. These drawbacks are of some concern when incorporating a second parameter, but are nearly insurmountable if the vector consists of more than two parameters. This is because the number of bins increases exponentially with the number of IM parameters, and thus the average number of records in each bin decreases very quickly to zero even with a large record set. This so-called *curse of dimensionality* [42, 43] affects all of the considered methods to some extent, but is especially problematic here. The method's ability to use the same set of records at all IM_1 levels is desirable in some situations, such as when one is performing IDA, but the approach can only be practically used with a two-parameter vector and large bin sizes (which is perhaps not very restrictive for applications, although it is a shortcoming for exploratory research).

Scale records to a specified level of IM_1, selecting records to match the desired distribution of secondary parameters

With this approach, ground motions are carefully selected to match the target $IM_2|IM_1$ distribution *at each* IM_1 *level*. This is done presently, with IM_1 as spectral acceleration and IM_2 as magnitude and/or distance [44]. The target conditional distribution is obtained from disaggregation of probabilistic seismic hazard analysis results, and ground motions with matching magnitude and distance values are selected and used for analysis. This method has also been applied in recent research projects with the ε parameter used here [35, 45]. Note that this is a special case of the

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re-weighting method described in the previous section, where each ground motion has an equal weight because they were carefully selected to match the target distribution.

The conditional distribution of $IM_2|IM_1$ often changes as the value of IM_1 changes (e.g. large positive ε values become more common as the spectral acceleration level increases), so ground motions must be re-selected at differing IM_1 levels. This adds an extra step in the record-selection process, and makes IDA methods more complicated because the records change as the IM level changes. The limited number of available ground motions may also provide a practical impediment to the adoption of this procedure. Once the selection is complete, however, analysis can be performed using simple scalar IM procedures.

Comparison of results from hybrid methods

EDP hazard curves are computed using the two hybrid methods and compared to basic scalar and vector methods in Figure 8. The structural model and record set used in previous examples are used here, with the exception of the special record-selection scheme, where records are re-selected at each Sa level to match the ε distribution obtained from disaggregation (record-selection details are given in [21]). All three of the methods that account for the effect of ε result in a lowering of the drift hazard curve. Although the agreement is not perfect, the methods are all capturing the effect of ε. Thus, the hybrid methods based on careful record selection or re-weighting can achieve similar results to the vector IM method that uses regression on ε. This result also shows that the scalar IM Sa(0.8 s) is biased, or ‘insufficient,’ at large levels of EDP (i.e. incorporating ε in the prediction changes the answer, suggesting that the scalar IM consisting of Sa alone does not provide complete information about the effect of the ground motions). Further evidence for this conclusion has been presented elsewhere [35, 36].

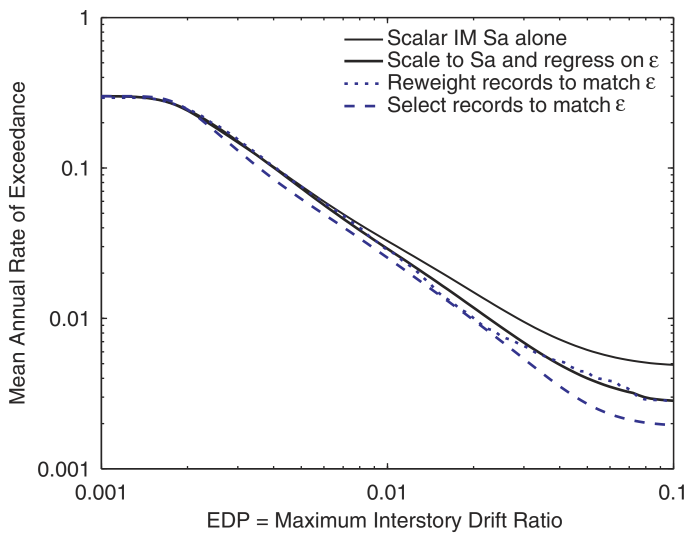

Figure 8. Comparison of drift hazard results using the scalar IM stripes procedure, vector-IM-based regression procedure, the hybrid special record selection procedure, and the hybrid re-weighting procedure.

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CONCLUSIONS

A variety of methods for obtaining the distribution of structural response (EDP) as a function of either a scalar or vector IM have been presented and discussed. The possibilities are listed

Table I. Summary of EDP|IM estimation methods.

Method	Pros	Cons	Ease of generalization to vector IMs
Linear regression on a cloud	Possible to avoid record scaling. Generally requires fewer records than other methods. Compatible with closed-form solutions for drift hazard	Standard assumptions (linear conditional mean and constant variance) are often not appropriate over large ranges of IM; relaxation of the approximations is possible but removes some of the advantages of this choice	Multiple regression on vector IMs is a simple extension. Collinearity among predictors may be a problem
Parametric distributions on IM stripes	Fewer parametric assumptions than with clouds. IM dependence can vary by IM level	Typically requires more dynamic analyses than cloud methods	Regression on stripes is less restrictive than regression on all IM parameters simultaneously, while also limiting the required number of dynamic analyses
Empirical CCDF on IM stripes	No parametric assumptions about response distribution	Requires a significant number of records for estimation	Curse of dimensionality is a significant problem. Application to vectors requires a significant number of records, carefully selected
IM capacity from IDAs	Requires fewer runs than IM stripes if IDAs are formed carefully	Requires a few extra steps to create IDAs and interpolate to compute capacity distributions	Generalization to vectors is straightforward
Hybrid method: Scale ground motions to IM_1, while re-weighting the data to match the target distribution of other IM parameters	Achieves the gains of vector IM methods. Records do not need to be re-selected at each IM_1 level	Weights for results from some records will be zero or nearly zero, essentially throwing away data. The curse of dimensionality is a problem for vectors with many parameters	Simpler processing than with explicit vector procedures
Hybrid method: Scale ground motions to IM_1, while carefully selecting the records to match other IM parameters	Achieves the gains of vector IM methods while requiring only the scalar-IM processing procedure	Requires careful record selection. Records likely need to be re-selected at differing IM_1 levels	Simpler processing than with explicit vector procedures

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in Table I, along with brief summaries of their positive and negative attributes. In general, one must make trade-offs between the accuracy of the estimates and the amount of data required for estimation.

For research activities investigating the effectiveness of a potential vector IM, an effective approach consists of scaling records to the primary IM parameter (e.g. spectral acceleration at the first-mode period of the structure), and then using regression analysis to measure the effect of the additional IM parameters. This method does not severely restrict the functional form of the mean response versus IM relationship, while also not requiring excessive numbers of structural analyses to be performed.

Cloud methods, which use multiple linear regression on all IM parameters simultaneously, require the fewest number of dynamic analyses and most effectively avoid the 'curse of dimensionality,' but the model often needs to be limited to a narrow range of IM levels where the linear prediction model is reasonable, or assumptions of linearity need to be relaxed. The smaller number of required dynamic analyses has led several researchers to adopt this approach for seismic reliability computations. At the other extreme, nonparametric methods such as empirical CDFs are potentially very accurate, but may require a prohibitive number of dynamic analyses, especially for IM vectors containing many parameters.

Once the relationship between structural response (EDP) and the IM parameters has been determined using one of the above methods, the results can be incorporated with ground motion hazard curves to obtain explicit estimates of seismic reliability. Computation of ground motion hazard for scalar IMs is well developed using probabilistic seismic hazard analysis (PSHA) tools. Vector ground motion hazard for some vector IMs (such as the one consisting of spectral acceleration and ε considered here) can be obtained from scalar hazard curves combined with standard deaggregation results. In other cases, such as for IMs consisting of spectral acceleration values at multiple periods, special vector-valued PSHA computations are needed. While vector-valued PSHA is not yet common, the needed approaches and software have been developed. As these tools become more widely available, the methods described above should provide analysts with a range of options for using vector-valued IMs to predict probabilistic structural response.

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REFERENCES

1. Bazzurro P, Cornell CA. Seismic hazard analysis of nonlinear structures I: methodology. *Journal of Structural Engineering* 1994; **120**:3320–3344.
2. Cornell CA, Krawinkler H. Progress and challenges in seismic performance assessment. *PEER Center News*, vol. 3, 2000. http://peer.berkeley.edu/news/2000spring/performance.html
3. Deierlein GG. Overview of a comprehensive framework for earthquake performance assessment. *PEER 2004/05*, In *International Workshop on Performance-Based Seismic Design Concepts and Implementation*, Bled, Slovenia, Fajfar P, Krawinkler H (eds), 2004; 12. http://peer.berkeley.edu/Products/PEERReports/reports-2004/reports04.html
4. Han SW, Wen YK. Method of reliability-based seismic design I: equivalent nonlinear systems. *Journal of Structural Engineering* 1997; **123**:256–263.

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5. Bazzurro P, Cornell CA, Shome N, Carballo JE. Three proposals for characterizing Mdof nonlinear seismic response. *Journal of Structural Engineering* 1998; **124**:1281–1289.
6. Jalayer F, Beck JL, Porter KA. Effects of ground motion uncertainty on predicting the response of an existing Rc frame structure. *Thirteenth World Conference on Earthquake Engineering*, Vancouver, Canada, 2004; 10.
7. Kramer SL. *Geotechnical Earthquake Engineering*. Prentice-Hall: Upper Saddle River, NJ, 1996; 653.
8. McGuire RK. *Seismic Hazard and Risk Analysis*. Earthquake Engineering Research Institute: Berkeley, 2004; 240.
9. Krawinkler H (ed.). In *Van Nuys Hotel Building Testbed Report*: *Exercising Seismic Performance Assessment*. Pacific Earthquake Engineering Research Center. University of California at Berkeley: Berkeley, CA, 2004.
10. Jalayer F. Direct probabilistic seismic analysis: implementing non-linear dynamic assessments. *Ph.D. Thesis*, Department of Civil and Environmental Engineering, Stanford University, Stanford, CA, 2003; 244. http://www.stanford.edu/group/rms/ (accessed 14/3/05).
11. Shome N, Cornell CA. Probabilistic seismic demand analysis of nonlinear structures. *RMS-35*, RMS Program, Stanford, CA, 1999; 320. http://www.stanford.edu/group/rms/
12. Bazzurro P, Cornell CA. Vector-valued probabilistic seismic hazard analysis. *Seventh U.S. National Conference on Earthquake Engineering*, Earthquake Engineering Research Institute, Boston, MA, 2002; 10.
13. Benjamin JR, Cornell CA. *Probability*, *Statistics*, *and Decision for Civil Engineers*. McGraw-Hill: New York, 1970; 684.
14. Luco N, Cornell CA. Structure-specific scalar intensity measures for near-source and ordinary earthquake ground motions. *Earthquake Spectra* 2007, in press.
15. Vamvatsikos D, Cornell CA. Developing efficient scalar and vector intensity measures for IDA capacity estimation by incorporating elastic spectral shape information. *Earthquake Engineering and Structural Dynamics* 2005; **34**:1573–1600. http://dx.doi.org/10.1002/eqe.496
16. Baker JW, Cornell CA. Choice of a vector of ground motion intensity measures for seismic demand hazard analysis. In *Proceedings of the 13th World Conference on Earthquake Engineering*, Vancouver, Canada, 2004; 15.
17. Luco N, Manuel L, Baldava S, Bazzurro P. Correlation of damage of steel moment-resisting frames to a vector-valued set of ground motion parameters. *Ninth International Conference on Structural Safety and Reliability*, Rome, Italy, 2005; 8.
18. Rice JA. *Mathematical Statistics and Data Analysis* (2nd edn). Duxbury Press: Belmont, CA, 1995; xx, 602, A49.
19. Lehmann EL, D'Abrera HJM. *Nonparametrics*: *Statistical Methods Based on Ranks* (Rev. 1st edn). Prentice-Hall: Upper Saddle River, NJ, 1998; 463.
20. Neter J, Kutner MH, Nachtsheim CJ, Wasserman W. *Applied Linear Statistical Models* (4th edn). McGraw-Hill: Boston, MA, 1996; 1408.
21. Baker JW, Cornell CA. Vector-valued ground motion intensity measures for probabilistic seismic demand analysis. *Report #150*, John A. Blume Earthquake Engineering Center, Stanford, CA, 2005; 321. http://blume.stanford.edu/Blume/Publications.htm
22. Cornell CA, Jalayer F, Hamburger RO, Foutch DA. Probabilistic basis for 2000 SAC federal emergency management agency steel moment frame guidelines. *Journal of Structural Engineering* 2002; **128**:526–533.
23. FEMA 350. *Recommended Seismic Design Criteria for New Steel Moment-Frame Buildings*. *SAC Joint Venture*. Prepared for the Federal Emergency Management Agency, Washington, DC, 2000.
24. FEMA 351. *Recommended Seismic Evaluation and Upgrade Criteria for Existing Welded Steel Moment-Frame Buildings*. *SAC Joint Venture*. Prepared for the Federal Emergency Management Agency, Washington, DC, 2000.
25. Aslani H. Probabilistic earthquake loss estimation and loss deaggregation in buildings. *Ph.D. Thesis*, Department of Civil and Environmental Engineering, Stanford University, Stanford, CA, 2005.
26. Mackie K, Stojadinovic B. Seismic demands for performance-based design of bridges. *PEER 2003-16*, Pacific Earthquake Engineering Research Center, University of California at Berkeley, Berkeley, CA, 2003; 150.
27. Shome N, Cornell CA. Structural seismic demand analysis: consideration of 'Collapse'. *Eighth ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability*, University of Notre Dame, South Bend, IN, 2007; 7.
28. Bazzurro P, Luco N. Do scaled and spectrum-matched near-source records produce biased nonlinear structural responses? *Proceedings of 8th National Conference on Earthquake Engineering*, San Francisco, CA, 2006.
29. Shome N, Cornell CA, Bazzurro P, Carballo JE. Earthquakes, records, and nonlinear responses. *Earthquake Spectra* 1998; **14**:469–500.

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30. Iervolino I, Cornell CA. Record selection for nonlinear seismic analysis of structures. *Earthquake Spectra* 2005; **21**:685–713. http://link.aip.org/link/?EQS/21/685/1
31. Deutsch CV, Journel AG. *Gslib Geostatistical Software Library and User's Guide* (Version 2.0. edn). Oxford University Press: New York, 1997; 369.
32. Vamvatsikos D, Cornell CA. Applied incremental dynamic analysis. *Earthquake Spectra* 2004; **20**:523–553.
33. Kennedy RP, Short SA, Merz KL, Tokarz FJ, Idriss IM, Power MS, Sadigh K. Engineering characterization of ground motion—task I: effects of characteristics of free-field motion on structural response. *NUREG/CR-3805*, U.S. Nuclear Regulatory Commission, Washington, DC, 1984.
34. Bazzurro P, Cornell CA. Seismic hazard analysis of nonlinear structures I: applications. *Journal of Structural Engineering* 1994; **120**:3345–3365.
35. Baker JW, Cornell CA. Spectral shape, epsilon and record selection. *Earthquake Engineering and Structural Dynamics* 2006; **35**:1077–1095.
36. Baker JW, Cornell CA. A vector-valued ground motion intensity measure consisting of spectral acceleration and epsilon. *Earthquake Engineering and Structural Dynamics* 2005; **34**:1193–1217.
37. Bazzurro P. *Probabilistic Seismic Demand Analysis*. Department of Civil and Environmental Engineering, Stanford University: Stanford, CA, 1998; 329. http://www.stanford.edu/group/rms/
38. Abrahamson NA. Nonstationary spectral matching program RSPMATCH. *PG&E Internal Report*, 1993.
39. Silva WJ. WES RASCAL code for synthesizing earthquake ground motions. *US Army Corps of Engineers State-of-the-Art for Assessing Earthquake Hazards in the United States Report 24*, *Paper S-73-1*. 1987.
40. McGuire RK, Silva WJ, Costantino CJ. Technical basis for revision of regulatory guidance on design ground motions hazard- and risk-consistent ground motion spectra guidelines. *NUREG/CR-6728*, Division of Engineering Technology Office of Nuclear Regulatory Research, U.S. Nuclear Regulatory Commission, Washington, DC, 2001.
41. Carballo JE, Cornell CA. Input to nonlinear structural analysis: modification of available accelerograms for different source and site characteristics. *Sixth U.S. National Conference on Earthquake Engineering*, Seattle, WA, 1998; 13.
42. Bellman RE. *Adaptive Control Processes*: *A Guided Tour*. Princeton University Press: Princeton, NJ, 1961; 255.
43. Hastie T, Tibshirani R, Friedman JH. *The Elements of Statistical Learning*: *Data Mining*, *Inference*, *and Prediction*. Springer: New York, 2001; 533.
44. Stewart JP, Chiou S-J, Bray JD, Graves RW, Somerville PG, Abrahamson NA. Ground motion evaluation procedures for performance-based design. *PEER 2001-09*, Pacific Earthquake Engineering Research Center, University of California at Berkeley, Berkeley, CA, 2001; 229.
45. Goulet C, Haselton C, Mitrani-Reiser J, Beck JL, Deierlein GG, Porter KA, Stewart JP. Performance of a code-conforming reinforced-concrete frame building—Part I: ground motion selection and structural collapse simulation. In *Proceedings of the 8th National Conference on Earthquake Engineering*, San Francisco, California, 2006; 10.

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