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Conditional Mean Spectrum: Tool for ground motion selection

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Abstract

A common goal of dynamic structural analysis is to predict the response of a structure subjected to ground motions having a specified spectral acceleration at a given period. This is important, for example, when coupling ground motion hazard curves from probabilistic seismic hazard analysis with results from dynamic structural analysis. The prediction is often obtained by selecting ground motions that match a target response spectrum, and using those ground motions as input to dynamic analysis. The commonly used Uniform Hazard Spectrum (UHS) is shown here to be an unsuitable target for this purpose, as it conservatively implies that large-amplitude spectral values will occur at all periods within a single ground motion. An alternative, termed a Conditional Mean Spectrum (CMS), is presented here. The CMS provides the expected (i.e., mean) response spectrum, conditioned on occurrence of a target spectral acceleration value at the period of interest. It is argued that this is the appropriate target response spectrum for the goal described above, and is thus a useful tool for selecting ground motions as input to dynamic analysis. The Conditional Mean Spectrum is described, its advantages relative to the UHS are explained, and practical guidelines for use in ground motion selection are presented. Recent work illustrating the impact of this change in target spectrum on resulting structural response is briefly summarized.
KEY WORDS: ground motions; record selection; uniform hazard spectrum; conditional mean spectrum; epsilon

Introduction

A common goal of dynamic structural analysis is to predict the response of a structure subjected to ground motions having a specified spectral acceleration (*Sa*) at a given period. This spectral acceleration is often large, as it may correspond to some low probability of exceedance such as 10% or 2% in 50 years. Conditioning on *Sa* at only one period is desirable, because probabilistic assessments benefit greatly from having a direct link to a ground motion hazard curve—for spectral acceleration at a single period—obtained from Probabilistic Seismic Hazard Analysis (e.g., Bazzurro and Cornell 1994; Cornell et al. 2002; Cornell and Krawinkler 2000). The structural response prediction is often obtained by selecting ground motions that match some corresponding target response spectrum, and using those ground motions as input to dynamic analysis.

If structural response is to be estimated by selecting ground motions to match a target response spectrum, one must find the "typical" response spectrum associated with the specified large-amplitude *Sa* value at a single period. This paper describes a procedure for computing this spectrum, and illustrates how it can be used for ground motion selection. The paper also discusses why the

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more-common Uniform Hazard Spectrum is not an appropriate target spectrum for this particular response prediction problem. The resulting target spectrum obtained using this approach (the "Conditional Mean Spectrum") maintains the probabilistic rigor of PSHA, so that consistency is achieved between the PSHA and the ground motion selection. This enables one to make quantitative statements about the probability of observing the structural response levels obtained from dynamic analyses that utilize this spectrum; in contrast, the UHS does not allow for such statements. In probabilistic engineering assessments, this rigor is a significant benefit that likely justifies the slight changes in the analysis approach relative to traditional ground motion selection approaches.

This spectrum calculation procedure has been proposed previously (Baker and Cornell 2006b), and several recent publications have studied the impact of this approach on structural response results obtained from dynamic analysis. Findings from that work are briefly summarized, but the focus of this paper is on providing further suggestions for using this new target spectrum as a ground motion selection tool, and discussing insights that have arisen from recent experience using this spectrum.

The Uniform Hazard Spectrum versus real ground motions

Figure 1a shows the U.S. Geological Survey Uniform Hazard Spectrum (UHS) with a 2% probability of exceedance in 50 years design spectrum for a site in Riverside, California (latitude/longitude = 33.979/-117.335). This UHS is approximately replicated by the design spectra in building codes. To illustrate the similarity, the MCE design spectrum for this site is also shown in Figure 1a, as computed using ASCE/SEI 7-05 guidelines (American Society of Civil Engineers 2005). This site is used for illustration because it has high amplitude design ground motions but does not meet the requirements for the code to apply a "deterministic cap," so the MCE spectrum is comparable to a 2% in 50 years probabilistic UHS.

This Uniform Hazard Spectrum is constructed by enveloping the spectral amplitudes at all periods that are exceeded with 2% probability in 50 years, as computed using probabilistic seismic hazard analysis (PSHA). PSHA also provides information about the earthquake events most likely to cause occurrence of the target spectral amplitude at a given period. Suppose we are analyzing a structure with a first-mode period of 1 second, and are thus interested in the 2% in 50 years Sa(1s) value of 0.89g seen in Figure 1a. Figure 2 shows the deaggregation distribution of magnitudes, distances, and ε's ("epsilons") that will cause the occurrence of Sa(1s)=0.89g at this site. (Figure 3 shows the same result but at periods of 0.2s and 2.0s, illustrating that UHS spectral amplitudes at these three periods are caused by somewhat differing earthquake events.) At the 1 second period shown in Figure 2, the mean causal magnitude (M) is 7.03, the mean causal distance (R) is 12.2 km and the mean causal ε is 2.02. The median predicted spectrum associated with an earthquake having magnitude 7.03 and distance 12.2 km is shown in Figure 1b (computed using Abrahamson and Silva 1997). The median Sa(1s) is clearly much smaller than the Sa(1s)=0.89g amplitude associated with this deaggregation; the difference can be quantified by the ε parameter. This parameter is defined as the number of standard deviations by which a given lnSa value differs from the mean predicted lnSa value for a given magnitude and distance. Mathematically, this is written

$$\varepsilon(T) = \frac{\ln Sa(T) - \mu_{\ln Sa}(M,R,T)}{\sigma_{\ln Sa}(T)} \tag{1}$$

where $\mu_{\ln Sa}(M,R,T)$ and $\sigma_{\ln Sa}(T)$ are the predicted mean and standard deviation, respectively, of lnSa at a given period, and $\ln Sa(T)$ is the log of the spectral acceleration of interest. The first two parameters are computed using ground motion models (also sometimes called attenuation models) (e.g., Abrahamson and Silva 1997). Note that $\varepsilon(T)$ is formulated in terms of lnSa values because Sa values are well represented by lognormal distributions; this formulation results in $\varepsilon(T)$ being a normal random variable with zero mean and unit standard deviation (also called a "standard" normal random variable). Because of this lognormal distribution, it can also be shown that the exponential of $\mu_{\ln Sa}(M,R,T)$ is the *median* value of (non-log) Sa. Thus when "median Sa" is used in calculations, that is because it corresponds to the underlying mean of ln*Sa*. The $\mu_{\ln Sa}(M, R, T)$ term in equation 1 is often a function of additional parameters such as site conditions and rupture mechanism, but those terms are omitted from the notation for brevity (for the computations here, additional required parameters can be approximately inferred from knowledge regarding site conditions and regional seismicity).

Coming back to the example, the mean ε(1s) of 2.02 indicates that the *Sa*(1s)=0.89g amplitude is caused by ground motions that are, on average, approximately two standard deviations larger than the median predicted ground motions from the causal earthquake event. This can be seen in Figure 1b, where the median + 2σ predicted spectrum is approximately equal to the *Sa*(1s)=0.89g amplitude from the UHS.

To illustrate that this ε variation is a real phenomenon, Figure 4 shows the response spectra from 20 real ground motions with approximately *M*=7 and *R*=12km (more precisely, $6.7 < M < 7.1$ and $5\,\text{km} < R < 21\,\text{km}$). The median of these spectra are close to the predicted median spectrum, but there is significant scatter in the spectra. One of the spectra, plotted using a heavier line, has an *Sa*(1s) approximately equal to the 0.89g of interest here, indicating that is has an ε(1s) value of approximately two. While this spectrum has a large amplitude at 1s, it is not equally large (relative to the median) at all periods. This illustrates one reason why a Uniform Hazard Spectrum (which is similar to the median + 2σ spectrum, and would be identical if the *M*=7.03 and *R*=12.2km earthquake was the only earthquake occurring at the site) is not representative of individual ground motion spectra: individual spectra are unlikely to be equally above-average at all periods. It is well-appreciated that a UHS envelopes contributions from multiple magnitude/distance contributors to hazard (Figure 3; Bommer et al. 2000; Naeim and Lew 1995; Reiter 1990), but enveloping over ε's can be an even more significant effect in many cases (Baker and Cornell 2005a; Baker and Cornell 2006b).

Given that the uniform hazard spectrum is thus not representative of the spectra from any individual ground motion, it will make an unsatisfactory ground motion selection target in many cases. In the following section, we will study more carefully the properties of real spectra, and use the results to formulate an alternative target spectrum.

Characterizing the response spectra of real ground motions

Consider the example response spectrum highlighted in Figure 4. It is shown again in Figure 5, along with the median spectrum prediction for ground motions having its particular magnitude and distance. (Note that the median spectra in Figures 3 and 4 differ slightly; Figure 4 shows the median for the target *M* and *R*, while Figure 5 shows the median for the *M* and *R* of the example ground motion, since ε is computed with respect to the latter *M* and *R*.) Recall from equation 1 that the ε value for a ground motion at a given period is defined as the number of standard deviations by which the log of the ground motion's spectral value differs from the mean log prediction. We see in Figure 5 that the example ground motion's spectrum is slightly more than two standard deviations larger than the median prediction at 1s (more precisely, the ground motion's ln*Sa* is two standard deviations larger than the mean ln*Sa* prediction); exact calculations show that ε(1s)=2.3. Similarly, the spectrum has ε(0.2s)=1.2 and ε(2s)=1.4, because it is 1.2 and 1.4 standard deviations larger than the median predication at 0.2 and 2 seconds, respectively. We can perform this computation for many ground motions, to see how their ε values probabilistically relate to each other at various periods.

Figure 6a illustrates this type of data, obtained from ground motions in the NGA database (Chiou et al. 2008). Each point in the figure represents the ε(1s) and ε(2s) values observed from a single ground motion. The ε(1s)=2.3 and ε(2s)=1.4 values are highlighted in the figure, to illustrate where the ground motion of Figure 5 is located in Figure 6a. There is a strong correlation between these ε(1s) and ε(2s) values (ρ = 0.75), but the two are not identical. Figure 6b shows similar data for ε(1s) and ε(0.2s), illustrating that ε's for those two periods show weaker correlation (ρ = 0.44) than the data in Figure 6a.

For the example site above, deaggregation showed that the target ε(1s) value was approximately equal to 2. The question is then, what are the associated ε values at other periods, given that we know ε(1s)=2? We can then use the data from Figure 6 to determine the distribution of ε(2s) associate with a "2ε" value at 1s. The distributions highlighted in Figure 6 show that when ε(1s)=2, ε(0.2s) and ε(2s) tend to be less than 2, but greater than 0 (the exact method for computing these distributions will be explained later).

To build a target spectrum from this information, we can use the expected (mean) value of ε at other periods, given that we know the value of the original ε at the period of interest. Probability calculations show that the expected ε value at any other period is equal to the original ε multiplied by the correlation coefficient between the two ε values. The empirical correlation coefficients from the data in Figure 6a and b are 0.75 and 0.44, respectively. The average ε(2s) is thus $0.75 \cdot \varepsilon$(1s) = 1.5, and the average ε(0.2s) is $0.44 \cdot \varepsilon$(1s) = 0.88. These *conditional mean* values of ε(2s) and ε(0.2s), given various values of ε(1s), are plotted in heavy lines in Figure 6; the lines in Figures 5a and 5b have slopes of 0.75 and 0.44, respectively.

We can use these ε(2s)=1.5 and ε(0.2s)=0.88 values to compute the associated spectral acceleration values at those two periods by solving equation 1 for $Sa(T)$, and can repeat the process at all periods to build a full response spectrum. Figure 7 shows this spectrum. It has a peak near 1s because the ε values are highly correlated at closely spaced periods, and decays towards the median spectrum (ε=0) at large and small periods as the correlation of the ε values with ε(1s) decreases. Reassuringly, this peaked spectrum roughly matches the response spectrum of the example ground motion that naturally had the M, R and Sa(1s) amplitude of interest. This new response spectrum is termed the "Conditional Mean Spectrum" (or CMS), as it consists of the *mean* values of the *spectrum* at all periods, *conditional* on an Sa value at a single period.

A simple procedure for computing the Conditional Mean Spectrum

The calculations involved in obtaining the Conditional Mean Spectrum are not difficult. To summarize the approach in an easily reproducible format, a step-by-step calculation procedure is presented in this section.

Step 1: Determine the target Sa at a given period, and the associated M, R and ε

To begin this computation, we identify a target Sa value at a period of interest. Let us denote the initial period of interest T^* (it is often equal to the first-mode period of the structure of interest, but it could be any other period of interest). In the example calculation above, T^* was 1s. It is also necessary to determine the magnitude, distance and $\varepsilon(T^*)$ values associated with the target $Sa(T^*)$. If the target $Sa(T^*)$ is obtained from PSHA, then the M, R and $\varepsilon(T^*)$ values can be taken as the mean M, R and $\varepsilon(T^*)$ from deaggregation (this information provided by the U.S. Geological Survey, as seen in Figure 2). In the case where one would like to perform this calculation for a scenario M, R and Sa, the associated ε would simply be the number of standard deviations by which the target Sa is larger than the median prediction given the M and R (often ε=1 in deterministic evaluations of this type, corresponding to the "median + 1σ Sa").

Step 2: Compute the mean and standard deviation of the response spectrum, given M and R

Next, we compute the mean and standard deviation of log spectral acceleration values at all periods, for the target M, R, etc.

$$\mu_{\ln Sa}(M,R,T) \tag{2}$$

$$\sigma_{\ln Sa}(T) \tag{3}$$

where $\mu_{\ln Sa}(M,R,T)$ and $\sigma_{\ln Sa}(T)$ are the predicted mean and standard deviation, respectively, of lnSa at period T, as defined previously in equation 1. These terms can be computed using existing ground motion models, and several online calculation tools exist to aid in obtaining these values (e.g., *http://www.opensha.org* and *http://peer.berkeley.edu/products/rep_nga_models.html*). For the calculations above, an example of this mean and standard deviation was shown graphically in Figure 5.

Step 3: Compute ε at other periods, given $\varepsilon(T^*)$

In this step we compute the "conditional mean" ε as illustrated in Figure 6, but for many periods. The conditional mean ε at other periods can be shown to equal $\varepsilon(T^*)$, multiplied by the correlation coefficient between the ε values at the two periods

$$\mu_{\varepsilon(T_i)|\varepsilon(T^*)} = \rho(T_i, T^*)\varepsilon(T^*) \tag{4}$$

where $\mu_{\varepsilon(T_i)|\varepsilon(T^*)}$ denotes the mean value of $\varepsilon(T_i)$, given $\varepsilon(T^*)$. Predictions of the required correlation coefficient, $\rho(T_i, T^*)$, have been pre-calculated in previous studies, so users of this procedure can obtain the needed correlations using a simple predictive equation. One prediction, valid for periods between 0.05 and 5 seconds, is

$$\rho(T_{\min}, T_{\max}) = 1 - \cos\left(\frac{\pi}{2} - \left(0.359 + 0.163 I_{(T_{\min}<0.189)} \ln\frac{T_{\min}}{0.189}\right)\ln\frac{T_{\max}}{T_{\min}}\right) \tag{5}$$

where $I_{(T_{\min}<0.189)}$ is an indicator function equal to 1 if $T_{\min} < 0.189$ s and equal to 0 otherwise, and where $T_{\min}$ and $T_{\max}$ denote the smaller and larger of the two periods of interest, respectively (Baker and Cornell 2006a). A more refined (but more complicated) correlation model, valid over the wider period range of 0.01 to 10 seconds, is also available (Baker and Jayaram 2008), but equation 5 is nearly equivalent if only periods between 0.05 and 5 seconds are of interest (the Baker and Jayaram model was used to produce the figures above, so that spectra could be computed at periods as short as 0.01s, but equation 5 is shown here because of its greater simplicity).

Step 4: compute Conditional Mean Spectrum

The CMS can now be computed using the mean and standard deviation from Step 2 and the conditional mean ε values from Step 3. Substituting the mean value of $\varepsilon(T_i)$ from equation 4 into equation 1 and solving for ln*Sa*(T) produces the corresponding conditional mean value of ln*Sa*(T_i), given ln*Sa*(T^*)

$$\mu_{\ln Sa(T_i)|\ln Sa(T^*)} = \mu_{\ln Sa}(M, R, T_i) + \rho(T_i, T^*)\varepsilon(T^*)\sigma_{\ln Sa}(T_i) \tag{6}$$

where $\mu_{\ln Sa}(M, R, T_i)$ and $\sigma_{\ln Sa}(T_i)$ were obtained using equations 2 and 3, $\rho(T_i, T^*)$ was obtained using equation 5, and M, R and $\varepsilon(T^*)$ were identified in Step 1. The exponential of these $\mu_{\ln Sa(T_i)|\ln Sa(T^*)}$ values gives the CMS, as plotted in Figure 7.

In conclusion, the Conditional Mean Spectrum calculation requires only existing ground motion models and PSHA results, plus two additional simple formulas (equations 5 and 6). The ground motion predictions from Step 2 are typically cumbersome to compute by hand, but they can easily be incorporated into a simple computer program to perform the complete calculation procedure. While this procedure is not as widely implemented as UHS calculations, it is simpler to compute a CMS than a UHS.

Ground motion selection

Once the CMS is computed, it can be used to select ground motions for use in dynamic analysis of structures. The CMS tells us the mean spectral shape associated with the *Sa*(T^*) target, so ground motions that match that target spectral shape can be treated as representative of ground motions that naturally have the target *Sa*(T^*) value.

To find ground motions matching a target CMS, one must first identify the period range over which the CMS should be matched. This period range would ideally include all periods to which the structural response is sensitive. The period range may include the periods of higher modes of vibration (e.g., typically in frame buildings, $T_2 \cong T_1 / 3$ and $T_3 \cong T_1 / 5$, where T_i is the period of the i^{th} mode of vibration) as well as longer periods that are seen to affect a nonlinear structure whose first mode period has effectively lengthened. A period range from $0.2T_1$ to $2T_1$ is often effective for mid-rise buildings. This $0.2T_1$ to $2T_1$ range is similar to the $0.2T_1$ to $1.5T_1$ range specified by ASCE 7-05, but statistical studies suggest that nonlinear buildings are often sensitive to response spectra at periods longer than $1.5T_1$ (Baker and Cornell 2008; Cordova et al. 2001; Haselton and Baker 2006; Vamvatsikos and Cornell 2005).

Measuring match with the target spectrum

Once a period range of interest has been identified, a library of ground motions can be examined to identify those that most closely match the target CMS. One effective criterion for determining the similarity between a ground motion and the CMS is the sum of squared errors (*SSE*) between the logarithms of the ground motion's spectrum and the target spectrum

$$SSE = \sum_{j=1}^{n} \left(\ln Sa(T_j) - \ln Sa_{CMS}(T_j) \right)^2 \tag{7}$$

where $\ln Sa(T_j)$ is the log spectral acceleration of the ground motion at period T_j, and $\ln Sa_{CMS}(T_j)$ is the log CMS value at period T_j from equation 6. The periods T_j should cover the period range identified in the previous section, and in the author's experience, 50 T_j values per order of magnitude of periods is sufficient to identify ground motions with a reasonably smooth match to the target spectrum. For example, if periods from $0.2T_1$ to $2T_1$ are considered, then the periods span one order of magnitude and thus at least 50 periods within this range should be considered in equation 7. The difference of the logarithms of *Sa* values is used in equation 7 because earlier calculations of the target spectrum use ln*Sa*, but if the sum of squared errors of (non-log) *Sa* values is used instead, there will not be a significant impact on the ground motions identified as providing the best match.

To select ground motions, equation 7 can be evaluated for each ground motion under consideration, and the ground motions with the smallest *SSE* values selected. This approach is more effective if we also allow for scaling of the ground motions. Scaling can be used to make the ground motion spectral amplitudes approximately equal the target amplitude, and then equation 7 can be used to identify which of the scaled ground motions most closely match the target. In this case, $Sa(T_j)$ in equation 7 would denote the spectral acceleration of the *scaled* ground motion at period T_j.

The scale factor for a given ground motion can be chosen in several ways. The simplest method is to scale each ground motion so that its $Sa(T^*)$ matches the target $Sa(T^*)$ from the CMS. In this case, the scale factor would simply be the ratio between the target $Sa(T^*)$ and the unscaled ground motion's $Sa(T^*)$

$$\text{scale factor} = \frac{Sa_{CMS}(T^*)}{Sa(T^*)} \tag{8}$$

This approach is simple, and produces ground motions whose $Sa(T^*)$ values exactly match the target value upon which all CMS calculations are based. An alternative approach is to scale each ground motion so that the average response spectrum over the periods of interest is equal to the average of the target spectrum over the same periods. In this case, a given ground motion's scale factor is

$$\text{scale factor} = \frac{\sum_{j=1}^{n} Sa_{CMS}(T_j)}{\sum_{j=1}^{n} Sa(T_j)} \tag{9}$$

Figure 8 shows example ground motions selected to match a target CMS, using these two scaling methods. Figures 7a and 7b show ground motions selected to match the CMS, after scaling using equations 8 and 9, respectively. Figure 8a shows spectra having a characteristic "pinch" at T^* because the scaling ensures that they are all equal at that point, but otherwise the spectra in the two sub-figures are comparable. Because the scaling using equation 8 is slightly simpler, produces ground motions exactly matching the target $Sa(T^*)$, and does not significantly reduce the match to the target spectrum at other periods, it is the recommended scaling procedure for use with this approach.

The basic premise of ground motion scaling is sometimes questioned, as it is a modification of ground motions with no obvious physical justification. Empirically, however, it has been observed that ground motions selected and scaled to match the CMS produce displacements in buildings that are comparable to displacements produced by unscaled ground motions, unlike ground motions scaled using some other common approaches (Baker and Cornell 2005b; Goulet et al. 2008; Luco and Bazzurro 2007). This suggests that the scaling procedures outlined here will likely not impact the resulting structural responses.

Extensions of the basic selection procedure

One potential modification to the above selection and scaling procedure is to weight mismatches of Sa values at certain periods more than others in the calculation of equation 7. This could be done if one knew that the structure was more sensitive to spectral values at certain periods, but it therefore requires more in-depth knowledge of the structure's behavior than is typically known prior to performing dynamic analysis. Further, unless the weights for the various periods vary dramatically, weighting typically has little impact on the ground motions selected for use. Given the small impact on the ground motions selected, and the additional information required, this modification is unlikely to be useful for most applications.

It is a simple matter to exclude ground motions that are deemed undesirable for other reasons (e.g., they have magnitudes or distances that are grossly different than the corresponding targets, very large required scale factors, or inappropriate spectral values at periods other than those considered explicitly in the matching procedure). One can either exclude such ground motions prior to computing matches using equation 7, or one can evaluate all available ground motions using equation 7 and then remove undesirable ground motions from the small set identified as closely matching the CMS. These secondary ground motion properties are often less important to structural response than the spectral values considered in equation 7, but they can easily be considered in this manner as long as they do not restrict the pool of potential ground motions so severely that the only remaining ground motions have a poor match to the CMS.

When selecting multi-component ground motions, one can perform this procedure by defining the Sa in the above equations as the geometric mean of the two horizontal components, and computing the target CMS and individual ground motion spectra using this geometric mean Sa. The correlations between ε values at multiple periods have been found to be identical for both single-component Sa values and geometric mean Sa values, so equation 6 is valid in either case (Baker and Jayaram 2008). Additionally, correlations between vertical and horizontal Sa values are available in Baker and Cornell (2006a), so it is also possible to compute conditional mean values of vertical Sa amplitudes, given some target horizontal $Sa(T^*)$, using the same procedure. Ground motions can then be selected based on the match between the ground motions' geometric mean Sa values and the target geometric mean CMS, as well as between the ground motion and target vertical Sa values if desired.

Choice of T^* for conditioning

The entire CMS procedure starts from a design Sa value at the specified period T^*, and the remaining spectrum is computed conditioned on that $Sa(T^*)$. This creates a potential challenge: the conditioning creates Sa values at other periods that are always less "extreme" than $Sa(T^*)$. If the structural response parameter of interest is driven primarily by excitation at a period other than T^*, ground motions selected to match a CMS conditioned at T^* may produce inappropriately low responses.

Typically, probabilistic performance-based assessments choose T^* as the first-mode period of the structure for predicting peak displacements of first-mode dominated structures (e.g., Bazzurro and Cornell 1994; Cornell et al. 2002), but this choice of T^* is not always appropriate. For example, floor accelerations and upper-story shear forces may be more sensitive to higher-mode excitation than to first-mode excitation.

If one is interested in multiple structural response parameters, driven by excitation at differing periods, or if one is unsure of the period of excitation most important to a particular structural response parameter, it may be useful to construct conditional mean spectra conditioned on *Sa* values at multiple periods. For each CMS, a separate set of ground motions would be selected and used for analysis. The resulting sets of analyses could be inspected to identify which T^* was most important, by identifying which corresponding CMS produced the largest values of a given structural response parameter. These response values associated with the most important T^* would then be used as design values. The design values for differing response parameters may thus not come from the same CMS. Figure 9 shows conditional mean spectra computed using three T^* values; used together to select multiple sets of ground motions, these spectra might serve as a replacement for a UHS over the period range covered by the T^* values. One other potential CMS that could be included in this approach is a CMS conditioned on exceedance of *Sa* values averaged over some period range; this computation is not significantly more complicated than the one above, and is described by Baker and Cornell (2006b).

There is an important implication underlying this multiple-spectrum approach: it is not possible to select a single set of ground motions that represents an equivalent hazard level for all periods, while also maintaining a spectral shape representative of spectra from real ground motions. As seen in Figure 4, the ground motion with the largest *Sa* at 1s is not the ground motion with the largest *Sa* at 0.2s or 2s. Different ground motions will be responsible for high amplitudes at varying periods, so it is helpful to have some information about the periods of interest if one would like to efficiently select ground motions using this approach. This poses obvious practical problems when one would like to select ground motions prior to having a structural design completed, or if one would like to use a fixed set of ground motions for analyzing several sets of structures. The Uniform Hazard Spectrum may be a desirable tool in those cases because it is invariant to the periods being considered, with the tradeoff that it is conservative in enveloping design *Sa* values at all periods. Analysts are thus faced with a tradeoff between the convenient but conservative results obtained using the UHS, or the elimination of conservatism at the expense of additional required analyses when using multiple Conditional Mean Spectra.

The motivation for using multiple Conditional Mean Spectra is similar to the motivation for more general load-combination rules in structural analysis. Using ground motions matched to the Uniform Hazard Spectrum, which considers peak spectral amplitudes at all periods simultaneously, is analogous to simultaneously applying peak wind loads, peak snow loads and peak live loads simultaneously. Using multiple Conditional Mean Spectra is analogous to considering each peak load type individually, while applying (relatively smaller) values of the other loads types that are likely to been seen at the same point in time. There are rigorous structural reliability justifications for most load combination rules, however, and a comparably rigorous derivation for CMS combinations is still in development.

Impact of the CMS on structural response

Before using the CMS as a target spectrum, it is important to consider its practical impact relative to the UHS or other targets. Several recent studies have investigated the extent to which this impact is important when trying to predict response of structures. The studies vary somewhat in the way that they treat the issue—by varying the shape of the target response spectrum directly, or studying it indirectly via $\varepsilon(T^*)$, which was seen above to indicate the resulting spectral shape. The studies suggest that the impact of using the CMS (instead of the UHS or other similar spectrum) varies depending upon the characteristics of the structure being analyzed, the seismicity of the region considered, and the probability level associated with the target $Sa(T^*)$.

The dynamic characteristics of a structure are important because they will affect the extent to which the structure is influenced by variations in spectral values at a range of periods. The peak responses of an elastic single-degree-of-freedom oscillator with period T^* will be identical whether subjected to UHS-matched ground motions or CMS-matched ground motions, provided the $Sa(T^*)$ values are equal in both cases (because by definition their response will be proportional to $Sa(T^*)$). On the other hand, nonlinear multi-degree of freedom systems may be sensitive to excitation at a wide range of periods and thus will be sensitive to the target response spectrum used for selecting ground motions. It has been empirically confirmed that ductile and higher-mode-sensitive structures are more sensitive to consideration of the CMS (Applied Technology Council 2008; Haselton 2006).

The seismicity of the region considered and the ground motion probability level of interest jointly affect the impact of the CMS. The important factor is the probability of *earthquake* occurrence relative to the probability level associated with the target *ground motion Sa*. If exceedance of the target $Sa(T^*)$ is roughly as probable as occurrence of the dominant earthquake (i.e., the earthquake M/R that contributes most significantly to exceedance of the target Sa), then the design $Sa(T^*)$ will be comparable to the median predicted Sa associated with the dominant earthquake. This means that $\varepsilon(T^*)$ will be approximately equal to zero, and equation 4 thus suggests that ε values at all periods will approximately equal zero; that is, the CMS will approximately equal the median spectrum, rather than having the "peak" at T^* seen in Figure 7. If, however, the probability level considered for design is much smaller than the probability of the dominant earthquake in the region, then by definition the design Sa is "rare" relative to the median Sa from the dominant earthquake, and the CMS will take the peaked shape of the example shown in Figure 7. More concretely, in high seismic regions such as coastal California, it is not unreasonable for the 2% in 50 year $Sa(T^*)$ level to be caused by earthquakes with a 20% probability of occurrence in 50 years. The factor-of-ten reduction in probability between the Sa exceedance and the earthquake occurrence means that the Sa amplitude has only a 10% probability of exceedance, given occurrence of the earthquake. A 10% probability of exceedance corresponds to an ε of approximately 1.3, because ε has a standard normal distribution. This highly seismic case is similar to the example above, where the mean ε was 2. Conversely, in low-seismicity regions such as parts of the Eastern United States, it is possible that the 2% in 50 year $Sa(T^*)$ level is caused by an earthquake with a 2% probability of occurrence in 50 years, and thus the CMS would not differ significantly from the median spectrum. Further, if the same earthquake event is the dominant contributor to hazard at all periods, then the UHS would also look like the median spectrum for this event. Similarly, if the target probability associated with the Sa level is increased, decreasing the target ground motion amplitude, then the UHS and CMS will approach each other. Most work to date has focused on the importance of the CMS in highly seismic regions and for low probability Sa levels; further work is needed to understand the importance of this issue in low-seismicity regions.

Because the CMS effect is more pronounced for rare ground motions, it is important to consider when predicting the safety of buildings against collapse (which is typically caused by very high amplitude ground motions). The ATC-63 project, which modeled the collapse safety of structures designed to modern building codes, found that accounting for the effect of the CMS increased the median spectral acceleration that a building could withstand prior to collapsing (the "median collapse capacity") by up to 60%, relative to analyses with ground motions having response spectra similar in shape to the UHS (Applied Technology Council 2008). Other researchers have found that varying the target spectral shape from one associated with $\varepsilon(T^*)=0$ to one associated with $\varepsilon(T^*)=2$ resulted in a 40% to 80% increase in median collapse capacity (Goulet et al. 2007; Haselton and Baker 2006; Liel 2008; Zareian 2006), depending upon the structure considered. Because the rate of occurrence of ground motions decreases rapidly as the amplitude of the ground motion increases, these increases in collapse capacity translated into an order-of-magnitude reduction in the predicted annual probability of collapse. When dealing with non-collapse responses, studies under similar conditions observed that neglecting this ε effect often results in an overestimation of mean structural response by 30% to 60% (Baker and Cornell 2006b; Goulet et al. 2008; Haselton et al. 2008). In all cases, consideration of either the CMS target or the target $\varepsilon(T^*)$ was found to be important.

Variability of structural response

While the proposed approach has been shown to produce accurate estimates of *mean* structural response given $Sa(T^*)$, the variability in response is also of interest in many assessments. For this purpose, the above procedure will require some modification. The problem with the above procedure is that the CMS is only a *mean* spectrum, and does not address the variability in the response spectrum for a given $Sa(T^*)$. This response spectrum variability will in turn affect observed variability in structural responses. As seen graphically in Figure 6, the variability in ε values at other periods is dependent on their correlation with $\varepsilon(T^*)$. Probability calculations show that the conditional standard deviation of ε at some period T_i is related to this correlation through the following equation

$$\sigma_{\varepsilon(T_i)|\varepsilon(T^*)} = \sqrt{1-\rho^2(T_i,T^*)} \tag{10}$$

where $\rho(T_i,T^*)$ is defined in equation 5. As an aside, the distributions superimposed on Figure 6 are normal distributions with mean values defined by equation 4 and standard deviations defined by equation 10; this conditional distribution results from the observation that the ε values have a bivariate normal distribution (Jayaram and Baker 2008).

Because $\varepsilon(T_i)$ is the only uncertain parameter in the prediction of the response spectrum (when conditioning on $Sa(T^*)$, M and R), we can write the standard deviation of $\ln Sa(T_i)$ as

$$\sigma_{\ln Sa(T_i)|\ln Sa(T^*)} = \sigma_{\ln Sa(T_i)}\sqrt{1-\rho^2(T_i,T^*)} \tag{11}$$

This conditional standard deviation is shown in Figure 10, along with the ground motions selected to match the CMS and shown in Figure 8a. If the spectra of the selected ground motions properly represented the target conditional standard deviation, approximately 1/3 of them would lie outside of the +/- σ lines shown on the plot (because the log spectral values are normally distributed), but almost no spectral values are observed outside of those lines in Figure 10. The variability in the selected ground motions has been artificially suppressed because equation 7 identified ground motions whose spectra each closely match the mean spectrum. While in principle one could select ground motions to match both the target mean and standard deviation, in practice there are two challenges. First, finding ground motions that match the mean spectrum can be accomplished by examining each ground motion individually, but the standard deviation of a set of ground motion spectra can only be computed by considering the complete set. Because it is often not feasible to evaluate every possible combination of ground motions from a large library, the search procedure becomes more complex. Second, in order to find an optimal set of ground motions, one must specify the relative importance of matching the mean versus matching the standard deviation of the target spectrum, but this is difficult to do in a defensible manner without further study to understand the effect of mismatch of the mean and standard deviation on resulting structural response estimates.

A second issue that in principle adds variability to the spectrum at other periods is that these predictions were made using only mean values of $M/R/\varepsilon$ from deaggregation, while Figure 2 illustrates that in general a variety of magnitudes and distances may contribute to exceedance of the $Sa(T^*)$ level of interest. The variability in causal $M/R/\varepsilon$ values will theoretically introduce additional variability into the response spectrum prediction, but test calculations suggest that this additional variability is likely insignificant in any realistic situation (Baker and Cornell 2005b, Appendix E). A related question with these inputs is whether modal (as opposed to mean) $M/R/\varepsilon$ values should be used to compute the CMS; modal values are often used for ground motion selection, and mean values may not even correspond to a physically realizable earthquake event (Bazzurro and Cornell 1999). The motivation for using mean values in this application is that we are not interested in these values directly, but only in their impact on the resulting response spectrum. A Taylor Series expansion can be used to show (e.g., Benjamin and Cornell 1970) that the mean value of a function of random inputs can be computed by evaluating the function using the *mean* inputs, so long as the function is approximately linear over the range of likely input values (where here the inputs are $M/R/\varepsilon$ and the function is the ground motion model that predicts *Sa*). The same property does not hold when using modal values of the inputs.

In the only study to date of the CMS variance and its impact on structural response, it was seen that ground motions selected to match only the mean spectrum (using the approach proposed here) produced the same mean peak structural displacements response as those selected to match both the mean and variability of the target spectrum. Matching only the mean, however, resulted in an underestimation of the standard deviation of peak displacements by 30% to 50% (Baker and Cornell 2005b, Chapter 6). If proper variability in response is desired, the approach proposed here thus requires some modification. At present, variability in the spectra of selected ground motions has only been obtained through ad hoc modifications of the above selection approach; future research should soon provide a rigorous and general solution.

While the problem of capturing structural response variability is not resolved here, the common alternative of matching ground motions to a target UHS also does not satisfactorily capture variability in structural response (and it is much less clear how one would do it with the UHS, given that there is no analogous conditional standard deviation of *Sa* values for that case). In addition, the (variability suppressing) method proposed here is actually desirable in cases where only the mean structural response is of interest, because suppression of response variability makes it possible to precisely determine mean response using a smaller number of ground motions.

Conclusions

An approach has been described that allows one to compute the expected response spectrum associated with a target spectral acceleration (*Sa*) value at a single period, using knowledge of the magnitude, distance and ε value that caused occurrence of that target *Sa*. For large-amplitude (ε>0) *Sa* levels, this spectrum has a peak at the period used for conditioning (T^*), and decays to relatively lower amplitudes at periods that differ greatly from the conditioning period. The result, termed a Conditional Mean Spectrum (CMS), can be used as a target spectrum for ground motion selection when performing dynamic analysis of structures. A step-by-step procedure was presented for computing this spectrum, and for selecting and scaling ground motions to match this spectrum. The level of effort required to implement this procedure is comparable to the effort required to obtain ground motions that match a Uniform Hazard Spectrum, and no significant new procedures are required beyond those needed to compute the Uniform Hazard Spectrum.

Several arguments were presented regarding why the CMS is a useful target for ground motion selection. The alternative Uniform Hazard Spectrum is significantly conservative for some purposes: the stated probability level associated with a UHS is the probability of exceeding *any single* spectral value, but the probability of simultaneously exceeding *all* spectral values from a UHS is much smaller (and is also unknown). The structural responses from ground motions matching the more probabilistically consistent CMS are thus significantly smaller than the responses from ground motions matching the UHS and having the same $Sa(T^*)$ level. Unlike results obtained using a UHS, ground motions selected and scaled to match the CMS produce structural responses comparable to unscaled ground motions that naturally have the target $Sa(T^*)$.

Some challenges still remain for implementing this approach, relating to implementation for structures sensitive to excitation at multiple periods and accurate quantification of variability in response. Work is in progress to more completely address those challenges, but recent experience with this approach suggests that even in its current form it is a useful tool with several advantages relative to the alternative UHS.

Notation

The following symbols are used in this paper:

ε = normalized residual from a ground motion model prediction

ρ = correlation coefficient

$\mu_{\ln Sa}$ = predicted (by a ground motion model) mean value of log spectral acceleration

$\sigma_{\ln Sa}$ = predicted (by a ground motion model) standard deviation of log spectral acceleration

M = earthquake magnitude

R = distance from earthquake source to the site of interest

Sa = spectral acceleration

T^* = primary period of interest for computing the CMS

T_i = i^{th} fundamental period of vibration of a structure

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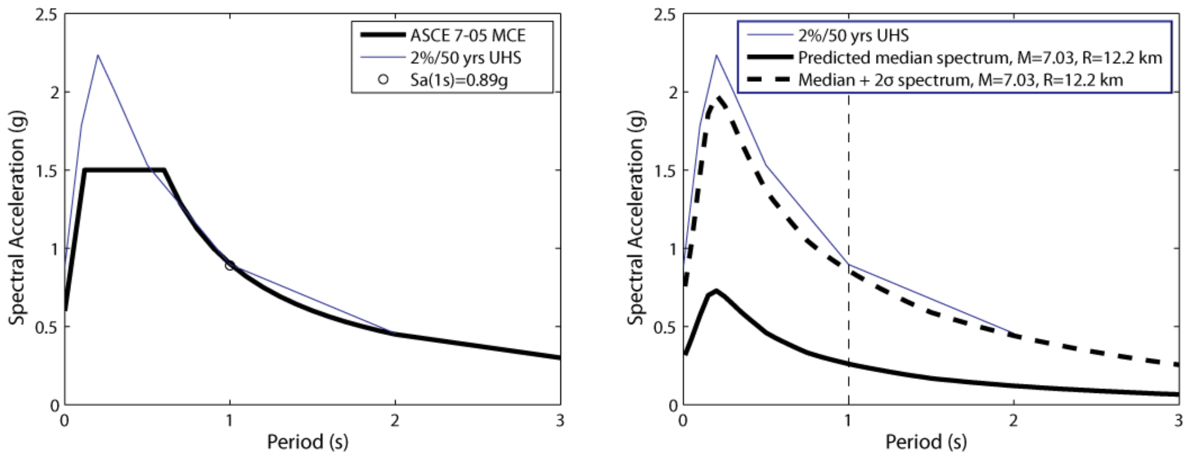


(a) (b)

Figure 1. (a) Building code MCE design spectrum and probabilistic Uniform Hazard Spectrum for a site in Riverside, California. (b) Probabilistic Uniform Hazard Spectrum for Riverside, along with the predicted median spectrum and median + 2σ spectrum associated with an M=7.03, R=12.2 km event (Abrahamson and Silva 1997).

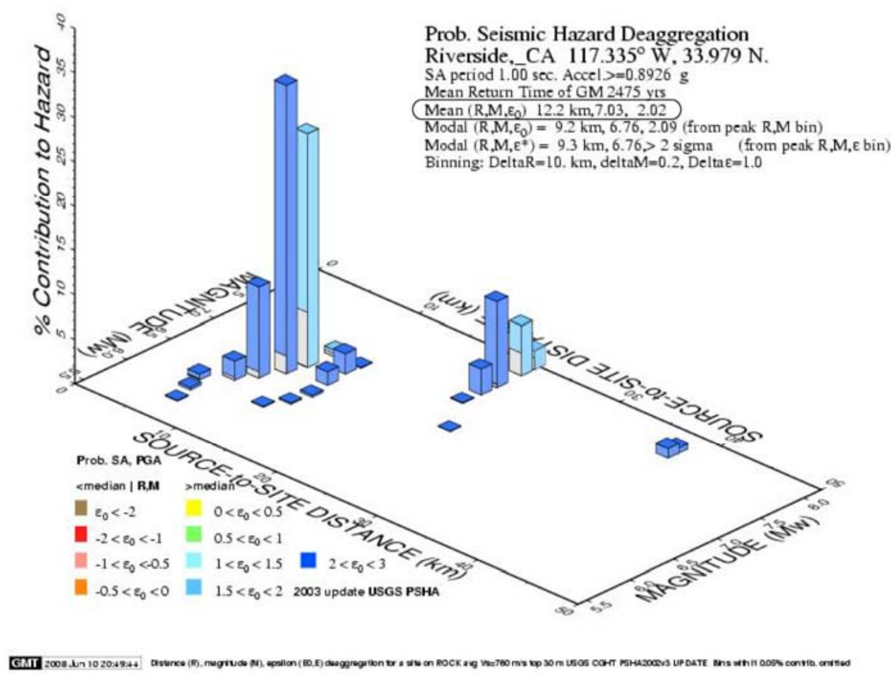
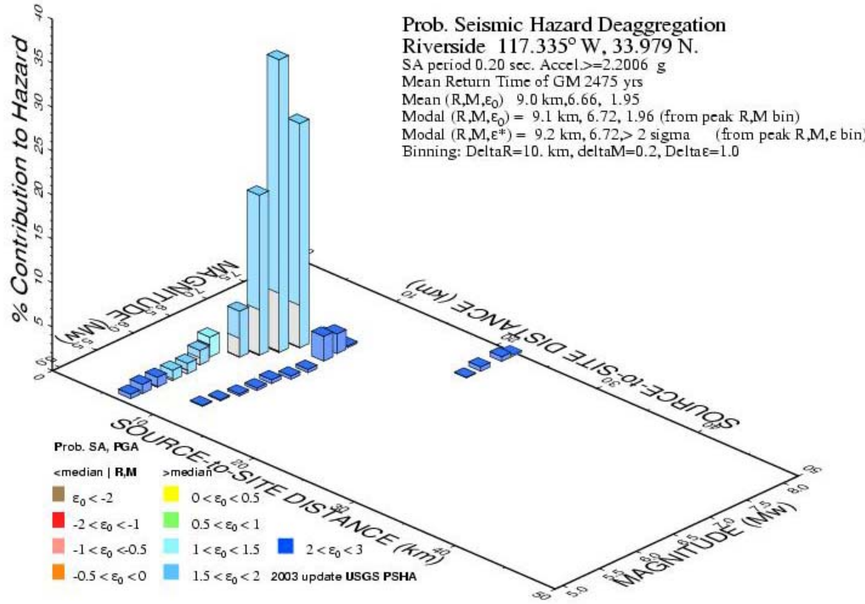
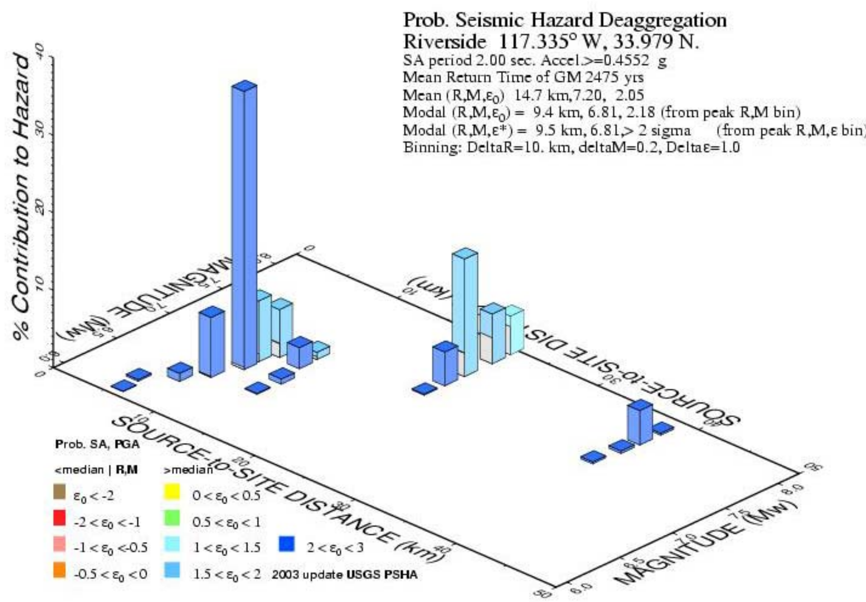


Figure 2. PSHA deaggregation for Riverside, given Sa(1s)>0.89g. (Figure from USGS Custom Mapping and Analysis Tools, http://earthquake.usgs.gov/research/hazmaps/interactive/, 2008. Emphasis on mean deaggregation values added by the author.)



(a)



(b)

Figure 3. PSHA deaggregation for Riverside, given exceedance of the Sa values with 2475 year return periods (a) at a period of 0.2s, (b) at a period of 2.0s. (Figure from USGS Custom Mapping and Analysis Tools, http://earthquake.usgs.gov/research/hazmaps/interactive/, 2008.)

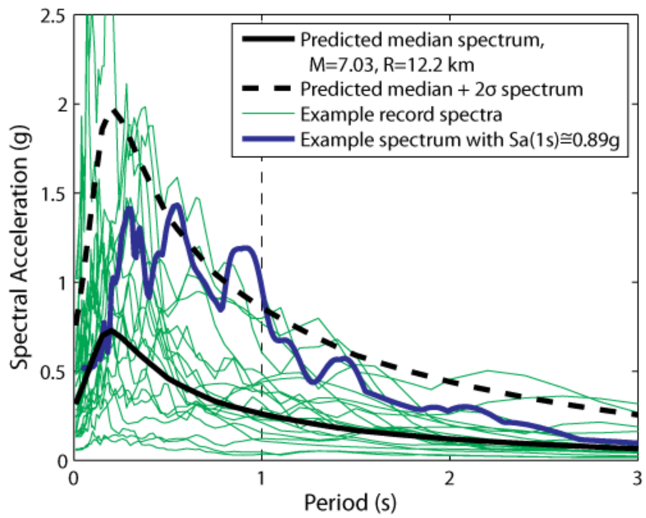


Figure 4. Response spectra from real ground motions having approximately magnitude = 7 and distance = 12 km. The example spectrum shown with a heavier line is the Castaic Old Ridge Route recording from the M = 6.7 Northridge earthquake, recorded on a Class C site with a closest distance to the fault rupture of 20 km.

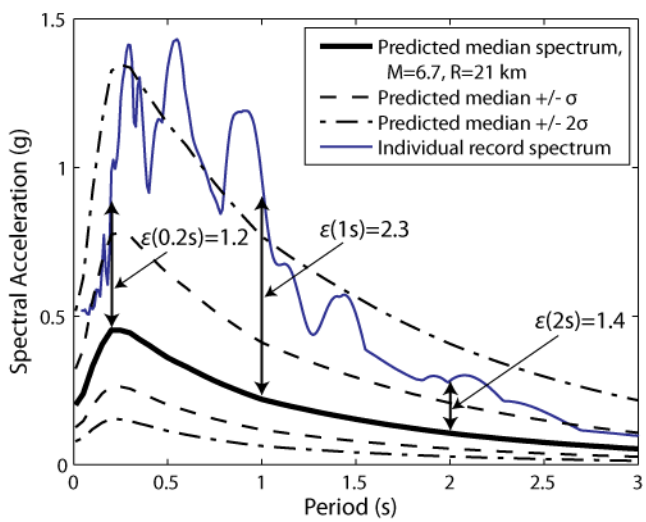


Figure 5. Response spectra from the example Castaic Old Ridge Route ground motion, used to illustrate calculation of ε values at three periods. Note that the +/- σ bands are not symmetric around the median because they are +/- σ values of lnSa, rather than (non-log) Sa.

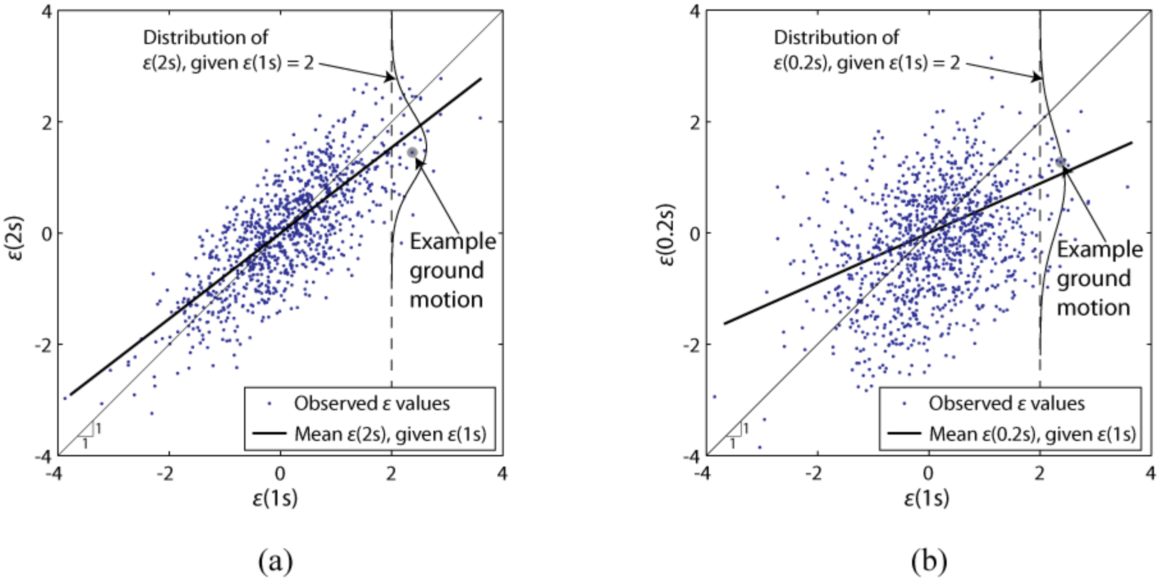


Figure 6. Scatter plots of ε values from a large suite of ground motions. The points associated with the ground motion in Figure 5 are highlighted. (a) ε(1s) versus ε(2s). (b) ε(1s) versus ε(0.2s).

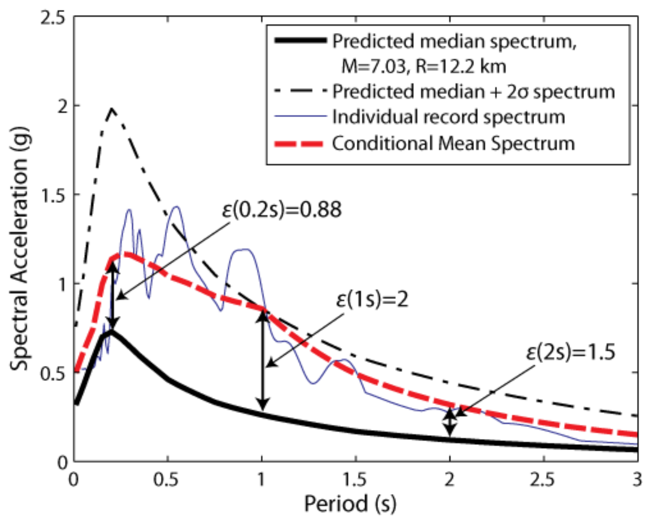
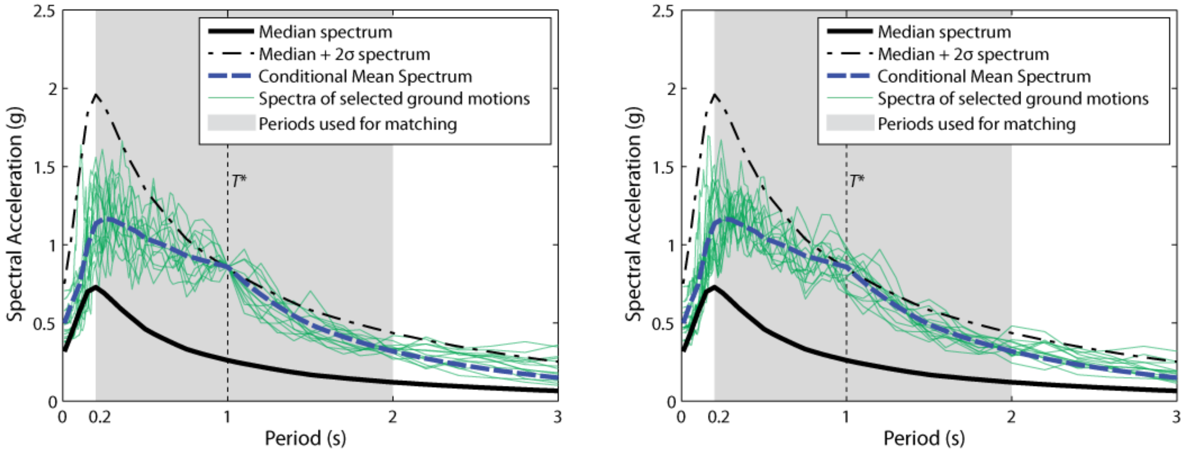


Figure 7. Conditional mean values of spectral acceleration at all periods, given *Sa*(1s), and the example Castaic Old Ridge Route ground motion.



(a) (b)

Figure 8. Conditional mean spectrum for the Riverside example site (with T^*=1s), and response spectra from ground motions selected to match this target spectrum. (a) Ground motions selected after scaling spectra to match the target $Sa(T^*)$. (b) Ground motions selected after scaling spectra to match the CMS over the entire period range considered.

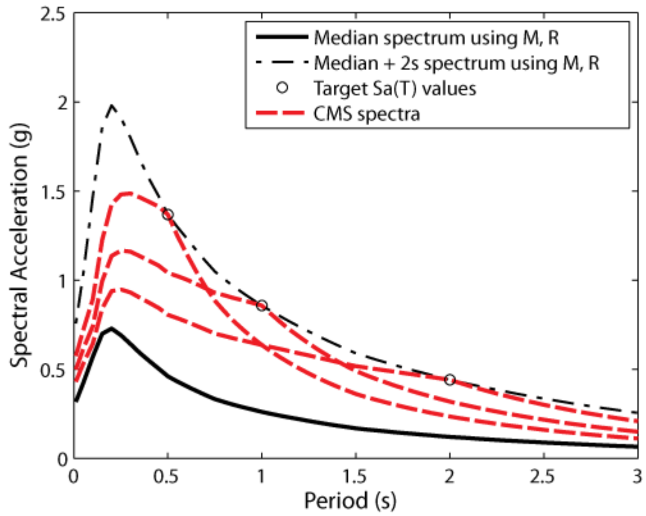


Figure 9. Conditional Mean Spectra, conditioned on Sa values at several periods, but having an equal probability of exceedance.

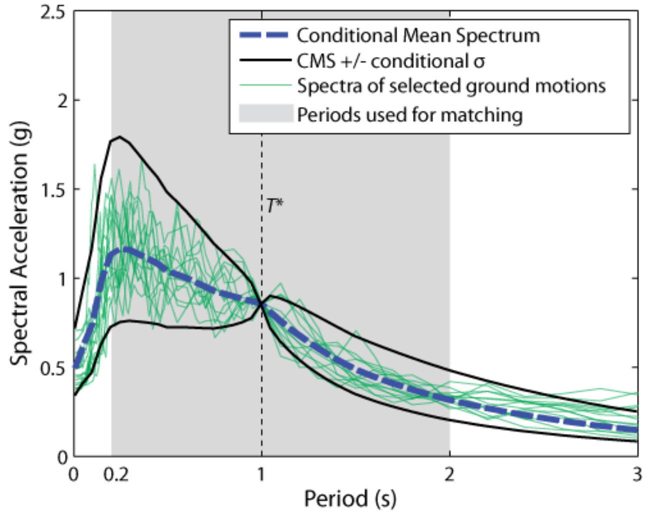


Figure 10. CMS (with T*=1s), the CMS +/- the conditional standard deviation from equation 11, and the response spectra from ground motions selected previously to match the CMS.