Which Spectral Acceleration Are You Using?

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Analysis of the seismic risk to a structure requires assessment of both the rate of occurrence of future earthquake ground motions (hazard) and the effect of these ground motions on the structure (response). These two pieces are often linked using an intensity measure such as spectral acceleration. However, earth scientists typically use the geometric mean of the spectral accelerations of the two horizontal components of ground motion as the intensity measure for *hazard* analysis, while structural engineers often use spectral acceleration of a single horizontal component as the intensity measure for *response* analysis. This inconsistency in definitions is typically not recognized when the two assessments are combined, resulting in unconservative conclusions about the seismic risk to the structure. The source and impact of the problem is examined in this paper, and several potential resolutions are proposed. This discussion is directly applicable to probabilistic analyses, but also has implications for deterministic seismic evaluations. [DOI: 10.1193/1.2191540]

INTRODUCTION

Calculation of the risk to a structure from future earthquakes requires assessment of both the probability of occurrence of future earthquakes (hazard) and the resulting response of the structure due to earthquakes (response). The analysis of hazard is typically performed by earth scientists (e.g., seismologists or geotechnical engineering scientists), while the analysis of response is typically performed by structural engineers. The results from these two specialists must then be combined, and this is often done by utilizing an intensity measure (IM) (Banon et al. 2001, Cornell et al. 2002, Moehle and Deierlein 2004). Earth scientists provide the probability of occurrence of varying levels of the IM (through hazard maps or site-specific analysis), and structural engineers estimate the effect of an earthquake with given levels of the IM (using dynamic analysis or by associating the IM with the forces or displacements applied in a static analysis).

Spectral acceleration, *Sa*, is the most commonly used intensity measure in practice today for analysis of buildings. This value represents the maximum acceleration that a ground motion will cause in a linear oscillator with a specified natural period and damping level. (In fact, the true measure is pseudospectral acceleration, which is equal to spectral displacement times the square of the natural frequency, but the difference is often negligible and the name is often shortened to simply "spectral acceleration.") But *Sa* is often defined differently by earth scientists and structural engineers. The difference originates from the fact that earthquake ground motions at a point occur in more than

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one direction. While structural engineers often use the *Sa* caused by a ground motion along a single axis in the horizontal plane, earth scientists often compute *Sa* for two perpendicular horizontal components of a ground motion, and then work with the geometric mean of the *Sa*'s of the two components. Both definitions of *Sa* are valid. However, the difference in definitions is often not recognized when the two pieces are linked, because both are called "spectral acceleration." Failure to use a common definition may introduce an error in the results.

In this paper, the differences in these two definitions are examined, along with the reasons why earth scientists and structural engineers choose their respective definitions. Examples of the use of these definitions are presented, along with the potential impact of failing to recognize the discrepancy. Several procedures for addressing the problem are examined, and the relative advantages and disadvantages of each are considered. Analysis is sometimes performed for each axis of a structure independently, and other times an entire 3-D structural model is analyzed at once. Both of these cases are considered, and consistent procedures for each are described. These procedures should be helpful for analysts performing seismic risk assessments of structures.

SPECTRAL ACCELERATION: TWO DEFINITIONS

TREATMENT OF SPECTRAL ACCELERATION BY EARTH SCIENTISTS

The earth scientist's concern with spectral acceleration is in predicting the distribution of spectral acceleration at a site, given an earthquake with a specified magnitude, distance, faulting style, local soil classification, etc. This prediction is made in the form of an attenuation model. Many attenuation models are empirically developed using analysis of recorded ground motions (see Abrahamson and Silva 1997, Boore et al. 1997, Campbell 1997, Sadigh et al. 1997, and Spudich et al. 1999, among many others). There is scatter in this recorded data (due to path effects, variation in stress drop, and other factors that are not captured by the attenuation model), which must be dealt with during development of the attenuation model.

The observed variability in spectral acceleration is well represented by a lognormal distribution (Abrahamson 1988, 2000). Thus, attenuation models work with the mean and standard deviation of the logarithm of *Sa*, which can be represented by a Gaussian distribution. The broad variability of the distribution hinders estimation of the mean value of $\ln Sa$ needed for the attenuation law. The log *Sa*'s of two perpendicular components of the ground motion are thus averaged, reducing the variance and allowing the mean value of $\ln Sa$ to be estimated with greater confidence. For example, in Figure 1 it is seen that arbitrary-component spectra vary more about the estimated mean than their geometric mean does.

The exponential of the mean of the logarithms of two numbers is termed the "geometric mean" because it is the square root of their product (this is also the same as the SRSS spectral values referred to in section 9.5.7.2.2 of ASCE 2002). For conciseness, we will refer to the geometric mean of spectral acceleration of two components as $Sa_{g.m.}$, and the spectral acceleration of an arbitrary component will be referred to as Sa_{arb}. The logarithms of these values will be referred to as $\ln Sa_{g.m.}$ and $\ln Sa_{arb}$, respectively. The

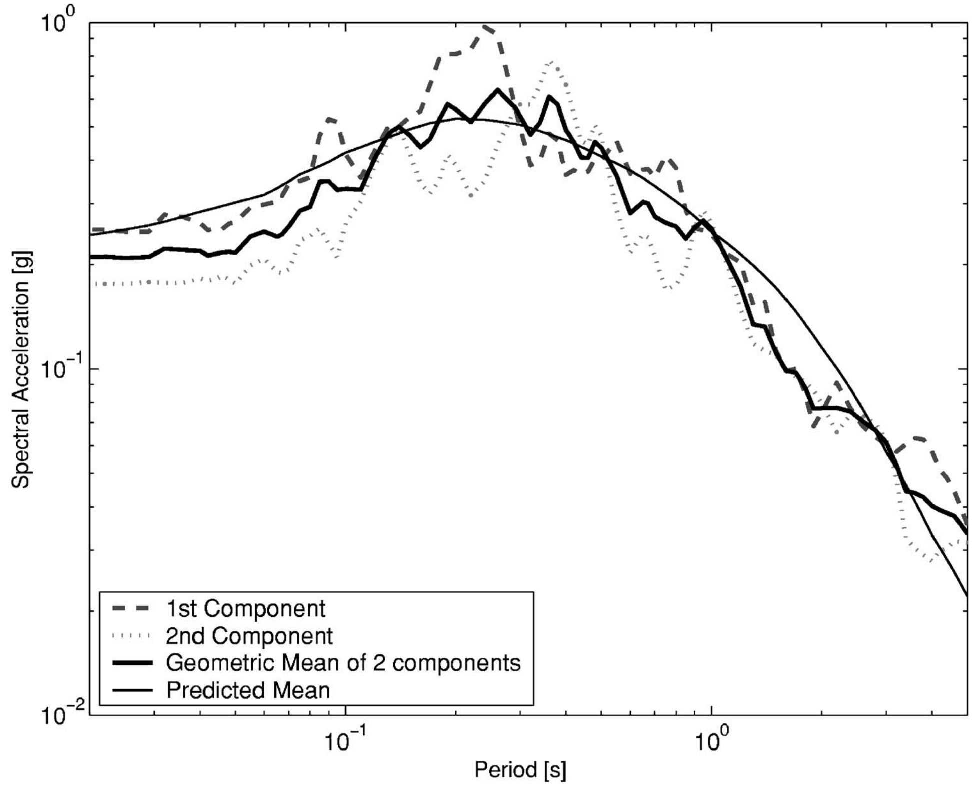

Figure 1. Response spectra from the magnitude 6.2 Chalfant Valley earthquake recorded at Bishop LADWP, 9.2 km from the fault rupture. Response spectra for the two horizontal components of the ground motion, the geometric mean of the response spectra, and the predicted mean for the given magnitude and distance using the prediction of Abrahamson and Silva (1997).

terms Sa and $\ln Sa$ will be used to refer to spectral acceleration and its logarithm, without specification as to which definition is used. And the standard deviation of $\ln Sa$ will be referred to as the "dispersion" of Sa, following common practice elsewhere. It is noted again that these values are functions of the period and damping level specified, but this is not stated explicitly in the notation because consideration of a particular period and damping are not needed for this discussion.

Attenuation models typically provide a predicted mean and standard deviation for the conditional random variable $\ln Sa_{g.m.}$, given an earthquake magnitude, distance, etc. These estimates for $\ln Sa_{g.m.}$ can be made directly from the data, because the averaging of the two components transformed the observed data into values of $Sa_{g.m.}$. For a given earthquake, the mean of the conditional random variable $\ln Sa_{arb}$ is equal to the mean of $\ln Sa_{g.m.}$. But the standard deviation of $\ln Sa_{arb}$ is greater than that of $\ln Sa_{g.m.}$ by a factor that could be as large as $\sqrt{2}$ if the two components were uncorrelated (because the standard deviation of the mean of 2 uncorrelated random variables with common standard deviation σ is equal to $\sigma/\sqrt{2}$). Calculating the standard deviation of $\ln Sa_{arb}$ thus takes an additional step of going back to the non-averaged data and examining the standard deviation there. Some researchers (e.g., Boore et al. 1997, Spudich et al. 1999) have taken this step, but many others have not because it was not recognized as important. However, the difference in standard deviations is in fact relevant for ground motion hazard analysis, as will be seen in the next section.

Deterministic Ground Motion Hazard Analysis

The effect of the standard deviation of $\ln Sa$ is easily seen in deterministic seismic hazard analysis. Often in a deterministic hazard analysis, a target spectral acceleration for a "Maximum Considered Event" is computed by specifying a scenario event (magnitude and distance), and then computing the value of $\ln Sa$, at a given period and damping level, that is one standard deviation greater than the mean prediction for that event (Reiter 1990, Anderson 1997). But the value of the standard deviation depends upon whether $Sa_{g.m.}$ or Sa_{arb} is being used as the IM. Because of its greater dispersion (logarithmic standard deviation), the target value of Sa_{arb} will thus be larger than that for $Sa_{g.m.}$. So the target spectral acceleration depends on the definition of Sa being used, even though both definitions have the same mean value of $\ln Sa$. For a "mean plus one sigma" ground motion, Sa_{arb} will thus be larger than that for $Sa_{g.m.}$ by a factor of $\exp(\sigma_{\ln Sa_{arb}} - \sigma_{\ln Sa_{g.m.}})$. For example, using the model of Boore et al. (1997, Boore 2005), this difference is exp(0.047) at a period of 0.8 seconds with 5% damping, implying that if Sa_{arb} is to be used as the IM, the target spectral acceleration would be about 5% larger than if $Sa_{g.m.}$ is used.

Another method used in deterministic hazard maps is to take as the hazard value 150% of the median spectral acceleration value for a characteristic event (ASCE 2002). One of the justifications for the 150% rule is that this will capture a reasonable fraction of the Sa values that could result from occurrence of this characteristic event. However, the fraction captured will vary based on which of the two definitions is used. Consider Sa at a period of 0.8 seconds. Per the Boore et al. (1997, Boore 2005) attenuation relationship used above, Sa_{arb} has a 23% chance of exceeding 150% of the median Sa value given the event, while $Sa_{g.m.}$ has a 21% chance of exceeding 150% of the median Sa given the event. Thus the level of conservatism resulting from this rule varies slightly depending on the Sa definition used. Spectral acceleration is merely a tool used to simplify the analysis problem; therefore the factor of safety should not vary based on the definition used. In principle the 150% rule for $Sa_{g.m.}$ should be a "156% rule" for Sa_{arb}, in order to provide the same level of conservatism.

Probabilistic Ground Motion Hazard Analysis

The variation in standard deviation is also seen in probabilistic seismic hazard analysis. In Figure 2, hazard curves for the two definitions of Sa are shown for a hypothetical site 8 kilometers away from a recurring magnitude 6.5 earthquake (this simple hazard environment is representative of sites near a single large fault). Again the attenuation model of Boore et al. (1997, Boore 2005) is used, because it provides dispersions for

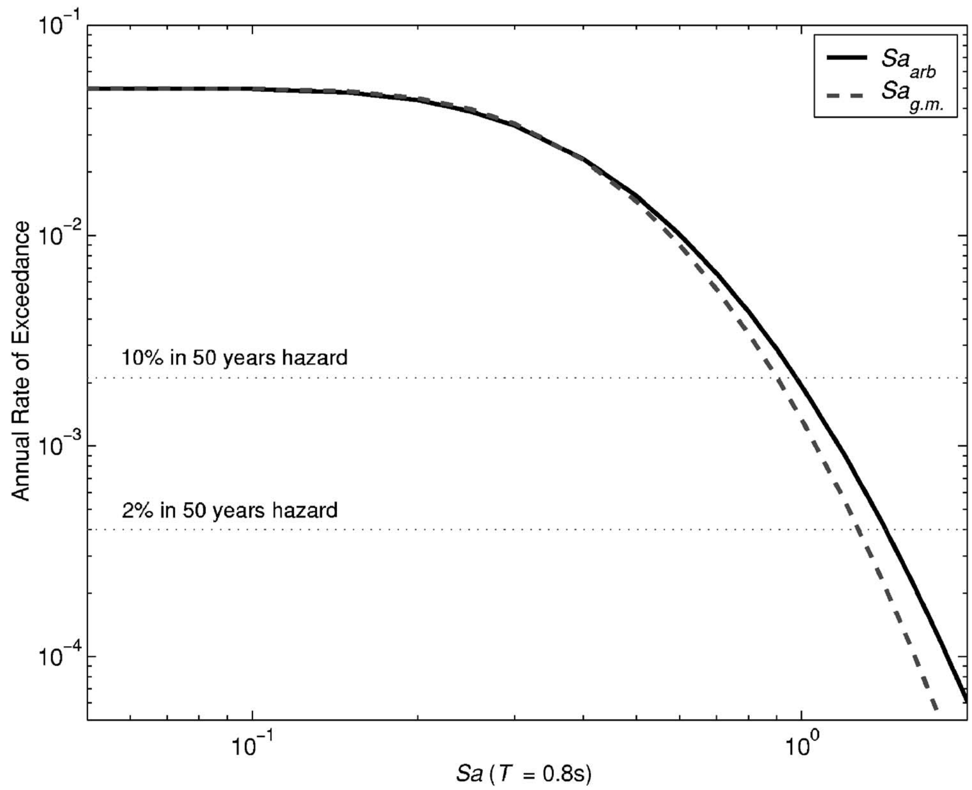

Figure 2. Ground motion hazard from a recurring magnitude 6.5 earthquake at a distance of 8 km, for Sa_{arb} and $Sa_{g.m.}$ at a period of 0.8 seconds with 5% damping.

both Sa_{arb} and $Sa_{g.m.}$. We see that the hazard curve for Sa_{arb} is greater than $Sa_{g.m.}$ due to the larger dispersion in Sa_{arb}. This is because ground motion hazard, especially at long return periods, is driven by ground motions that are larger than average. So even though Sa_{arb} and $Sa_{g.m.}$ have the same median value for each magnitude/distance considered, the larger-than-average values for Sa_{arb} will be greater than those for $Sa_{g.m.}$. This is the same effect as is seen in the deterministic hazard analysis case. (It is also true that the smaller-than-average values will be smaller for Sa_{arb} than for $Sa_{g.m.}$, but these events make a relatively smaller contribution to hazard, so the larger-than-average and smaller-than-average events are not offsetting.) For this site the Sa_{arb} with a 2% probability of exceedance in 50 years is 12% larger than the corresponding $Sa_{g.m.}$. The hazard for sites near multiple faults is simply a weighted sum of hazard curves similar to that shown in Figure 2, so we expect hazard curves at all sites to show this pattern.

The conclusion to be drawn from these examples is that the ground motion hazard is dependent on the definition of spectral acceleration. Even though $Sa_{g.m.}$ and Sa_{arb} have the same median value (for a given magnitude, distance, etc.), the difference in dispersion will cause a difference in ground motion hazard.

This result is important, because it means that ground motion hazard cannot be used interchangeably for both Sa_{arb} and $Sa_{g.m.}$. This brings to light a problem with the U.S. Geological Survey maps of spectral acceleration hazard (Frankel et al. 2002). The maps are produced using results from several attenuation models, some of which emphasize the dispersion in Sa_{arb}, and some of which provide only dispersion for $Sa_{g.m.}$. The U.S. Geological Survey has used the dispersions emphasized by the models' authors, resulting in a mix of both definitions being used. Thus the current maps are not strictly interpretable as the ground motion hazard for either Sa_{arb}, or $Sa_{g.m.}$. This will be addressed in future revisions to the maps, in light of the new recognition of the importance of this issue (Frankel 2004).

TREATMENT OF SPECTRAL ACCELERATION BY STRUCTURAL ENGINEERS

Structural engineers also utilize spectral acceleration as a basis for analysis of structural response. Let us first consider analysis of a single two-dimensional frame of a structure—a common situation in practice. In this case, only a single horizontal component of earthquake ground motion is needed for analysis. Therefore, spectral acceleration is computed only for the selected component at a period equal to the elastic first-mode period of the structure, and that is used as the intensity measure. In most cases, no distinction is made between the two components of a ground motion, so using a single component in this case is equivalent to using Sa_{arb} as the intensity measure. To compute $Sa_{g.m.}$ using both horizontal components of the ground motion, but then use only one of the components, the stronger or the weaker, for analysis would only introduce unnecessary scatter into the relationship between the IM and structural response.

This increased scatter resulting from prediction with $Sa_{g.m.}$ is illustrated in Figure 3. Prediction of response of a structure is made using both Sa_{arb} and $Sa_{g.m.}$. A model of an older seven-story reinforced concrete frame, described by Jalayer (2003), is used for analysis. Sixty unscaled recorded ground motions were used to perform nonlinear dynamic analysis. Figure 3 shows that the prediction of the mean log maximum interstory drift ratio, denoted as $\ln \theta$, is very similar for both intensity measures, but use of $Sa_{g.m.}$ as the IM results in increased dispersion of θ relative to the use of Sa_{arb}, as was anticipated above. The larger dispersion implies that there is greater uncertainty in the estimate of median response (i.e., if $Sa_{g.m.}$ is used as the IM, a greater number of analyses would need to be performed to achieve the same confidence in the mean $\ln \theta$). Thus the use of Sa_{arb} as the IM is preferable for the structural engineer in order to minimize the number of nonlinear dynamic analyses performed.

Many examples of the use of Sa as an intensity measure exist in the literature. For example, modal analysis (Chopra 2001), the SAC/FEMA methodology (SAC 2000a, b, c), and incremental dynamic analysis (Vamvatsikos and Cornell 2002) all use Sa as a predictor of structural response in some cases. In virtually every application of these procedures, Sa_{arb} (as opposed to $Sa_{g.m.}$) is used as the intensity measure for analysis of a single frame of a structure.

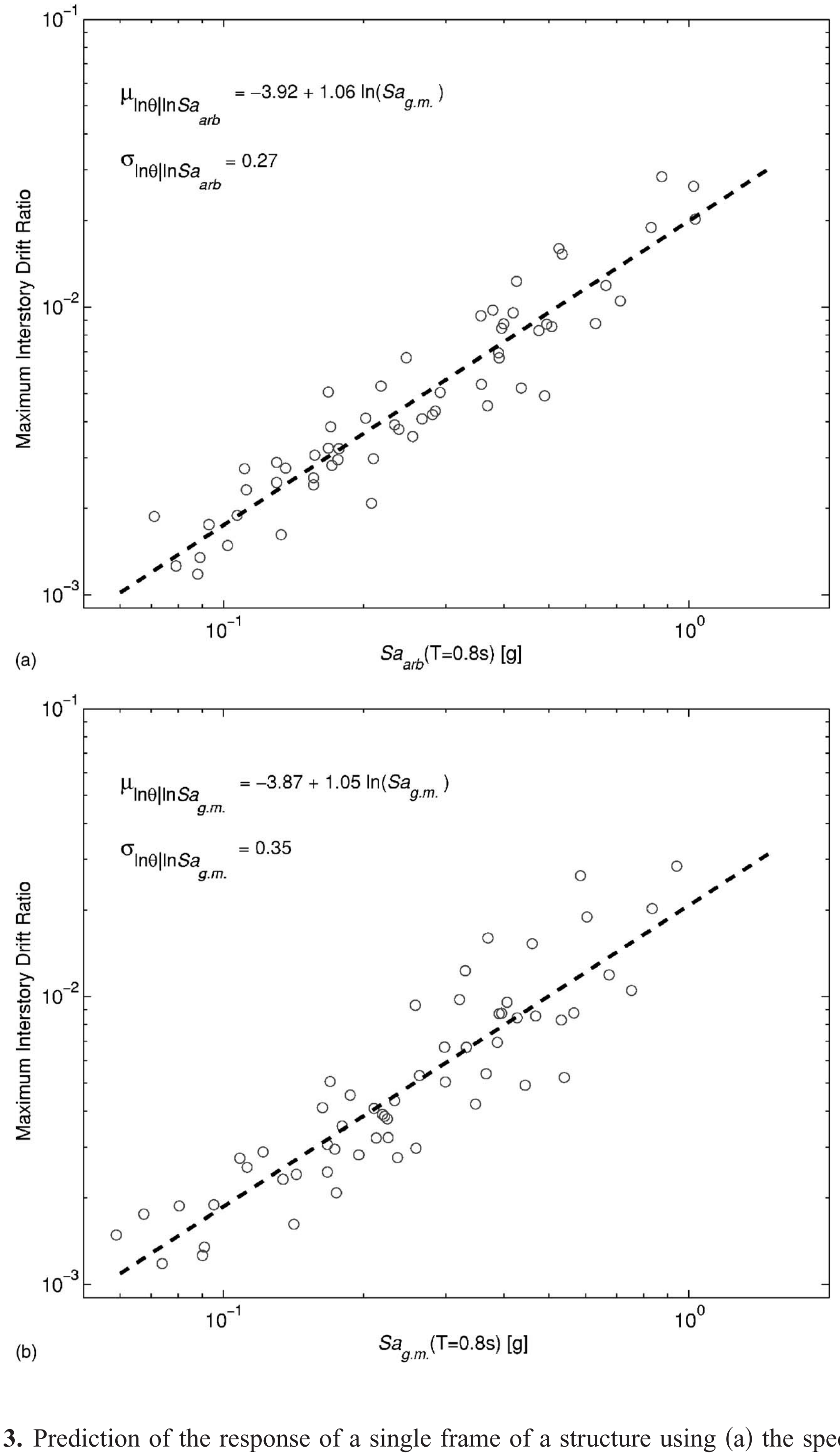

Figure 3. Prediction of the response of a single frame of a structure using (a) the spectral acceleration of the ground motion component used (Sa_{arb}), and (b) the spectral acceleration of the average of both components ($Sa_{g.m.}$).

INCORRECT INTEGRATION OF HAZARD AND RESPONSE

Spectral acceleration hazard is coupled with response analysis during performance-based analysis procedures (e.g., Cornell and Krawinkler 2000, Cornell et al. 2002). In past application of these procedures, frequently the ground motion hazard analysis has been unwittingly performed with $Sa_{g.m.}$ (to utilize existing attenuation models), and the response analysis has performed with Sa_{arb} (to minimize dispersion in the response prediction), resulting in the inconsistency discussed in this paper. Examples where the authors know only too intimately that a hazard analysis based on $Sa_{g.m.}$ was inadvertently coupled with a response analysis based on Sa_{arb} include Baker and Cornell (2004), Jalayer and Cornell (2003), Yun et al. (2002), and Shome and Cornell (1999). In the work of others it is seldom clear because the question was not discussed, but it can be suspected that if the hazard analysis were based on the USGS hazard maps or popular attenuation laws such as Abrahamson and Silva (1997) and Sadigh et al. (1997), where the only reported dispersion is that for $Sa_{g.m.}$, then an inconsistency is likely to exist.

In addition, other design and analysis procedures (e.g., ASCE 2000, 2002), utilize the U.S. Geological Survey maps of spectral acceleration hazard (Frankel et al. 2002) to attain target *Sa* values at which the performance of the structure should be checked. Although in this case there is no explicit statement of the reliability of a structure analyzed in this manner, *Sa* is still used as a link between hazard and response. Thus it is preferable to define *Sa* consistently in both the hazard and response. Possibilities for a consistent treatment of the problem are discussed in the following section.

VALID METHODS OF COMBINING HAZARD AND RESPONSE

For performance-based analysis procedures, it is necessary that the median and dispersion of response at a given IM level be consistent with the IM definition used for hazard analysis. This can be achieved in several ways, the choice of which may depend in part on the situation and available information. Three proposed solutions for use in analyzing a structure along a single axis are outlined below. The common characteristic of each method is that the IM used for hazard analysis and the IM used for response analysis are consistently defined.

1. CALCULATE THE GROUND MOTION HAZARD FOR Sa_{arb}

With this method, the structural response analysis described above is unchanged, but the ground motion hazard analysis is performed for the consistent intensity measure, Sa_{arb}. This allows for the estimation of structural response with less dispersion than when $Sa_{g.m.}$ is used (e.g., see Figure 3). And once more attenuation models are developed with dispersion for Sa_{arb}, the hazard analysis is no more difficult than hazard analysis for $Sa_{g.m.}$. The disadvantage is that few current attenuation models provide the dispersion for Sa_{arb}, meaning that many models, and the resulting hazard analysis, cannot be used without modification.

2. PREDICT STRUCTURAL RESPONSE USING $Sa_{g.m.}$

With this method, the ground motion hazard is unchanged from current $Sa_{g.m.}$-based practice. Instead, the response analysis is modified, using $Sa_{g.m.}$ as the IM rather than Sa_{arb}. To do this, one would compute the IM of a record as the geometric mean of Sa of the two components of the ground motion, even though only one component will be used for analysis. This method has the advantage of not requiring new attenuation laws or hazard analysis. Unfortunately, it will introduce additional dispersion into the response prediction, as was seen in Figure 3, and hence will be less efficient as an IM.

3. PERFORM HAZARD ANALYSIS WITH $Sa_{g.m.}$, RESPONSE ANALYSIS WITH Sa_{arb}, AND INFLATE THE RESPONSE DISPERSION

This method takes advantage of the fact that the median structural response for a given Sa level is the same whether $Sa_{g.m.}$ or Sa_{arb} is used. Only the dispersion is increased if $Sa_{g.m.}$ is used, as was seen in Figure 3. Structural response is thus performed using Sa_{arb} (as per standard practice) to obtain the median response. Then the dispersion in response is inflated to reflect that which would have been seen if $Sa_{g.m.}$ had been used as the intensity measure instead. An estimate of the amount by which the dispersion should be increased can be obtained using a first-order approximation. For this procedure, we assume the following model for the relationship between spectral acceleration and response:

$$\ln \theta = a + b \ln Sa_{arb} + \epsilon_{arb} \tag{1}$$

$$\ln \theta = a + b \ln Sa_{g.m.} + \epsilon_{g.m.} \tag{2}$$

where θ is the structural response value of interest, a and b are coefficients to be estimated from the data using least-squares regression, and ϵ's are zero-mean random variables (note that a and b have the same expected value in Equations 1 and 2, as is demonstrated in the Appendix—Equations 3 and 11—and as is supported by the empirical estimates in Figure 3). The model is seen to fit well in Figure 3, as in many other cases (at least locally). We are interested in estimating the standard deviation of $\ln \theta$ given $Sa_{g.m.}$, in the case where we know only the standard deviation of $\ln \theta$ given $\ln Sa_{arb}$. From Equation 13, we can find the ratio of the two conditional standard deviations. For the example data set here, $\rho_{\ln Sa_x, \ln Sa_y} = 0.797$ and $\rho_{\ln \theta_x, \ln Sa_x} = 0.942$, implying a ratio between standard deviations of 1.34. Thus the predicted conditional standard deviation for the example problem would be $1.34^{*}0.27 = 0.36$, approximately matching the standard deviation in 2b (0.35).

The SAC procedure (SAC 2000a, b, c) uses the model of structural response adopted in Equation 1, but with $b=1$. So an analysis using the SAC procedure would be a natural candidate for this method. One would simply perform analysis using Sa_{arb} as before, but inflate the dispersion in structural response using Equation 13 before continuing with the SAC methodology.

In the short term, this third method is attractive because it leaves existing hazard and response procedures unmodified, and instead makes a correction before the two analyses

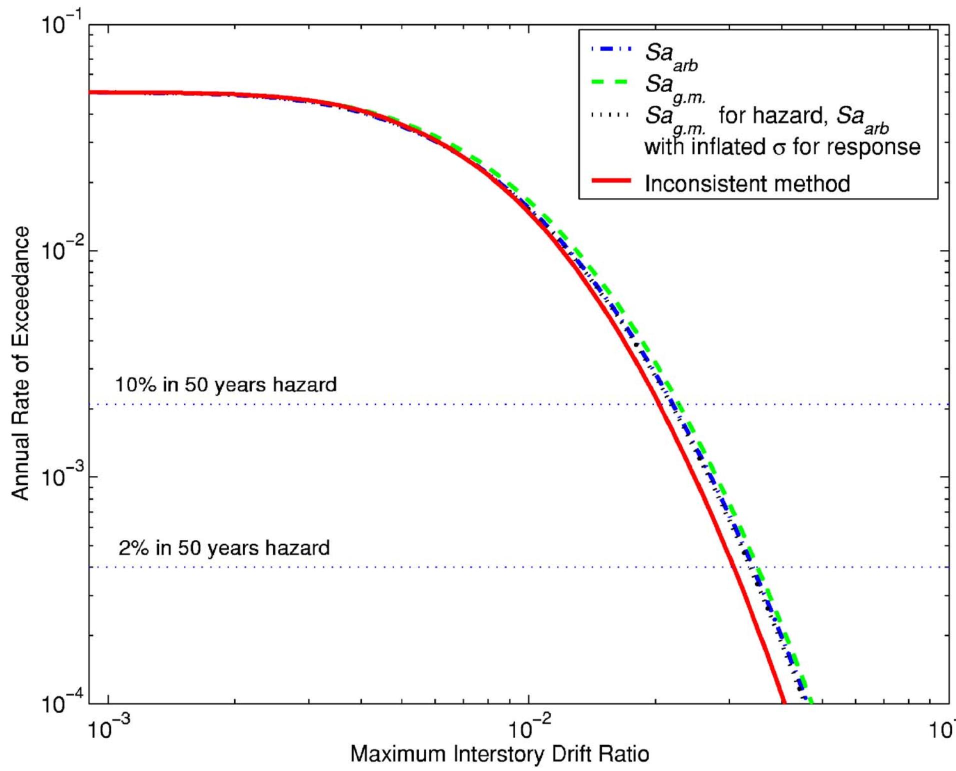

Figure 4. Drift hazard as computed using the three methods proposed above and the inconsistent method.

are combined. However, in the long term one of the two more direct methods using a consistent IM for both hazard and response would be more expeditious.

RESULTS FROM THE PROPOSED METHODS

All three of the methods described should result in the same answer for the probability of exceedance of a given limit state in the structure, aside from the inherent variability in the answer resulting from the statistically uncertain estimates of hazard and response (Baker and Cornell 2003). Additional methods can also be conceived using alternative intensity measures, but the above methods are expected to be the simplest and most similar to current practices.

To illustrate the results of the proposed methods, a drift hazard analysis is performed using the three proposed methods and the previous inconsistent method. This analysis combines the ground motion hazard from Figure 2 with the structural response analysis from Figure 3 to determine the rate of exceeding a given response level in the structure. The procedure for computing this drift hazard curve is described by Bazzurro et al. (1998). The results are displayed in Figure 4. It is seen that the three proposed methods produce comparable results, while the inconsistent method results are unconservative (e.g., the drift level exceeded with a 2% probability in 50 years is underestimated by approximately 10%). While the magnitude of the error is not overwhelmingly large, it nonetheless represents an easily correctible systematic flaw in the procedure.

In many applications today, the drift hazard curve is not computed. Rather, the ground motion hazard is used to specify a target spectral acceleration to use in analyzing structural response (i.e., the spectral acceleration associated with a 2% probability of exceedance in 50 years). An inconsistent approach will cause a comparable bias in these calculations as well.

ANALYSIS OF 3-D STRUCTURAL MODELS: COMBINING HAZARD AND RESPONSE

When analyzing a 3-D structural model, both horizontal components of the ground motion are used, so the above procedures using a single component are not necessarily applicable. In this case, as before, the concern is that the intensity measures used in the hazard analysis and response analysis should be consistent. Fortunately, the preferred method for analysis in this case is also a method that is apparently often used in practice. Several potential procedures are discussed here:

1. USE $Sa_{g.m.}$ AS THE INTENSITY MEASURE

In this case, ground motion hazard analysis is performed for $Sa_{g.m.}$, as is standard practice today. The intensity measure used for structural response is also $Sa_{g.m.}$, computed for the two components of ground motion used in the analysis. This method is in use today (e.g., Stewart et al. 2001) and appears to be the most straightforward for 3-D structures. For this reason, it is currently recommended by the authors when a scalar intensity measure is used (see below). The preferred choice of an IM in the case where the two axes of the structure have different fundamental periods has not been extensively examined. In the absence of further research, one obvious possibility is to use $Sa_{g.m.}$ at an intermediate period (e.g., the geometric mean of the two periods).

2. USE Sa_{arb} AS THE INTENSITY MEASURE

When using Sa_{arb} as an intensity measure in this case, it is necessary to specify the component of the ground motion being measured, as there are now two horizontal components used in the analysis, each with a differing value of Sa. If the objective is only a scalar drift hazard curve (e.g., there is only a single response parameter of interest), then the practitioner may obtain the most efficient estimate by performing the regression analysis shown in Figure 3 one time for each candidate IM (e.g., for Sa_{arb} oriented along the "*x-x*" axis of the structure, for Sa_{arb} oriented along the "*y-y*" axis of the structure, and for $Sa_{g.m.}$). Because all three IM choices should lead to the same answer (in the limit with a very large sample of dynamic analyses), the engineer is free to choose the IM that results in the most efficient estimation. That is, the IM that results in the smallest standard deviation of response prediction. In some cases, the optimal IM will be apparent a priori: if the response parameter of interest is a drift in the *x-x* axis, then it is likely that the optimal IM is Sa_{arb} oriented along the *x-x* axis. In other cases, for example, when assessing the axial force in a corner-column of a structure or when there is significant torsion in the structure, the optimal IM may be less obvious.

In some cases it is necessary to select a common IM for estimation of more than one response parameter simultaneously. For example, in loss estimation procedures associated with performance-based engineering it is often desirable to know the probability distribution of a set of story drifts and floor accelerations *simultaneously*. In this case, it may again be useful to consider several candidate IMs and examine the trade-offs in efficiency. For instance, Sa_{arb} oriented along the *x-x* axis is likely to estimate the responses along the *x-x* axis efficiently but the responses along the *y-y* axis less efficiently and vice versa for Sa_{arb} along the *y-y* axis. Choosing an IM in this case will depend upon the relative importance of the various response parameters of interest, and an understanding of which IM is most efficient for predicting the important response parameters. Note that the results for the less-important response parameters will not be incorrect, but only estimated with less statistical precision.

The above procedure appears to be valid in the case where no record-scaling is used as part of the response predictions. If the records are scaled before performing the structural analysis, the scaling procedure must be carefully considered. Previous studies of the implications of scaling a single component of ground motion may not be applicable to scaling of two components. The authors are particularly concerned about the choice of a scale factor for the orthogonal component of a ground motion when a selected component has been scaled by a specified factor. This problem is currently under investigation.

3. USE A VECTOR INTENSITY MEASURE REPRESENTING THE TWO COMPONENTS INDIVIDUALLY

This approach uses a two-parameter intensity measure, consisting of the spectral accelerations in both the *x-x* and *y-y* directions. Vector-valued ground motion hazard analysis (Bazzurro and Cornell 2002) is used to compute the joint hazard for the spectral acceleration values of the two components of ground motion. Response prediction can then be an explicit function of the two components independently. This approach should reduce the dispersion in structural response and may be useful in some situations (e.g., the two axes of the structure have differing periods, or Sa_{arb} along a given axis is not effective at estimating responses along the opposite axis). However, this method is not ready for widespread adoption until use of vector ground motion hazard analysis becomes more common.

APPLICATION TO CURRENT PRACTICE

The approaches described above rationally combine the uncertainty in both ground motion hazard and structural response, but they differ from current U.S. building code–based design practice (i.e., ASCE 2002). When dynamic time-history analyses are utilized in practice today, a suite of ground motions (typically three or seven) is scaled to a target response spectrum (obtained using the deterministic or probabilistic hazard analysis methods described above). These motions are then used to analyze a structure, and evaluation is based on either the maximum structural response (if fewer than seven motions are used) or the average response (if at least seven motions are used). But again, the definition of spectral acceleration is not stated explicitly. An inconsistent basis (e.g., $Sa_{g.m.}$ spectra and Sa_{arb} scaling) is to be discouraged. As shown above, the use of $Sa_{g.m.}$ for two-dimensional analysis will result in lower target spectra than when Sa_{arb} is used, but higher variation among the structural response. If fewer than seven records are used, then the higher variation of structural response values will be implicitly (but not accurately) captured by the current rule because the maximum response value is likely to be larger. If, however, seven records are used and the average structural response is taken, then there is no penalty paid for the higher variation of structural response that results from using $Sa_{g.m.}$ rather than Sa_{arb}. Therefore, consistent use of $Sa_{g.m.}$ would be somewhat unconservatively biased with seven or more records under the current rule. The ideal solution to this inconsistency would be to incorporate the structural response uncertainty explicitly (as is done explicitly in, e.g., SAC [2000a, b, c] and Banon et al. [2001]). Short of this, the code should require that target response spectrum be based on Sa_{arb} when at least seven records are used for two-dimensional analysis, so that the analysis will include the extra variability in the ground motion intensity. In this case structural engineers should explicitly request that the hazard analysis and the scaled ground motion records be based on the Sa_{arb} definition of Sa. For three-dimensional analysis, $Sa_{g.m.}$ is probably the natural choice for the reasons outlined in the previous section.

CONCLUSIONS

Although intensity measure–based analysis procedures have proven to be useful methods for linking the analyses of earth scientists and structural engineers, care is needed to make sure that the link does not introduce errors into the analysis. Two definitions of "spectral acceleration" are commonly used by analysts, and the distinction between the definitions is not always made clear. Because of this, a systematic error has been introduced into the results from many risk analyses, typically resulting in unconservative conclusions. For an example site and structure located in Los Angeles, the error resulted in a 12% underestimation of the spectral acceleration value exceeded with a 2% probability in 50 years, and a 10% underestimation of the structure's maximum interstory drift ratio exceeded with a 2% probability in 50 years.

This problem is, however, merely one of communication, and not a fundamental flaw with the intensity measure approach. It is not difficult to use intensity measures in ways that produce correct results. For analysis of a single frame of a structure, the authors see three paths to the correct answer: (1) use Sa_{arb} for both parts of the analysis; (2) use $Sa_{g.m.}$ for both parts of the analysis; and (3) perform hazard analysis with $Sa_{g.m.}$, and structural response analysis with Sa_{arb}, but inflate the dispersion in the structural response prediction to represent the dispersion that would have been seen if $Sa_{g.m.}$ had been used. If a three-dimensional model of a structure is to be analyzed, the most straightforward method is to use $Sa_{g.m.}$ as the intensity measure for both the ground motion hazard and the structural response. In the absence of a single standard procedure, both earth scientists and structural analysts are encouraged to explicitly state which *Sa* definition they are using for evaluation, in the interest of transparency.

The methods described above will all produce valid estimates of the annual frequency of exceeding a given structural response level. In the future it would be desirable to have attenuation models that estimate the dispersion of both $Sa_{g.m.}$ and Sa_{arb}, in order to allow flexibility in the definition of the spectral acceleration used for analysis. Finally, vector-based methods of hazard and response analysis should improve upon the current situation in the future.

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APPENDIX: SLOPES AND STANDARD DEVIATIONS OF REGRESSION PREDICTIONS

This appendix explores in more detail the prediction of structural response as a function of either spectral acceleration of an arbitrary component or an average component. Consider a set of earthquake ground motions consisting of two components. We will refer to these two components as the "X" and "Y" components for clarity (the common assumption of no preferential orientation of motion is made here, which is typically valid when near-fault directivity effects are not present). Now consider the probabilistic distribution of the spectral acceleration values of these ground motion components. Logarithms of spectral acceleration and structural response are used, to take advantage of the linear relationships in the logarithmic domain often observed between these variables (e.g., Figure 3 and 5).

The $\ln Sa$ values of the x and y components have means, denoted as $\mu_{\ln Sa_x}$ and $\mu_{\ln Sa_y}$, and standard deviations, denoted as $\sigma_{\ln Sa_x}$ and $\sigma_{\ln Sa_y}$. Because there is no preferential direction to these motions, $\mu_{\ln Sa_x}=\mu_{\ln Sa_y}$ and $\sigma_{\ln Sa_x}=\sigma_{\ln Sa_y}$ (although our *estimates* of these values for a particular data set might not be exactly equal, the underlying *true values* are assumed to be). We can compute a correlation coefficient between the two components, denoted as $\rho_{\ln Sa_x,\ln Sa_y}$. The dependence between $\ln Sa_x$ and $\ln Sa_y$ is purely linear due to the lack of preferential orientation, as can be seen, for example, in Figure 5. We make the further mild assumption that the conditional variances of $\ln Sa_y$ given $\ln Sa_x$ and $\ln Sa_x$ given $\ln Sa_y$ are constant.

Consider now the analysis of a structural frame, oriented along the x axis of the ground motions. We perform nonlinear dynamic analysis with the x component of each ground motion to calculate a set of structural response values. The logarithmic response values have a mean, $\mu_{\ln \theta_x}$, and a standard deviation, $\sigma_{\ln \theta_x}$. There is a relationship between $\ln Sa_x$ (log spectral acceleration in the x direction) and $\ln \theta_x$ (log response of the structure, oriented in the x direction) that can be represented by a linear correlation, and

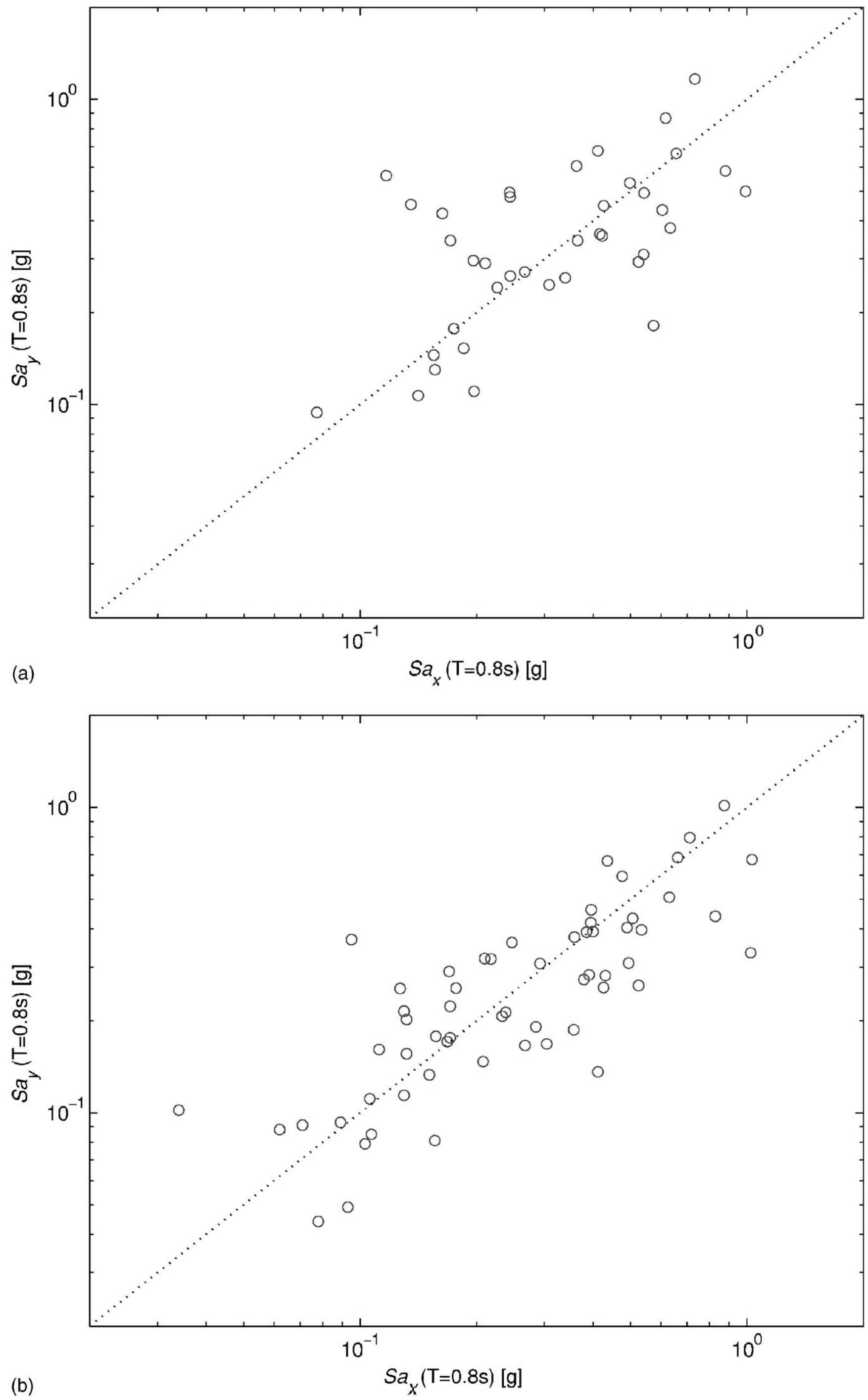

Figure 5. Samples of $(\ln Sa_x, \ln Sa_y)$ pairs from (a) a set of ground motions with magnitude $\approx$6.5 and distance $\approx$8 km, and (b) a set of ground motions with a wider range of magnitudes and distances, used to perform the structural analyses displayed in Figure 3.

measured with a correlation coefficient denoted as $\rho_{\ln\,\theta_x,\ln\,Sa_x}$. This linear relationship is represented by, for instance, the regression line shown in Figure 3a. But sometimes the relationship between ground motion intensity and structural response is represented as a function of the geometric mean of the *Sa*'s of the two components, as in Figure 3b. We can represent this relationship with the correlation coefficient denoted as $\rho_{\ln\,\theta_x,\ln\,Sa_{g.m.}}$. We make the assumption that the dependence between $\ln\theta_x$ and $\ln Sa$ is purely linear and the conditional variance of $\ln\theta_x$ given $\ln Sa$ is constant (for both $\ln Sa_x$ and $\ln Sa_{g.m.}$). We are interested in the relationship between the slopes and standard deviations of prediction errors resulting from prediction using these two predictors $\ln Sa_x$ and $\ln Sa_{g.m.}$.

First, recalling that $\ln Sa_{g.m.}$ is the average of $\ln Sa_x$ and $\ln Sa_y$, we note that the spectral acceleration of the *y* component has no direct physical effect on the response of the frame oriented in the *x* direction (indeed, the *y* component of the ground motion is not even used in analysis). By itself, the parameter $\ln Sa_y$ does have, however, an indirect ability to predict θ_x simply due to its correlation with the predictor $\ln Sa_x$. Therefore, while $\ln\theta_x$ and $\ln Sa_y$ are statistically correlated, it is true that *given* $\ln Sa_x$, $\ln Sa_y$ provides no *additional* information about the distribution of $\ln\theta_x$. That is, $\ln\theta_x$ and $\ln Sa_y$ are conditionally independent given $\ln Sa_x$.

Using the above information, we can calculate conditional means and variances, using best linear predictors. For example, the mean value of $\ln\theta_x$ given $\ln Sa_x$ is

$$\mu_{\ln\,\theta_x|\ln\,Sa_x=x} = \left(\mu_{\ln\,\theta_x} - \frac{\rho_{\ln\,\theta_x,\ln\,Sa_x}\sigma_{\ln\,\theta_x}\mu_{\ln\,Sa_x}}{\sigma_{\ln\,Sa_x}}\right) + \left(\frac{\rho_{\ln\,\theta_x,\ln\,Sa_x}\sigma_{\ln\,\theta_x}}{\sigma_{\ln\,Sa_x}}\right)x \equiv a + bx \tag{3}$$

It can be shown that *a* and *b* are the expected values of the coefficients estimated from linear least squares regression of $\ln\theta_x$ on $\ln Sa_x$. The conditional variance of $\ln\theta_x$ given $\ln Sa_x$ is

$$\sigma^2_{\ln\,\theta_x|\ln\,Sa_x} = \sigma^2_{\ln\,\theta_x}(1 - \rho^2_{\ln\,\theta_x,\ln\,Sa_x}) \tag{4}$$

We are interested in computing the conditional mean and variance of $\ln\theta_x$ given $\ln Sa_{g.m.}$ for comparison, but several intermediate results are needed first. The marginal mean and variance of $\ln Sa_{g.m.}$ are

$$\mu_{\ln\,Sa_{g.m.}} = E[1/2(\ln Sa_x + \ln Sa_y)] = \mu_{\ln\,Sa_x} \tag{5}$$

$$\begin{aligned}\sigma^2_{\ln\,Sa_{g.m.}} &= \mathrm{Var}[1/2(\ln Sa_x + \ln Sa_y)] = 1/4(\sigma^2_{\ln\,Sa_x} + \sigma^2_{\ln\,Sa_y} + \rho_{\ln\,Sa_x,\ln\,Sa_y}\sigma_{\ln\,Sa_x}\sigma_{\ln\,Sa_y})\\ &= \sigma^2_{\ln\,Sa_x}\frac{1 + \rho_{\ln\,Sa_x,\ln\,Sa_y}}{2}\end{aligned} \tag{6}$$

The conditional variance of $\ln Sa_x$ given $\ln Sa_y$ is

$$
\begin{aligned}
\sigma^2_{\ln\theta_x|\ln Sa_y} &= \mathrm{E}[\mathrm{Var}[\ln\theta_x|\ln Sa_x,\ln Sa_y]|\ln Sa_y] + \mathrm{Var}[\mathrm{E}[\ln\theta_x|\ln Sa_x,\ln Sa_y]|\ln Sa_y] \\
&= \mathrm{E}[\sigma^2_{\ln\theta_x}(1-\rho^2_{\ln\theta_x,\ln Sa_x})|\ln Sa_y] \\
&\quad + \mathrm{Var}\left[\mu_{\ln\theta_x} + \rho_{\ln\theta_x,\ln Sa_x}\sigma_{\ln\theta_x}\left(\frac{\ln Sa_x - \mu_{\ln Sa_x}}{\sigma_{\ln Sa_x}}\right)\middle|\ln Sa_y\right] \\
&= \sigma^2_{\ln\theta_x}(1-\rho^2_{\ln\theta_x,\ln Sa_x}) + \rho^2_{\ln\theta_x,\ln Sa_x}\frac{\sigma^2_{\ln\theta_x}}{\sigma^2_{\ln Sa_x}}\mathrm{Var}[\ln Sa_x|\ln Sa_y] \\
&= \sigma^2_{\ln\theta_x}(1-\rho^2_{\ln\theta_x,\ln Sa_x}\rho^2_{\ln Sa_x,\ln Sa_y})
\end{aligned} \tag{7}
$$

This implies that the correlation coefficient between $\ln\theta_x$ and $\ln Sa_y$ is $\rho_{\ln\theta_x,\ln Sa_x}\rho_{\ln Sa_x,\ln Sa_y}$, showing that it is weaker than the correlation between $\ln\theta_x$ and $\ln Sa_x$, and only as strong as the correlation $\ln Sa_y$ has with $\ln Sa_x$ permits. This simple product form is a result. Next we compute the covariance between $\ln\theta_x$ and $\ln Sa_{g.m.}$:

$$
\begin{aligned}
\mathrm{Cov}[\ln\theta_x,\ln Sa_{g.m.}] &= 1/2\mathrm{Cov}[\ln\theta_x,\ln Sa_x] + 1/2\mathrm{Cov}[\ln\theta_x,\ln Sa_y] \\
&= 1/2(\rho_{\ln\theta_x,\ln Sa_x}\sigma_{\ln\theta_x}\sigma_{\ln Sa_x} + \rho_{\ln\theta_x,\ln Sa_x}\rho_{\ln Sa_x,\ln Sa_y}\sigma_{\ln\theta_x}\sigma_{\ln Sa_y}) \\
&= 1/2\rho_{\ln\theta_x,\ln Sa_x}\sigma_{\ln\theta_x}\sigma_{\ln Sa_x}(1+\rho_{\ln Sa_x,\ln Sa_y})
\end{aligned} \tag{8}
$$

Finally, using the results of Equations 6 and 8, we compute the correlation coefficient between $\ln\theta_x$ and $\ln Sa_{g.m.}$:

$$
\rho_{\ln\theta_x,\ln Sa_{g.m.}} = \frac{\mathrm{Cov}[\ln\theta_x|\ln Sa_{g.m.}]}{\sigma_{\ln\theta_x}\sigma_{\ln Sa_{g.m.}}} = \rho_{\ln\theta_x,\ln Sa_x}\sqrt{\frac{1+\rho_{\ln Sa_x,\ln Sa_y}}{2}} \tag{9}
$$

We are now ready to find the conditional mean of $\ln\theta_x$ given $\ln Sa_{g.m.}$:

$$
\mu_{\ln\theta_x|\ln Sa_{g.m.}=z} = \mu_{\ln\theta_x} + \rho_{\ln\theta_x,\ln Sa_{g.m.}}\sigma_{\ln\theta x}\left(\frac{z-\mu_{\ln Sa_{g.m.}}}{\sigma_{\ln Sa_{g.m.}}}\right) \tag{10}
$$

Substituting from Equations 5, 6, and 9 gives

$$
\mu_{\ln\theta_x|\ln Sa_{g.m.}=z} = \left(\mu_{\ln\theta_x} - \frac{\rho_{\ln\theta_x,\ln Sa_x}\sigma_{\ln\theta_x}\mu_{\ln Sa_x}}{\sigma_{\ln Sa_x}}\right) + \left(\frac{\rho_{\ln\theta_x,\ln Sa_x}\sigma_{\ln\theta_x}}{\sigma_{\ln Sa_x}}\right)z = a + bz \tag{11}
$$

where a and b are the same as in Equation 3. Therefore, *the expected slope and intercept of a regression analysis will be the same regardless of whether* $\ln Sa_{g.m.}$ or $\ln Sa_x$. *is used to predict* $\ln\theta_x$. Now consider the conditional variance of $\ln\theta_x$ given $\ln Sa_{g.m.}$:

$$\sigma^2_{\ln\,\theta_x|\ln Sa_{g.m.}} = \sigma^2_{\ln\,\theta_x}\left(1-\rho^2_{\ln\,\theta_x,\ln Sa_{g.m.}}\right) = \sigma^2_{\ln\,\theta_x}\left(1-\rho^2_{\ln\,\theta_x,\ln Sa_x}\frac{1+\rho_{\ln Sa_x,\ln Sa_y}}{2}\right) \quad (12)$$

Therefore, the conditional standard deviation of $\ln\theta_x$ given $\ln Sa_{g.m.}$ is greater than the conditional standard deviation given $\ln Sa_x$ by a factor equal to

$$\frac{\sigma_{\ln\,\theta_x|\ln Sa_{g.m.}}}{\sigma_{\ln\,\theta_x|\ln Sa_x}} = \sqrt{1+\frac{1-\rho_{\ln Sa_x,\ln Sa_y}}{2}\left(\frac{\rho^2_{\ln\,\theta_x,\ln Sa_x}}{1-\rho^2_{\ln\,\theta_x,\ln Sa_x}}\right)} \quad (13)$$

Noting that correlation coefficients always lie on the interval $[-1,1]$, we see that the ratio in Equation 13 is always greater than or equal to 1, and increases with decreasing $\rho_{\ln Sa_x,\ln Sa_y}$ or increasing $\rho_{\ln\,\theta_x,\ln Sa_x}$. The term $\rho_{\ln Sa_x,\ln Sa_y}$ is dependent on the record set in use, but typically falls between 0.8 and 0.9, depending on the range of magnitudes and distances of the records (letting the magnitude and distance vary in the record set increases the correlation between components relative to a recordset selected from a narrow range of magnitude and distance values, as is seen in Figure 5).

Note that this result only holds under the conditions described above. The most critical assumption is that response in the x direction is unaffected by ground motion input in the y direction. This may not be the case in, for example, torsionally coupled structures. In addition, the assumption of linear dependence between $\ln\theta_x$ and $\ln Sa$ may not always hold. In these cases, the simple inflation factor is not applicable.

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