Safety, Reliability, Risk, Resilience and Sustainability of Structures and Infrastructure
12th Int. Conf. on Structural Safety and Reliability, Vienna, Austria, 6–10 August 2017
Christian Bucher, Bruce R. Ellingwood, Dan M. Frangopol (Editors)

Model parameter uncertainties and correlations: quantification and assessment of impacts on seismic collapse risk

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Abstract: Quantification of seismic collapse risk of structures requires consideration of uncertainties in seismic loading and in modeling of structures. In this paper, we discuss recent efforts to quantify stochastic properties of structural modeling parameters. Results include development of a new technique to estimate stochastic dependence among model parameters, an assessment of the impacts of these correlations on structural reliability, and a discussion of the implications of these findings. The results further confirm the importance of considering model parameter uncertainties when evaluating structural collapse risk, as expected given the significant uncertainties associated with modeling this phenomenon. Further, the results illustrate several practical approaches for better estimating and propagating these uncertainties in practical structural response assessments (i.e., without dramatically changing analysis approaches currently used in earthquake engineering).

1 Introduction

Seismic risk analysis requires quantification and propagation of uncertainties in order to quantify the probability of adverse outcomes. Significant effort has been devoted to quantifying and assessing uncertainty in ground shaking and structural model parameters, but much less attention has been given to stochastic dependence among parameters [1]. A number of studies have performed risk analysis with assumed dependence among parameters [2, 3, 4, 5, 6].

To consider dependence, we utilized the concentrated plasticity model proposed by Ibarra et al. [7], and shown in Figure 1. The 'backbone' model has five parameters: capping plastic rotation ($\theta_{cap,pl}$), secant stiffness to 40% of the component yield moment (EI_{stf}), yield moment (M_y), capping moment (M_c), and post-capping rotation (θ_{pc}). A sixth parameter (γ) governs deterioration under cyclic loading. This model is often used when simulating sidesway collapse in frame structures [e.g., 8, 9, 10].

Predictive models for these parameters find their marginal distributions to be lognormal (with the exception of M_c which is strictly greater than M_y and so requires a mild transformation to be modeled as a lognormal random variable [11]. We make the additional mild assumption that the joint distribution of their logarithms is multivariate normal; dependence among parameters can then be quantified by correlation coefficients. We thus discuss correlation coefficients for model parameters exclusively below.

When modeling a complete structure, these concentrated plasticity elements are utilized at the ends of beams and columns (Figure 2). The structural model then requires the six component parameters to be specified at the many locations throughout the structure.

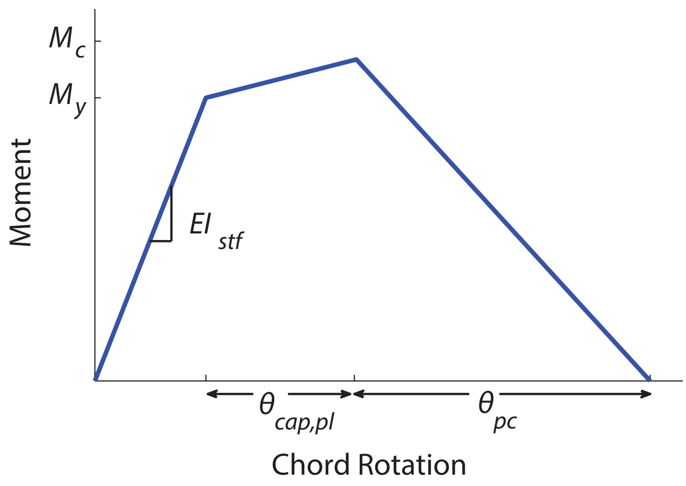

Figure 1: Ibarra et al. [7] model for moment versus rotation of a plastic hinge in a structure, with the five backbone model parameters labeled.

Predictive models for mean values and standard deviations of these parameters have been calibrated from component test data [12, 13], and predict values as a function of component geometry, axial load, etc. Of interest here is the correlation among parameter values. Note that correlation exists between parameters for a given component, as well as from component-to-component; we refer to these as within- and between-component correlations, respectively.

2 Estimating correlations

Gokkaya et al. [11] proposed the following method for estimating correlations. A set of observed parameter values from a large set of tests are considered, where there are groups of components analogous to a set of components in a building. Predictive equations for means and standard deviations of those parameter values are needed as well (in this study they were already available, but in principle could be developed in conjunction with the correlations).

For the data set considered below, there are multiple tests performed by individual laboratories, and here we refer to these as 'test groups.' Most of these test groups have specimens with similar dimensions, and with fixed steel yield strength and area ratio of longitudinal reinforcing steel. The primary differences among the tests within a test group are the level of axial load and transverse reinforcement. These variations are analogous to variations we anticipate among components in a real-world building. Given these features, we make the assumption that model parameter values observed from tests in a single test group will show similar stochastic dependence to model parameter values among components in a real building (resulting from similarities in environmental conditions, workmanship, etc.). With this assumption, we then estimate model parameter correlations within test groups, as described below, in order to estimate correlations for building models.

We compute prediction residuals by comparing the observations from the test data to model predictions:

$$\ln y_{ij}^{k} = \ln \hat{y}_{ij}^{k} + \tilde{\varepsilon}_{ij}^{k} \tag{1}$$

where i and j represent the test group and test number, respectively, and the superscript k indicates the random variable of interest. Random variable k from the test specified by i and j is associated with observed value y_{ij}^{k}, predicted value $\hat{y}_{ij}^{k}$, and residual $\tilde{\varepsilon}_{ij}^{k}$.

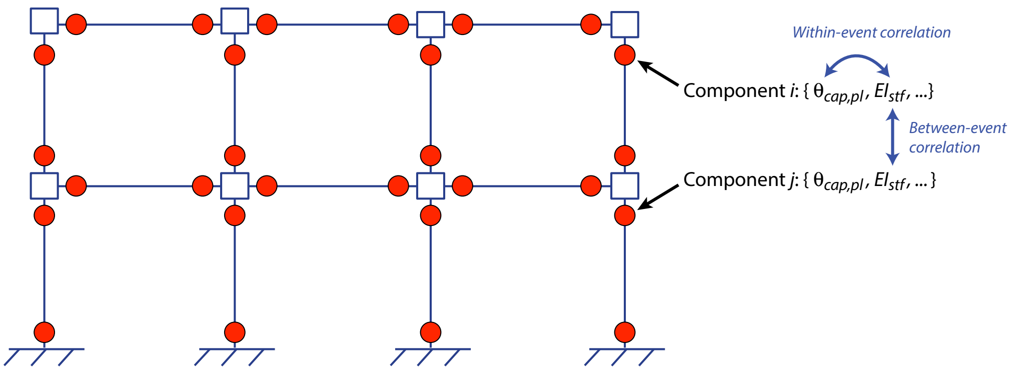

Figure 2: Illustration of locations in a structural model with concentrated plasticity elements, indicating within- and between-component correlation.

A one-way random effects model is applied to residuals from Equation 1 to assess the correlation structure of the model parameters [14]. The test groups are treated as a random effect, and logarithmic residuals of each random variable, $\tilde{\varepsilon}_{ij}^{k}$, are considered without any further transformation, leading to the following equation:

$$\begin{aligned} \ln\left(y_{ij}^{k}\right) - \ln\left(\hat{y}_{ij}^{k}\right) &= \tilde{\varepsilon}_{ij}^{k} \\ &= \mu^{k} + \alpha_{i}^{k} + \varepsilon_{ij}^{k} \end{aligned} \tag{2}$$

where μ^k is the mean of the data, and α^k and ε^k represent between- and within-test-group variability, respectively. The α^k and ε^k terms are independent random variables with zero means and variances σ_k^2 and τ_k^2, respectively. These variances are estimated from the regression procedure. From Equation 2, and the definition of correlation, the correlation coefficient for the logarithms of the model parameters k and k' within a given component is:

$$\rho_{\ln y_{ij}^{k}, \ln y_{ij}^{k'}} = \frac{\rho_{\alpha_i^k, \alpha_i^{k'}} \sigma_k \sigma_{k'} + \rho_{\varepsilon_{ij}^{k}, \varepsilon_{ij}^{k'}} \tau_k \tau_{k'}}{\sqrt{\sigma_k^2 + \tau_k^2}\sqrt{\sigma_{k'}^2 + \tau_{k'}^2}} \tag{3}$$

where all of the required correlation coefficients and standard deviations needed to evaluate this formula can be estimated from the random effects regression results. Similar results can be derived for between-component correlations [11].

When this approach was applied to concrete beam-column test data, between-component correlations of like parameters showed high values (i.e., M_y, $\theta_{cap,pl}$, EI_{stf}/EI_g and M_c/M_y have correlations of 0.7 or greater). This implies that values of these parameters across components will tend to take similar values.

Within a component, correlations of model parameters are small. Correlation between M_c/M_y and M_y, and between M_c/M_y and $\theta_{cap,pl}$, was approximately 0.3, and other parameter pairs had smaller correlations.

3 Impact on seismic reliability

3.1 Analysis procedure

To evaluate the impact of the above parameter correlations, seismic reliability analyses were performed. Monte Carlo simulations of building models were generated (where the six parameter values discussed above were randomized to have appropriate variance/covariance structure). Each realization of the structure was then subjected to an incremental dynamic analysis, increasing the amplitude of the ground motion (as measured by an intensity measure, *im*) until a failure was observed. For the results below, failure was defined as exceedance of a peak story drift ratio somewhere in the building. The *im* causing failure is influenced by variability in ground motion properties among ground motions with that *im* value, and by uncertainty in structural behavior (quantified by the parameter uncertainty discussed above). The analysis results are quantified by a fragility function estimating the probability of failure (F) at a given IM level, *im* ($P(F|IM = im)$), and parameterized here by a lognormal cumulative distribution function:

$$P(F|IM = im) = \Phi\left(\frac{\ln(im/\theta)}{\beta}\right) \tag{4}$$

where $\Phi()$ is a standard normal cumulative distribution function, and θ and β are distribution parameters estimated from the analysis data.
The mean annual frequency of failure (λ_f) is then obtained by integrating the collapse fragility function with the ground motion hazard curve for the site of interest [15], as given in Equation 5.

$$\lambda_f = \int_0^\infty P(F|IM = im)\left|\frac{d\lambda_{IM}(im)}{d(im)}\right| d(im) \tag{5}$$

where $\lambda_{IM}(im)$ is the mean annual rate of exceeding the ground motion *im* and $\frac{d\lambda_{IM}(im)}{d(im)}$ is the slope of the *im* hazard curve at *im*.

3.2 Analysis results

The above procedure was performed for a number of reinforced concrete frame buildings. For each building, a number of approaches to model parameter uncertainty were also considered. A ‘median model’ case set all parameters to their median predicted values, and considered no parameter uncertainty. A ’no correlation’ case considered all parameters uncertain, but uncorrelated (i.e., independent, given the multivariate Gaussian distribution considered). A ’full correlation’ case considered all parameters uncertain, and perfectly correlated. Finally, a ’partial correlation’ case considered all parameters uncertain, and partially correlated with correlation coefficients estimated from the procedure of Section 2. The partial correlation case is considered the best estimate of structural reliability, and the other cases were considered to evaluate the importance of incorporating correlations and to assess the accuracy of potential simplified correlation representations.
Figure 3 shows example results from this procedure. These results are for a modern four-story three-bay reinforced concrete frame. The building is assumed to be located in Los Angeles, was designed to satisfy 2003 IBC and ASCE 7-02 design standards [16, 17]. An OpenSEES model of the building, with a first-mode elastic period of 0.94s, was used for analysis [18, 2]. The 44 far-field ground motion components from FEMA-P695 were used as input ground motions [19]. These results show that parameter uncertainties are not particularly important for moderately nonlinear response (i.e., story drift ratios < 0.03), as all modeling approaches gave comparable answers. But for more severely nonlinear response, the results varied more dramatically. At a story drift ratio of 0.1 (generally associated with sidesway collapse), the predicted exceedance

rate varied by a factor of 2.5 between the median model case and the full correlation case. The (benchmark) partial correlation case produced results between these two extremes, and was best approximated by the no correlation case. A number of other structural performance metrics were considered (e.g., median and log-standard-deviation of response at a given *im* level), and the accuracy of simplified correlation representations varied depending upon the metric of interest. A broader set of analyses of this type produced a number of other findings [20]. The example building here had regular strength and stiffness distribution over its height, and so variations in parameter values generally did not change the collapse mechanism; for other structural models (e.g., those with a soft story) the collapse mechanism could change depending on a given realization of parameter values. This has important implications for structural engineers, as it indicates that a structural model with median model parameters may not indicate all plausible collapse mechanisms that could be present if one explicitly considers the uncertainty in the structural models representation of the building.

A general finding was that model uncertainty in general became more important as the degree of nonlinearity in the building increased. This is intuitive in that structural models (and associated model parameters) are less well understood at severe deformation demands. It also highlights that conclusions about the importance of model uncertainty require consideration of the limit state being considered; reference studies considering uncertainty in elastic model parameters, moderately nonlinear (or non-deteriorating) structures did not give an indication of the importance of model uncertainty on collapse capacity, as was observed in this study.

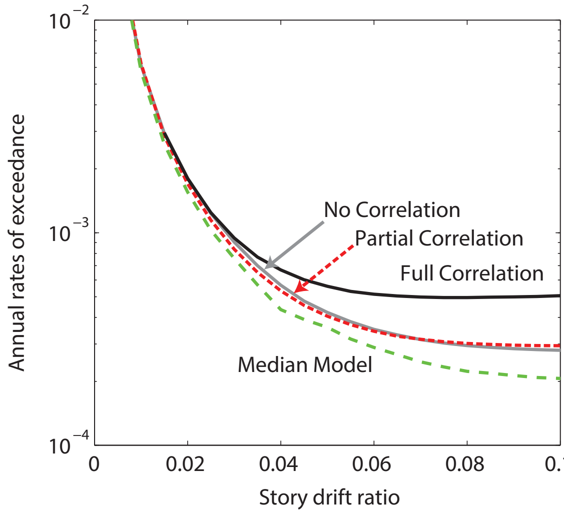

Figure 3: Mean annual frequency of exceedance of maximum story drift ratio using the considered correlation models (adapted from [11]).

4 Conclusions

This paper summarizes findings from recent studies on a new technique for estimating dependence among model parameters; this approach uses random effects modeling of parameters estimated from a database of experimental component tests, in order to quantify the degree to which parameter variability is shared across components or across parameters within a component. Groups of component tests that are conducted in similar conditions, and are investigating the impacts of particular properties of components that can effectively represent different locations in a structure, are suitable for this estimation approach. To illustrate the impacts of this modeling techniques, results from a case study analyses are presented. More detailed results and discussion are available in the papers cited above, and in [21].

Acknowledgement

We thank Dr. Shrey Shahi and Professor Art Owen for providing valuable feedback on mixed effects modeling. This work was supported by the National Science Foundation under NSF grant number CMMI-1031722. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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