Prediction of inelastic structural response using an average of spectral accelerations

M. Bianchini & P.P. Diotallevi
Dept. of Structural, Transport, Hydraulic, Survey & Territory Engrg., University of Bologna, Bologna, Italy

J.W. Baker
Dept. of Civil & Environmental Engrg., Stanford University, Stanford, USA

ABSTRACT: In probabilistic engineering assessment, the ground motion intensity measure (IM), which quantifies the severity of a seismic event, is used both as a scale factor for recorded ground motions in incremental dynamic analysis and as that parameter which defines the seismic hazard at a specified site. In this paper, the geometric mean of pseudo-spectral acceleration ordinates over a certain range of periods, $S_{a,avg}(T_1,\ldots,T_n)$, or briefly $S_{a,avg}$, is used as an optimal scalar IM to predict inelastic structural response of buildings subjected to recorded ground motions. This average of spectral values is a better predictor than both the elastic pseudo-spectral acceleration at fundamental period of structure and the peak ground acceleration, especially for inelastic structural systems. Furthermore, the seismic hazard at the site in terms of $S_{a,avg}$ as IM is simpler than the one performed for vector-values and inelastic IMs. Especially for multi-degree-of-freedom systems with long periods, $S_{a,avg}$ is very sensitive to higher-mode effects and it is able to account degrading behaviour. $S_{a,avg}$ is studied as a statistical predictor of structural response through its desirable properties and results are compared with traditional elastic-based scalar IMs.

1 INTRODUCTION

Performance-Based Earthquake Engineering (PBEE) is an approach which provides a set of useful tools to support seismic risk decisions and seismic performance through a probabilistic framework (Cornell & Krawinkler 2000). The performance is measured in terms of the amount of damage sustained by a building, when affected by earthquake ground motion, and the impacts of this damage on post-earthquake disposition of the building. While the general framework concerns all aspects of the performance-based engineering (including structural and non-structural design, construction quality assurance and maintenance of building integrity throughout its life cycle), this paper focuses on the structural aspects of the problem, by evaluation of the inelastic response of structural buildings.

The PBEE process has been provided with a robust methodology by the Pacific Earthquake Engineering Research (PEER) Centre, which directly incorporates the effects of uncertainty and randomness at each step of performance assessment procedure. This methodology defines the ground motion intensity measure (IM) concept as that characteristic parameter of earthquake ground motion that affects the engineering demands on structural systems. Classical examples of this indicator may be the peak values of ground shaking in terms of acceleration, velocity and displacement (a.k.a., PGA, PGV and PGD), or the elastic pseudo-spectral acceleration at the fundamental period of a structure, $S_a(T^{(1)})$, or briefly S_a. However, any other parameter can be an IM if it can be expressed as a function of mean annual frequency (MAF) of exceeding a certain level of ground motion parameter. In order to define the MAF of exceeding a certain level of IM for the area where the building is located, Probabilistic Seismic Hazard Analysis (PSHA) must be carried out in terms of the selected intensity measure (McGuire 2004). Then, structural response can be quantified by engineering demand parameters (EDPs), which are useful to predict damage to structural and non-structural components and systems (Whittaker et al. 2004). A possible choice could be the floor peak inter-storey drift angles $\theta_1,\ldots,\theta_n$ of a n-storeys structure, or their maximum, i.e., the maximum peak inter-storey drift angle, $\theta_{\max}$, defined as the peak over response time and maximum over the height of the structure. In reliability analysis, the hazard information can be combined with a certain EDP parameter prediction (given a selected IM) in order to assess the MAF of exceeding a specified value of that structural demand parameter: this link is formally expressed by Probabilistic Seismic Demand Analysis (PSDA, (Shome et al. 1998)). Hence, the choice of IM can affect the quality of the reliability result.

S_a has been used as a predictor and IM in seismic performance assessment, although significant variability in the structural response level has been observed for tall and long period buildings. This problem, which is named as inefficiency of S_a, is in part due to the fact that it does not reflect important higher mode spectral accelerations and any spectral shape (Abrahamson & Silva 1997). This question has been addressed by pairing it with a measure of spectral shape, ε, in a vector-valued IM (Baker & Cornell 2005a), $<S_a,\varepsilon>$. Nevertheless, in the light of PSDA approach, to a vector-valued IM must be associated a vector-valued PSHA to obtain the joint hazard curve, which has not yet been commonly applied, or using the conventional seismic disaggregation analysis (Bazzurro & Cornell 1999). It has been shown that the effectiveness of ε as a criterion to select ground motion records and to predict inelastic response of multi-degree-of-freedom systems is considerably greater than that of S_a alone for ordinary strong ground motions. In order to provide a good predictor of inelastic structural response for those buildings located both far from and near to earthquake source, a new scalar intensity measure based on the inelastic spectral displacement, S_{di}, has been recently developed. It has been demonstrated that, for structural systems dominated by the first mode of vibration, the S_{di} predictor is more convenient than $<S_a,\varepsilon>$, whereas, for those structures affected by higher-mode periods, it needs to be combined with a higher frequency elastic spectral displacement (Tothong & Luco 2007).

Especially for practical applications, the difficulties in working with a vector-valued or inelastic IM can be a barrier which is hard to overcome. It is well-known that structural response of multi-degree-of-freedom systems or inelastic systems is sensitive to multiple periods T_i (Baker & Cornell 2005b), so an intensity measure which averages elastic spectral acceleration values over a certain range of periods might be a useful and convenient predictor of structural response of inelastic systems. The concept of averaging spectral acceleration values over a certain period range was already anticipated in federal provisions, although it is more of a rough guide based on design spectrum to choose recorded ground motion rather than to define one predictor. In fact, both for two- and three-dimensional response history analysis procedures, many codes states that response spectra for the suite of motions is not less than the design response spectrum for the site, for periods ranging from $0.2 \cdot T^{(1)}$ to $1.5 \cdot T^{(1)}$ (SEI 2005). Nevertheless, this choice of period range has never been formally evaluated.

By comparing different response predictors, it was shown that correlation between damage, higher modes influence and elastic spectral acceleration were improved by averaging the spectral ordinates over an interval with bounds related to $T^{(1)}$. Depending on the multiplicity of degrees of freedom of structures, the higher vibration mode influence and the rate of degradation of the hysteresis nonlinear model, an average of spectral acceleration ordinates was studied as a efficient statistical predictor of inelastic response (Bianchini et al., in prep).

The present work aims to show the desirable properties of $S_{a,avg}(T_1,\ldots,T_n)$, or briefly $S_{a,avg}$, as a ground motion IM to be used in PSDA. $S_{a,avg}$ was defined as the geometric mean of the spectral acceleration ordinates on a set of n periods, i.e., the n-th root of the product of n elastic spectral values, and it is applied here to demand assessment of inelastic multi-degree-of-freedom systems. It is found that $S_{a,avg}$ can be used as a useful and practical predictor of structural response, compared with the most conventional elastic-based scalar IMs.

2 RESPONSE PREDICTION IN PBEE FRAMEWORK

2.1 *General considerations*

The aim of PSDA is to assess the structural performance of a given building by probabilistic assessment of the response under ground motion records. The latter is subsequently combined with information about seismic hazard at the site, calculated using PSHA, which provides the MAF of exceeding a specified level of IM value, $\lambda_{\mathrm{IM}}(im) = P[\mathrm{IM} > im]$.

In general, structural response can be simulated through EDPs using the chosen IMs and corresponding earthquake records. One method of calculating EDPs is through nonlinear incremental dynamic analysis (IDA), which predicts structural response under ground motion records incrementally scaled to different IM levels (Vamvatsikos & Cornell 2002). A collection of IDA curves parameterized on the same IMs and EDP and generated for the same structure under different recording defines the so-called IDA curve set. Then, median, 16%-ile and 84%-ile IDA are defined to summarize an IDA curve set. This is consistent also with the assumption to consider, for non-collapse data, the conditional response EDP (given an IM level) lognormal distributed around the mean and the standard deviation of their natural logarithms, respectively, called median and dispersion. By combination of the site-specific ground motion hazard curve with the structural response information, the MAF of exceeding a specified level of EDP value, $\lambda_{\mathrm{EDP}}(edp) = P[\mathrm{EDP} > edp]$, is obtained by the total probability theorem (Benjamin & Cornell 1970) as follow

$$\lambda_{\mathrm{EDP}}(edp) = \int_{im} G_{\mathrm{EDP|IM}}(edp, im)\left|\mathrm{d}\lambda_{\mathrm{IM}}(im)\right| \tag{1}$$

where $G_{\mathrm{EDP|IM}}(edp,im) = P[\mathrm{EDP} > edp \mid \mathrm{IM} = im]$ means the probability of exceeding a specified EDP level, edp, given a level of IM, im, or the complementary distribution function of EDP given IM. Equation (1) represents the classical form of PSDA proposed in PBEE framework. However, in order to compare the desirable properties of different IMs (i.e., their influence on $\lambda_{EDP}(edp)$ assessment), it is not necessary to assess the whole integral, but only the statistics on $G_{EDP|IM}$.

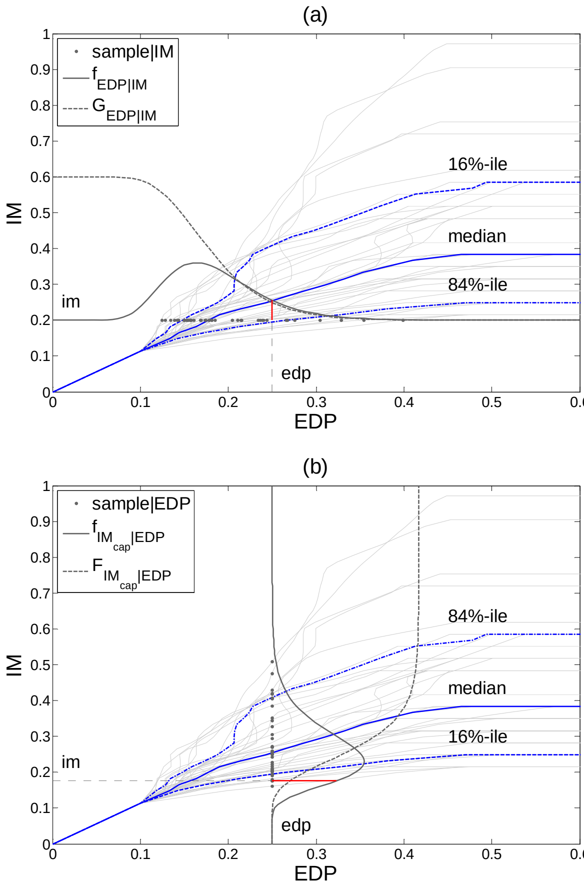

Figure 1. Distribution functions for (a) IM-based rule and (b) EDP-based rule of IDA curve set constituted of a suite of 40 recorded ground motions for a nonlinear single-degree-of-freedom systems. The area subtended by the bold red line represents (a) $G_{EDP|IM}(edp,im)$ and (b) $F_{IMcap|EDP}(im',edp)$.

The first integrand in Equation (1) assumes the so-called IM-based rule, which provides the distribution of demand that a given level of intensity can generate in the structure. In an IDA curve set it is hard to define a value that signals collapse for all IDA curves; in other words, prescribing a single point on the IDA curves that clearly divides them in two regions (non-collapse and collapse) does not result always feasible. This difficulty can be overcome by the EDP-based rule, which provides the distribution of intensities IM that are required to produce a level of damage, *edp*, or a given ductility level, μ. By using an EDP-based rule instead of the IM- one, the structural response hazard can be computed as

$$\lambda_{EDP}(edp) = \int_{im} F_{IM_{cap}|EDP}(im', edp)\left|d\lambda_{IM}(im')\right| \qquad (2)$$

where $F_{IMcap|EDP}(im',edp) = P[IM_{cap} < im' \mid EDP = edp]$ is the cumulative distribution function of IM capacity given EDP. The random variable IM_{cap} represents the distribution of IM values resulting in the structure (i.e., the structural capacity in terms of IM), having a certain EDP level. In general, the probability of exceeding a specified level of EDP given a level of IM does not match exactly the probability of not exceeding a level of IM_{cap} fixed and EDP value. This is particularly true if mean and standard deviation are used to summarize EDP- or IM- stripes. However, it has been shown that the 16%, 50%, and 84% fractiles given IM almost perfectly match, respectively, the 84%, 50%, and 16% fractiles given EDP (Vamvatsikos & Cornell 2004), and the results from Equations (1) and (2) will theoretically produce identical $\lambda_{EDP}(edp)$ results. Again, it follows that the random variable IM_{cap} given an EDP level can be considered lognormally distributed around its own median and dispersion. Figure 1 shows the similarity between (a) IM- and (b) EDP-based rule, from a statistical point of view. The two rules are applied to an IDA curve set of 40 recorded ground motions summarized by 16%, 50% (median), and 84% fractiles for a nonlinear single-degree-of-freedom systems. It should be noted that for the same EDP level and probability value $im \approx im'$, i.e., the two methods are indeed comparable.

2.2 *Desirable IM properties*

The EDP-based rule is here assumed to define statistical parameters as median, percentile and standard deviation of IDA curve set, and so assess the desirable properties of a selected IM in PSDA. A good IM is structure dependent, captures higher-mode effects and inelastic behaviour of buildings, and regards the frequency content of recorded ground motions. Strictly speaking, in order to ensure a reliable result of Equations (1) or (2), some features of the selected ground motion IM must be provided.

First of all, the IM is used to quantify the ground motion hazard at a site due to seismicity in the region, and from a probabilistic point of view this can be done through a ground motion prediction model. Hence, the feasibility of computing the seismic hazard in terms of a IM by an available and easy computable ground motion prediction model must be considered. Properly, it is possible to mention about the hazard computability property of a ground motion IM.

One of the most desirable properties of IM is the efficiency, which is defined as the standard deviation of IM_{cap} values associated with a given EDP level, $\sigma_{\ln(IMcap|EDP)}$. The standard error of the sample mean of $\ln(IM_{cap})$ for a specified EDP level is proportional to $\sigma_{\ln(IMcap|EDP)}$, with reducing the number of nonlinear dynamic analyses and earthquake records necessary to estimate the conditional distribution of IM_{cap} given EDP level. Fixing an EDP level, probability density functions and cumulative distribution functions of IM_{cap} can be shown and compared to choose which investigated IM has the best influence on $G_{\mathrm{IMcap|EDP}}$. It should be particularly noted how much the variation of the dispersion changes between the two proposals. In fact, observing dispersion index is the best tool to compare different IMs with possible different units. So, an efficient IM reduces the dispersion level of the distribution of $\mathrm{IM}_{\mathrm{cap}}$ associated at different ductility level, with the correspondent reduction of the uncertainty level associated to $\lambda_{\mathrm{EDP}}(edp)$ assessment.

Other two desirable IM properties are the sufficiency and the scaling robustness, which are better shown using IM-based rule. A sufficient IM is conditionally statistically independent of ground motion characteristics, such as magnitude M, distance R, ε parameter, etc. The idea that the ground motion characteristics given IM have a little systematic effect on the resulting seismic demand implies that the traditional form of application of the total probability theorem for PSDA is appropriate. In the integral form for assessing the MAF of exceeding an EDP level shown in Equation (1), there is no need to condition upon additional ground motion characteristics for a sufficient IM, i.e., $G_{\mathrm{EDP|IM}} \approx G_{\mathrm{EDP|IM},M,R,\varepsilon,\ldots}$. If an IM is not sufficient, the estimate of $G_{\mathrm{EDP|IM}}$ depends to some degree on which earthquake records are selected, thus ultimately altering the estimated seismic performance of the structure.

The last desirable property concerns the structural response which should be unbiased after the scaling operation to a value of scale factor equal to IM (i.e., if it is compared to the analogous responses obtained from un-scaled ground motions). If a selected IM is "robust with scaling", then the structural response for scaled earthquake records do not show any bias towards their scale factors. This characteristic has an important role in PSDA, where records are used to predict the probability of exceeding each value of EDP given the value of the IM through IDA process.

3 COMPUTATION OF $S_{a,avg}$

The description "average" of spectral accelerations can be interpreted in several ways, but Bianchini et al. (in prep.) defines it as the geometric mean of the spectral pseudo-acceleration ordinates at 5% of damping

$$S_{a,avg}\left(T_1,\ldots,T_n\right)=\left(\prod_{i=1}^{n} S_a\left(T_i\right)\right)^{1/n} \tag{3}$$

where $T_1,\ldots,T_n$ are the n periods of interest. Following the EDP-based rule, $S_{a,avg}$ becomes the proposed $\mathrm{IM}_{\mathrm{cap}}$. It should be noted that T_i does not mean the i-th natural period of vibration, but only the i-th value in the $T_1,\ldots,T_n$ set of periods. By taking logarithms of both sides of Equation (3), $S_{a,avg}$ can be expressed in the following form

$$\ln S_{a,avg}\left(T_1,\ldots,T_n\right)=\frac{1}{n}\sum_{i=1}^{n}\ln S_a\left(T_i\right) \tag{4}$$

Equation (4) simply computes the arithmetic mean of the logarithm of spectral accelerations. It is more convenient because ground motion prediction models quote the results of regression analyses in terms of natural logarithm of spectral accelerations.

Since multiple $\ln S_a(T_\mathrm{i})$ values are jointly Gaussian distributed (or similarly that $S_a(T_i)$ values are jointly lognormally distributed) as shown by Jayaram & Baker (2008), then a sum of them is still Gaussian, and it is fully described by the following expression of mean and variance

$$\mu_{\ln S_{a,avg}}=\frac{1}{n}\sum_{i=1}^{n}\mu_{\ln S_a(T_i)} \tag{5}$$

$$\sigma^2_{\ln S_{a,avg}}=\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{\ln S_a(T_i),\ln S_a(T_j)}\sigma_{\ln S_a(T_i)}\sigma_{\ln S_a(T_j)} \tag{6}$$

where $\mu_{\ln Sa(Ti)}$ and $\sigma_{\ln Sa(Ti)}$ are, respectively, the conditional mean and the standard deviation of $\ln S_a(T_i)$, available from popular ground motion attenuation models. It should be noted that the term conditional refers to the value for a given earthquake moment magnitude, site-to-source distance, site classification, etc. The term $\rho_{\ln Sa(Ti),\ln Sa(Tj)}$ in Equation (6) represents the correlation between the spectral shape of a single horizontal ground motion component at two different periods T_i and T_j. The correlation of spectral acceleration values was already computed by Baker & Jayaram (2008) from NGA ground motion models.

Bianchini et al. (in prep.) have proven that, for multi-degree-of-freedom systems, $S_{a,avg}$ can be calculated using ten points logarithmic spaced in the interval $T_1,\ldots,T_n$. Furthermore, they supposed that T_1 and T_n are unknown, but tied to the fundamental period of the structure, $T^{(1)}$. So, the average of spectral accelerations is calculated such that $T_1 = k_l T^{(1)}$ and $T_n = k_u T^{(1)}$, where k_l and k_u are constants specifying lower and upper bounds, respectively, relative to $T^{(1)}$. With these assumptions, Equation (3) can be written as

$$S_{a,avg}=\left[\underbrace{S_a\left(k_l T^{(1)}\right)\times\ldots\times S_a\left(k_u T^{(1)}\right)}_{\text{10 points log-spaced}}\right]^{1/10} \tag{7}$$

and Equation (4) as

$$\ln S_{a,avg}=\frac{1}{10}\left(\underbrace{\ln S_a\left(k_l T^{(1)}\right)+\ldots+\ln S_a\left(k_u T^{(1)}\right)}_{\text{10 points log-spaced}}\right) \tag{8}$$

The constant k_l is chosen to vary between $T_{low}/T^{(1)}$ and 1, whereas k_u between 1 and $T_{upp}/T^{(1)}$, where T_{low}

and T_{upp} are, respectively, the lower and the upper period of the elastic spectrum (which is constrained by the filter frequencies of the ground motions). For multi-degree-of-freedom systems, the lower bound $k_l T^{(1)}$ aims to assess the higher-mode influence on the dynamic behaviour of systems, while the upper one $k_u T^{(1)}$ accounts the response when the structure is damaged, and its effective period becomes lengthened. For structures first-mode dominated (i.e., for single-degree-of-freedom systems), which have just one predominant natural period of vibration, it does not make sense to speak of higher-mode contributions. So, the interval where $S_{a,avg}$ is calculated becomes simply ($T^{(1)}$,...,$k_u T^{(1)}$). Basically, when $k_l = k_u = 1$, $S_{a,avg}$ matches exactly S_a. Furthermore, if the system (whatever single- or multi- degree-of-freedom) is modelled with an elastic behaviour, then k_u becomes unimportant, because the range from $T^{(1)}$ onwards is the interval where the damaged structure inelastically modelled has its dynamic behaviour.

With regard to the efficiency property, results showed that k_u is always relevant for structures with a degrading nonlinear behaviour. In fact, for very stiff structures (i.e., referable to a single-degree-of-freedom system), k_u can be assumed equal 2.00, whereas of course $k_l = 1.00$. Similar trend was proven also for multi-degree-of-freedom systems, but it varies from 2.00 (low ductility level, say $\mu < 4$) to 3.00 (high ductility level, say $\mu \geq 4$). If systems are unaffected by vibration modes greater than the first one, k_l will be assumed equal 1.00. Concerning structures showing higher-mode effects, k_l oscillates around 0.25 for multi-degree-of-freedom systems with an important higher-mode influence, e.g., systems with combined shear-type and flexural-type deformation.

4 SIMULATED SYSTEMS AND GROUND MOTION RECORDS

Multi-degree-of-freedom systems are considered in this study adopting an hysteretic nonlinear model that includes strength and stiffness deterioration properties (Medina & Krawinkler 2003). The bilinear and peak oriented hysteretic model, which are normally used in seismic demand analysis to describe nonlinear behaviour of structures, are assumed to describe the response of multi-degree-of-freedom systems. They integrate an energy-based deterioration parameter, $\gamma_{s,c,k,a}$, that controls the cyclic deterioration modes (basic strength, post-capping strength, unloading stiffness, and accelerated reloading stiffness deterioration). The backbone curve without deterioration property is totally defined by the elastic initial stiffness, K_e, the yield strength, F_y, and the strain-hardening stiffness, $K_s = \alpha_s K_e$, where α_s represents the strain hardening. If deterioration is included, the backbone curve is completed by the ductility capacity (i.e., the ratio between the displacement at the peak strength, δ_c, and the yield displacement, δ_y) and the post-capping stiffness ratio, α_c, which controls the softening branch such that the post-capping stiffness can be written as $K_c = \alpha_c K_e$. For multi-degree-of-freedom systems, the critical damping is assumed to 5% value in accord with the viscous damping allowed in FEMA 356 (2000). A detailed description of backbone curve and hysteretic models can be found in (Ibarra et al. 2005). The resulting structural demand parameter (EDP) considered in this study is the maximum inter-storey drift angle, θ_{max}.

Thought the previous models are calibrated to describe the behaviour of a component, we assume that the response of multi-degree-of-freedom systems follows the same hysteresis and deterioration rules as a representative component. This is a simplifying assumption, as it is idealistic to assume that all components of structural system have the same deterioration properties and yield and deteriorate simultaneously. This assumption should be more acceptable when buildings are dominated by their elastic first-mode of vibration, i.e., when they can assumed as a single-degree-of-freedom systems.

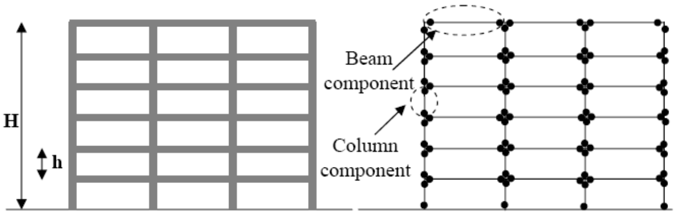

Figure 2. Multi-degree-of-freedom systems and the corresponding structural model: (left) geometry of moment-resisting frames and (b) their structural model (Zareian 2006).

The multi-degree-of-freedom systems considered in this work are constituted by two-dimensional regular frames of a three bays and several stories (Fig. 2), as modelled and analyzed by Ibarra et al. (2005). Three moment-resisting frames, as representative of low- and mid-rise structures and with a variety of structural properties, represent this set of multi-degree-of-freedom systems. They have six different number of stories, N = 6, 12 and 18 floors, and the fundamental period of structures is associated with this number. Structures are identified with their fundamental period $T^{(1)}$, such that $T^{(1)} = 0.1N$. The peak-oriented hysteretic model, which is considered at the beam ends and at the base of columns, is used for all structures. It utilizes $\gamma_{s,c,k,a} \rightarrow \infty$, as well as the following backbone curve parameters: α_s = 3%, $\delta_c/\delta_y = 4$, α_c = -10%. Each structure has three bays with story stiffness and strengths chosen to be representative of traditional structures. Especially when the height of the floor is noticeably high (say greater than 9), these multi-degree-of-freedom systems are particularly influenced to higher-mode excitations, because relative element stiffness are designed to obtain a straight line deflected shape for the first mode, and columns in a story and beams above them have the same moment of inertia.

Multi-degree-of-freedom systems are subjected to the same set of 40 ordinary ground motions recorded in California, and they are chosen as significant statistical sample of time histories. These records were used in Ibarra & Krawinkler (2005) and in Zareian (2006). They do not exhibit pulse-type near-fault characteristics and are recorded on stiff soil or soft rock, corresponding to soil type D according to FEMA 356 (2000). The source-to-site distance, R_{rup}, ranges from 13 to 40 km and the moment magnitude, M_w, from 6.5 to 6.9. Further requirements have been accounted, such as fault mechanism, presence of aftershocks, a bound on high-pass corner frequency, etc., because they could influence noticeably the recorded time history and the shape of the elastic and inelastic spectra (Bazzurro et al. 2004). The use of a single set of ground motion records is acceptable because it has been shown that inelastic response of systems is not greatly affected by M_w and R_{rup} (except for near-fault regions). Regarding the size of the set of GMs, the uncertainty associated with the estimated EDPs and collapse capacities may be quantified as a function of the number of data points evaluated in the form of confidence levels. The use of a set of 40 ground motion records provides estimates of the median that are within a one-sigma confidence band of 10% as long as the standard deviation of the natural logarithm of the collapse capacities or EDPs is less than $0.1(40)^{0.5} = 0.63$ (Tothong & Luco 2007).

5 RESPONSE PREDICTION USING $S_{a,avg}$

As previously mentioned, in order to obtain a reliable assessment of response prediction in PBEE framework, an IM must hold some desirable properties, such as hazard computability, efficiency, sufficiency and scaling robustness.

5.1 *Hazard computability property*

A ground motion prediction model can be easily developed for $\ln S_{a,avg}$ with an arbitrary set of periods $T_1,...,T_n$ using existing attenuation models. If a general attenuation law describes the ground motion IM as a function of magnitude, distance, site geology, etc. in terms of natural period of vibration, then it can be proved that the regression coefficients for $\ln S_{a,avg}(T_1,...,T_n)$ can be obtained simply by the mean of the regression coefficients for $\ln S_a(T_i)$. Thus, PSHA can be performed using $\ln S_{a,avg}$ as intensity measure in the same way of any single spectral acceleration value, using Equations (5) and (6). For example, Cordova et al. (2000) performed three hazard analyses for the Van Nuys site, using the Abrahamson and Silva (1997) ground motion prediction model for the prediction of average of spectral accelerations at two selected periods.

5.2 *Efficiency property*

Following IM-based rule, results obtained by comparison of popular elastic-based scalar IM such as peak ground acceleration, PGA, S_a and $S_{a,avg}$ are here presented in terms of the standard deviation of natural logarithms of the random variable IM_{cap} given a value of EDP, $\sigma_{\ln(IMcap|EDP)}$, i.e., the dispersion. Capacity ductility level μ, which can be associated with an EDP value, varies from 1 to 6, in order to observe the efficiency of the IMs for linear response up to significantly nonlinear response.

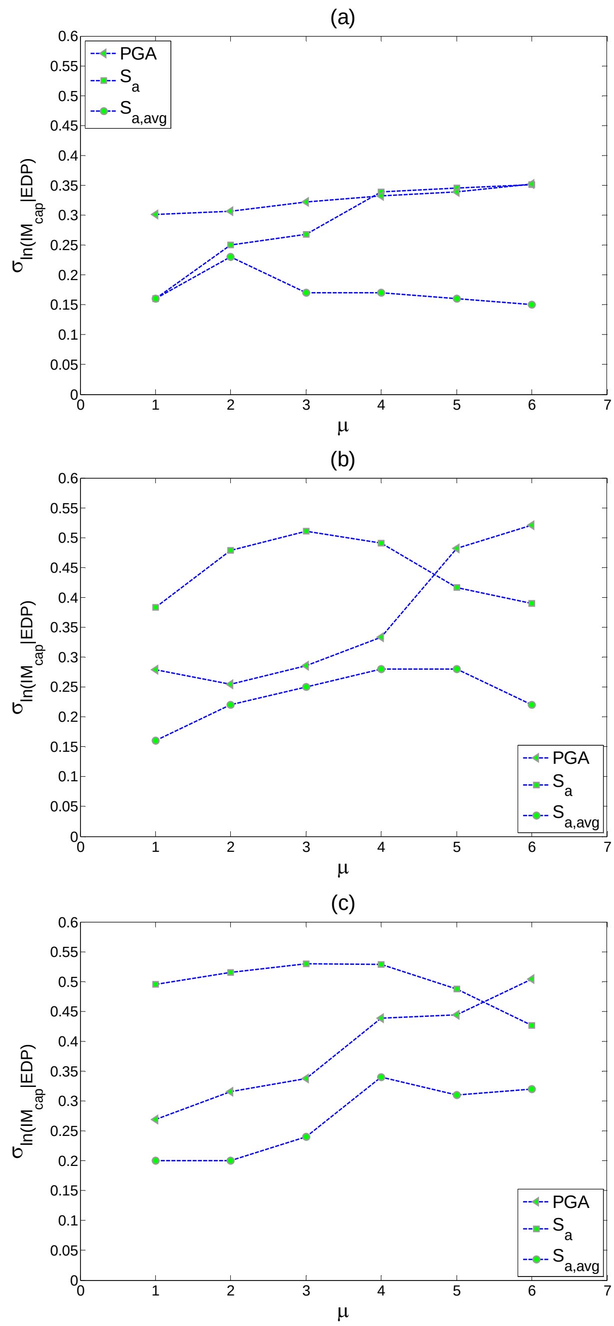

Figure 3. Comparison between different elastic-based scalar IMs in terms of $\sigma_{\ln(IMcap|EDP)}$ when IM = PGA, IM = S_a and IM = $S_{a,avg}$ for multi-degree-of-freedom systems with (a) $T^{(1)} = 0.6$ s and $N = 6$ floors, (b) $T^{(1)} = 1.2$ s and $N = 12$ floors, and (c) $T^{(1)} = 1.8$ s and $N = 18$ floors.

Figure 3 shows the trend for PGA, S_a and $S_{a,avg}$ for deteriorating multi-degree-of-freedom systems with (a) short, (b) medium and (c) long fundamental period. Results about PGA are extremely variables, because it is strongly insufficient and frail for scaling recorded ground motions. S_a appears an efficient predictor for structures with short period (a), but not for structures with medium and large periods (b, c). In every case, $S_{a,avg}$ shows its efficiency in PSDA approach, because it is always associated to small values of dispersion. As previously assessed, for the structure with the shortest period (a), it should be noted that the dispersion at elastic level (μ = 1) for S_a matches that one for $S_{a,avg}$, but it increases when capacity ductility level increases. When higher mode contributions become essential for the resulting structural behaviour and for high level of degradation, $S_{a,avg}$ appears to be the best solution as IM in PSDA (Bianchini et al., in prep.).

5.3 *Sufficiency property*

The sufficiency of an IM is evaluated by performing a regression analysis on the residuals of the PSDA results in terms of EDP|IM relative to the ground motion characteristic, magnitude M, distance R and ε parameter. A small p-value for the linear regression of the residuals of EDP|IM on M, or R, or ε is indicative of an insufficient IM, in which the slope of the linear trend (i.e., the coefficient of the regression estimate β_1) is statistically significant. Recall the p-value is defined as the probability of rejecting the null hypothesis in an analysis of variance, where the null hypothesis states that the coefficient of regression is zero. Smaller p-values indicate stronger evidence for rejecting the null hypothesis (higher statistical significance) and evidence of an insufficient IM. Hence, sufficiency property of an IM can be demonstrated in an absolute way, because it depends on how much β_1 moves away from zero. A small p-value indicated that it is very unlikely that true value of β_1 is zero, i.e., ground motion characteristic has a statistically significant effect on the structural response.

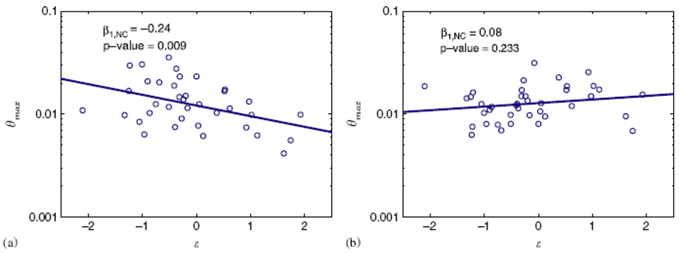

Figure 4. Sufficiency property of IM expressed as dependence of the maximum interstorey drift ratio (θ_{max} for non-collapse cases, NC) for a multi-degree-of-freedom system with $T^{(1)}$ = 1.8 s and N = 18 floors on the ground motion ε for a common value of (a) S_a and (b) $S_{a,avg}$. Regression estimate coefficients β_1 and p-value are shown in the figure, where bold lines show the regression fit.

It has been shown that S_a can be insufficient with respect to M_w and R_{rup} for tall, long-period structures and for near-source ground motions; in particular, S_a can be strongly insufficient with respect to the ground motion parameter ε (Baker & Cornell 2005a). This is proven in Figure 4a, which shows a plot of ln($\theta_{\max}$) versus ε jointly with a linear regression analysis by least-square-method for an inelastic multi-degree-of-freedom systems subjected to records scaled to a given S_a level. It should be noted that the slope of the linear trend between ln($\theta_{\max}$) and ε is statistically significant, as indicated by the small p-value for the estimated slope coefficient β_1. Figure 4b provides a linear regression analysis between ln($\theta_{\max}$) and ε scaling records in IDA to a comparable $S_{a,avg}$ level. The associated p-value shows the statistically insignificant of $S_{a,avg}$ by regards to ε parameter. Although not shown here, a similar trend in terms of sufficiency for $S_{a,avg}$ can be demonstrated with respect to M_w and R_{rup}.

5.4 *Scaling robustness property*

Similarly to the efficiency, the evaluation of the robustness of an IM is evaluated by performing a regression analysis on the residuals of the PSDA results in terms of EDP|IM relative to the scale factor used to perform IDA. The response for records scaled by different factors but to the same resulting IM level should not show a trend in responses versus scale factors. Such scaling robustness property is important because scaled records are often (as it was done in this work) used in PSDA to establish the first integrand in Equations (1) or (2) via IDA. For a inelastic multi-degree-of-freedom systems, Figure 5a demonstrates that scaling records to a value of S_a tends to result in biased structural responses that increase with increasing scale factors. This observation can be explained by the fact that records with large scale factors tend to have a smaller ε values. At the opposite, when using $S_{a,avg}$ as IM (see Fig. 5b) there is no statistically significant trend between ln($\theta_{\max}$) and the natural logarithm of the scale factor, indicating that $S_{a,avg}$ is robust with respect to scaling.

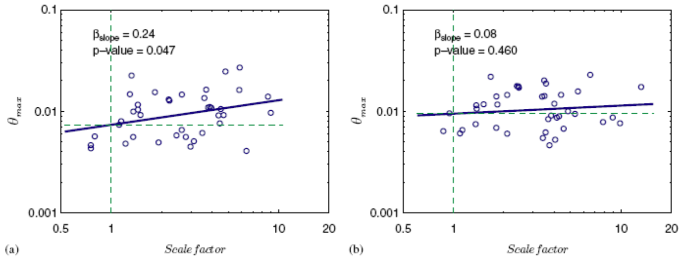

Figure 5. Scaling robustness property of IM expressed as maximum interstorey drift ratio (θ_{max}) for a multi-degree-of-freedom system with $T^{(1)}$ = 1.8 s and N = 18 floors versus scale factors for records scaled to a common value of (a) S_a and (b) $S_{a,avg}$. Regression estimate coefficients β_1 and p-value are shown in the figure, where bold lines show the regression fit and dashed ones the median θ_{max} predicted for un-scaled records.

CONCLUSIONS

The desirable properties of a ground motion IM based on an average of spectral acceleration ordinates in a given range of periods, $S_{a,avg}$, were analyzed in terms to the practicability in PSDA of inelastic multi-degree-of-freedom systems. In particular, $S_{a,avg}$ was calculated as the geometric mean of ten points logarithmic spaced in the interval $T_1,...,T_n$, where bounds are tied to the fundamental elastic period of structures, $T^{(1)}$. Such desirable properties, which leads to a definition of an optimal IM, can be summarized in hazard computability, efficiency, sufficiency and scaling robustness.

Since existing ground motion prediction models provide the natural logarithm of spectral values at a given period, it has been proven that the seismic hazard at the site can be computed for $S_{a,avg}$ simply averaging regression coefficients of attenuation laws in a given range of periods.

Using the so-called EDP-based rule, the MAF a level of an EDP can be obtained by the cumulative distribution function of the IM_{cap} given a value of EDP, jointly integrated with the hazard curve at the site. Using the latter formulation on resulting IDA curves of inelastic systems subjected to ground motion records, the variation of the standard deviation of $\ln(IM_{cap}|EDP)$ on several values of median ductility capacity for different elastic scalar-based IM was observed. By calibration of bounds of the interval, which accounts the influence of higher-modes and the rate of inelasticity, $S_{a,avg}$ became more efficient than traditional elastic scalar-based IMs for several level of median ductility capacity.

Linear regression analyses by least-square-method were conducted on the residuals of the PSDA results in terms of EDP|IM (IM-based rule) relative first to the ground motion characteristic, such as magnitude, distance and ε parameter, second to the scale factor used in IDA. This procedure was useful to assess sufficiency and scaling robustness properties of $S_{a,avg}$ compared with those resulting from S_a. The p-values of regression analyses indicates the statistically insignificance of $S_{a,avg}$ from both point of views, i.e., $S_{a,avg}$ could be considered also sufficient and robust to scaling process, then a practice predictor of inelastic structural response in PBEE approach.

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