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Statistical Tests of the Joint Distribution of Spectral Acceleration Values

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Abstract Assessment of seismic hazard using conventional probabilistic seismic hazard analysis (PSHA) typically involves the assumption that the logarithmic spectral acceleration values follow a normal distribution marginally. There are, however, a variety of cases in which a vector of ground-motion intensity measures are considered for seismic hazard analysis. In such cases, assumptions regarding the joint distribution of the ground-motion intensity measures are required for analysis. In this article, statistical tests are used to examine the assumption of univariate normality of logarithmic spectral acceleration values and to verify that vectors of logarithmic spectral acceleration values computed at different sites and/or different periods follow a multivariate normal distribution. Multivariate normality of logarithmic spectral accelerations are verified by testing the multivariate normality of interevent and intraevent residuals obtained from ground-motion models.

The univariate normality tests indicate that both interevent and intraevent residuals can be well represented by normal distributions marginally. No evidence is found to support truncation of the normal distribution, as is sometimes done in PSHA. The tests for multivariate normality show that interevent and intraevent residuals at a site, computed at different periods, follow multivariate normal distributions. It is also seen that spatially distributed intraevent residuals can be well represented by the multivariate normal distribution. This study provides a sound statistical basis for assumptions regarding the marginal and joint distribution of ground-motion parameters that must be made for a variety of seismic hazard calculations.

Introduction

Spectral acceleration values of earthquake ground motions are widely used in seismic hazard analysis. Conventional probabilistic seismic hazard analysis (PSHA) (e.g., Kramer, 1996) provides a framework for the probabilistic assessment of a single ground-motion parameter (such as the spectral acceleration computed at a single period). When implementing PSHA, it is typically assumed that the spectral acceleration follows a lognormal distribution marginally. There are, however, cases in which knowledge about the joint occurrence of several spectral acceleration values, corresponding to different periods, is required for hazard assessment (Bazzurro and Cornell, 2002). Additionally, a single earthquake can cause severe damage over a large area. Hence, when assessing the impact of earthquakes on a portfolio of structures or a spatially distributed infrastructure system, it is necessary to study the joint occurrence of spectral acceleration values at various sites in the region (Crowley and Bommer, 2006). Moreover, the knowledge of a vector of ground-motion intensity measures is useful in other practical applications that involve computation of the seismic response of a structure dominated by more than one mode (Shome and Cornell, 1999; Vamvatsikos and Cornell, 2005) or that involve joint prediction of structural and nonstructural seismic responses for loss estimation purposes, and it is useful in prediction of multiple demand parameters such as displacement and hysteric energy. In such cases, a vector of intensity measures needs to be considered, and hence, it is necessary to study the joint distribution of these intensity measures in observed ground motions.

Various empirical ground-motion models have been developed for estimating the response spectrum of a given ground motion (e.g., Abrahamson and Silva, 2007; Boore and Atkinson, 2007; Campbell and Bozorgnia, 2007; Chiou and Youngs, 2007). A typical ground-motion model has the form

$$\ln(Y) = \ln(\bar{Y}) + \varepsilon + \eta, \tag{1}$$

where Y denotes the ground-motion parameter of interest (e.g., $S_a(T_1)$, the spectral acceleration at period T_1); $\bar{Y}$ denotes the predicted (by the ground-motion model) median value of the ground-motion parameter (which depends on parameters such as magnitude, distance, period, and local soil conditions); ε denotes the intraevent residual, which is a random variable with zero mean and a standard deviation of σ; and η denotes the interevent residual, which is a random variable with zero mean and a standard deviation of τ. The standard deviations, σ and τ, are estimated during the derivation of the ground-motion model and are a function of the response period and, in some models, are a function of earthquake magnitude and distance from the rupture. Normalized intraevent residuals ($\tilde{\varepsilon}$) are obtained by dividing ε by σ. Similarly, η can be normalized using τ to obtain $\tilde{\eta}$.

The logarithmic spectral acceleration at a site due to an earthquake is usually assumed to be well represented by the normal distribution marginally (e.g., Kramer, 1996). Abrahamson (1988) performed rigorous statistical studies to verify the assumption that logarithmic peak ground acceleration (PGA) values follow the normal distribution marginally. Such rigorous studies have, however, not been performed on spectral accelerations. Moreover, the assumption of normality must be extended to the joint distribution of the logarithmic spectral accelerations, when performing vector-valued seismic hazard analysis (Bazzurro and Cornell, 2002; Baker and Cornell, 2006). When multiple ground-motion parameters are considered (for instance, Y_1 and Y_2), the ground-motion model equations take the following form:

$$\begin{aligned} \ln(Y_1) &= \ln(\bar{Y}_1) + \varepsilon_1 + \eta_1, \\ \ln(Y_2) &= \ln(\bar{Y}_2) + \varepsilon_2 + \eta_2, \end{aligned} \tag{2}$$

where $\bar{Y}_1$ and $\bar{Y}_2$ denote the predicted median values of the ground-motion parameters, ε_1 and ε_2 denote the intraevent residuals corresponding to the two parameters, and η_1 and η_2 denote the interevent residuals (η_1 equals η_2 if Y_1 and Y_2 denote $S_a(T)$ at two sites during the same earthquake). If Y_1 and Y_2 are spectral accelerations at two-closely spaced sites or spectral accelerations at two different periods at the same site, the residuals will not be independent (Baker and Cornell, 2006; Baker and Jayaram, 2008). Thus, an assumption of univariate normality does not necessarily imply joint normality between the residuals. There is a paucity of research work that examines the validity of assuming multivariate normality. This paper explores the validity of these assumptions using statistical tests for univariate and multivariate normality and a large library of spectral acceleration values from recorded ground motions.

The ground-motion model of Campbell and Bozorgnia (2007) is used in this study to compute the parameters shown in equations (1) and (2). The conclusions drawn from the work, however, did not change when the Boore and Atkinson (2007) ground-motion model was used as well. The spectral acceleration definition typically used in the next generation attenuation (NGA) ground-motion models is GMRotI50 (also known as GMRotI). This is the fiftieth percentile of the set of geometric means of spectral accelerations at a given period, obtained by rotating the as-recorded orthogonal horizontal motions through all possible nonredundant rotation angles (Boore *et al.*, 2006). The residuals used in this work are obtained based on this definition of the spectral acceleration.

The data for the analysis are obtained from the Pacific Earthquake Engineering Research (PEER) NGA database (see the Data and Resources section). In order to exclude records whose characteristics differ from those used by the ground-motion modelers for data analysis, only records used by the ground-motion model authors are considered in the tests for normality.

Testing the Univariate Normality of Residuals

This section discusses tests performed on the assumption that logarithmic spectral accelerations at a site due to a given earthquake are well represented by the normal distribution, marginally. A practical way to test the univariate normality of a data set is to inspect the normal *Q-Q* plot obtained from the data set by plotting the quantiles of the data sample against the corresponding quantiles of the theoretical normal distribution (e.g., Johnson and Wichern, 2007).

The following steps are involved in the construction of a normal *Q-Q* plot. Let $\mathbf{x}$ be a collection of n data values that need to be tested for normality. The data set is ordered (sorted in ascending order) to obtain $[x_{(1)}, x_{(2)}, \ldots, x_{(n)}]$ (such that $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$). When these sample quantiles $x_{(k)}$ are distinct (which is a reasonable assumption for continuously varying data), exactly k observations are less than or equal to $x_{(k)}$. The cumulative probabilities $p_{(k)}$ of each $x_{(k)}$ can be computed as $\frac{k}{n}$. It has been shown, however, that a continuity correction gives an improved $p_{(k)}$ estimate of $(k - 3/8)/(n + 1/4)$ (Johnson and Wichern, 2007), and hence, this definition of $p_{(k)}$ is used in this work. The normal *Q-Q* plot is obtained by plotting the ordered data samples against the theoretical normal quantiles corresponding to each of the probabilities $p_{(k)}$. The theoretical normal quantile corresponding to probability $p_{(k)}$ is obtained as $\mathbf{\Phi}^{-1}(p_{(k)})$, where $\mathbf{\Phi}^{-1}$ denotes the inverse of the cumulative normal distribution with the mean and the variance equaling the sample mean and the sample variance, respectively. If the data sample follows a normal distribution, the normal *Q-Q* plot will form a straight line with a slope of 45°, passing through the origin.

Results and Discussion

Normality tests are performed on intraevent and interevent residuals in order to verify the univariate normality of logarithmic spectral accelerations at a site due to an earthquake. The intraevent and the interevent residuals provided to us by the ground-motion model authors are used in the normality tests.

Intraevent Residuals. This section discusses results of the univariate standard normality tests performed on the normalized intraevent residuals ($\tilde{\varepsilon}$). As mentioned previously, $\tilde{\varepsilon}$ values are obtained by dividing the intraevent residuals (ε) by the standard deviations (σ) provided by the Campbell and Bozorgnia (2007) model.

Figure 1 shows the normal *Q-Q* plots of $\tilde{\varepsilon}$ computed at four different periods ranging between 0.5 and 10 sec, with the theoretical quantiles derived from the standard normal distribution (normal distribution with zero mean and unit variance). Long periods such as 10 sec may not be used in practice as often as short periods. These long periods are considered in this work, however, in order to cover the entire range of periods in which the ground-motion model used is applicable. Also shown in the figures are 45° lines passing though the origin. Deviation of the normal *Q-Q* plot from the 45° line indicates deviation from standard normality. It can be seen from Figure 1 that the normal *Q-Q* plots match reasonably well with the 45° lines in all four cases. This indicates that $\tilde{\varepsilon}$ can be considered to be univariate standard normal based on this data set. Note that while normality of $\tilde{\varepsilon}$ is assumed in PSHA, it is often assumed that the distribution is truncated. A typical decision would be to truncate the distribution at $\tilde{\varepsilon} = 2$ or 3 and not allow any larger $\tilde{\varepsilon}$ values (Bommer and Abrahamson, 2006). The tail of the marginal distribution needs to be studied in order to determine if this truncation of the normal distribution is reasonable. Figure 1 shows that $\tilde{\varepsilon}$ values larger than 2 are observed as often as would be expected from a nontruncated distribution. With the small data sets used, however, it is not possible to study the tail distribution beyond $\tilde{\varepsilon} = 3$.

A technique to obtain a larger number of samples at the tail of the distribution would be to pool the $\tilde{\varepsilon}$ values computed at different periods. The normalized residuals computed at various periods are shown to follow a standard normal distribution using the normal *Q-Q* plots in Figure 1. Hence, it can be inferred that quantiles of the pooled data set will match with the corresponding quantiles of a theoretical standard normal distribution. The pooled set has a larger number of data points in the tail, and hence, it is preferable to study the tail properties using the pooled data set rather than the individual data sets. Hence, 12,194 $\tilde{\varepsilon}$ values computed at 10 periods ranging from 0.5 to 10 sec are pooled together. The histogram of the pooled data set is shown in Figure 2 along with a scaled plot of the theoretical standard normal distribution. The figure shows that the data are in excellent agreement with the standard normal distribution, as expected based on the normal *Q-Q* plots shown in Figure 1. The normal *Q-Q* plot for the pooled data set is shown in Figure 3. It can be seen that the quantiles from the observed data

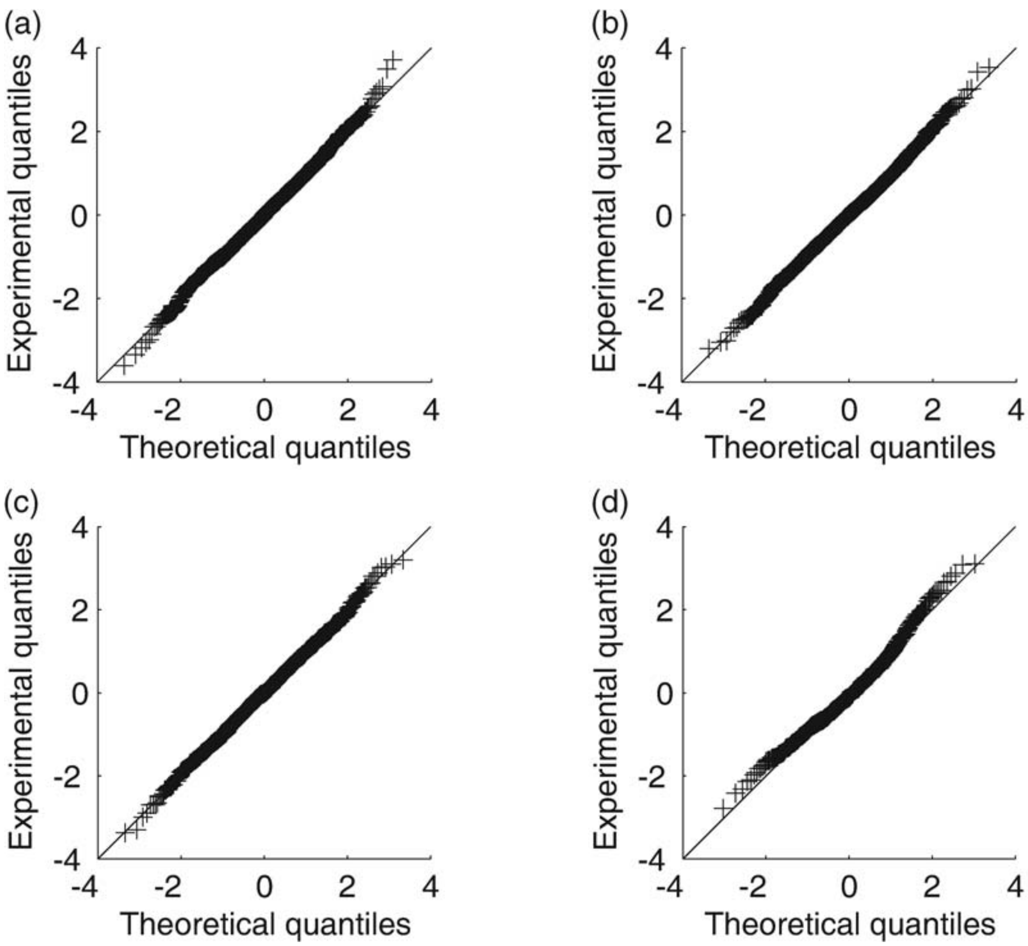

Figure 1. The normal *Q-Q* plots of the normalized intraevent residuals at four different periods. (a) $T = 0.5$ sec (1560 samples); (b) $T = 1.0$ sec (1548 samples); (c) $T = 2.0$ sec (1498 samples); and (d) $T = 10.0$ sec (507 samples).

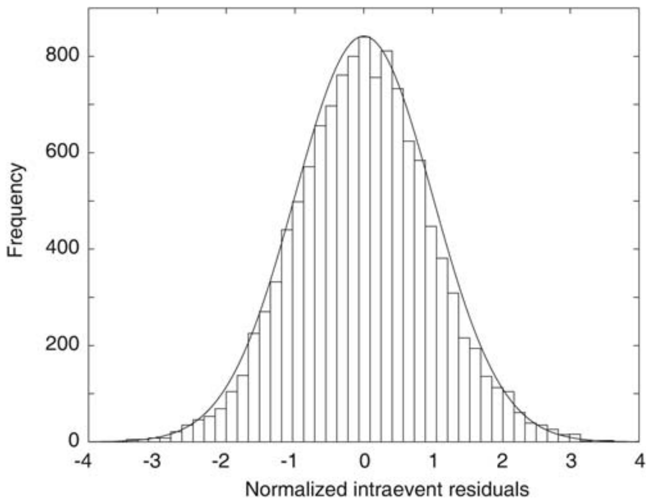

Figure 2. The histogram of the 12,194 pooled normalized intraevent residuals computed at 10 periods, with the theoretical standard normal distribution (scaled) superimposed.

match reasonably well with the theoretical quantiles up to $\tilde{\varepsilon}$ values of 3.5 or 4. Beyond $\tilde{\varepsilon} = \pm 4$, there is no longer enough data to study possible truncation. This large data set thus contradicts claims that an $\tilde{\varepsilon}$ truncation at less than 4 is reasonable and provides no evidence to support truncation at a larger value. This is consistent with the findings of other researchers examining large data sets (Bommer *et al.*, 2004; Abrahamson, 2006; Strasser *et al.*, 2008).

Interevent Residuals. According to the ground-motion model of Campbell and Bozorgnia (2007), the standard deviation of the interevent residuals (η) depends on the rock PGA at the sites. As a result, while the η values computed at any particular period are identical across all the sites during a given earthquake, the normalized interevent residuals ($\tilde{\eta}$) vary across sites even during a single earthquake (because the standard deviation, τ, with which they are normalized varies from site to site). This makes it impossible to use $\tilde{\eta}$ for the normality study. It is seen, however, using the records in the PEER NGA database (see the Data and Resources section) that over 90% of the standard deviations of η (obtained using the ground-motion model of Campbell and Bozorgnia [2007]) lie within a reasonably narrow interval (with an approximate range of 0.04). Hence, homoscedasticity (i.e., constant variance) of η is considered to be reasonable and so the η values are used as such, without normalization.

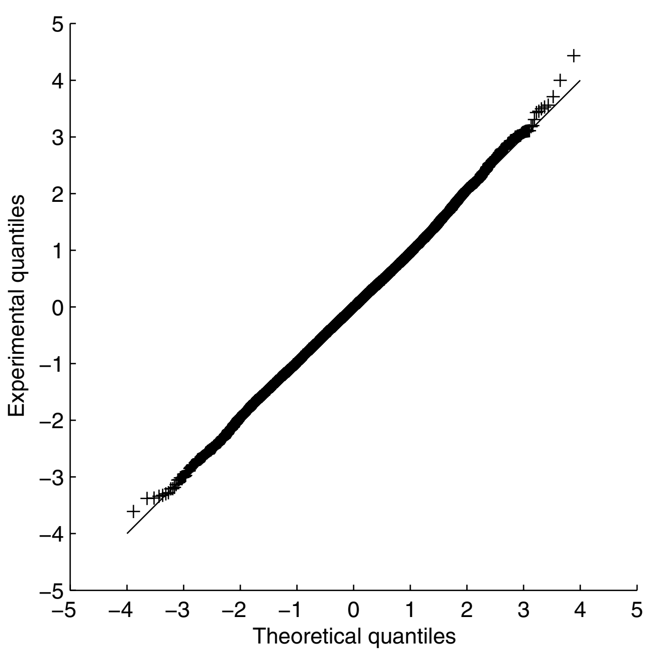

Figure 3. The normal *Q-Q* plot of the pooled set of normalized intraevent residuals.

Figure 4 shows the normal *Q-Q* plot obtained using the η values corresponding to four different periods. The theoretical quantiles are obtained using a normal distribution with zero mean and a standard deviation that equals the sample standard deviation (which does not equal 1 because the η values are not normalized). It is seen from Figure 4a–d that the normal *Q-Q* plots match reasonably well with the 45° straight lines, thereby indicating the univariate normality of interevent residuals.

Testing the Multivariate Normality Assumption for Random Vectors Using Independent Samples

In this section, several statistical tests are presented that can be used with observed ground-motion data to test the validity of the assumed multivariate normal distribution for logarithmic spectral accelerations.

A given ground motion will have spectral acceleration values that vary stochastically as a function of period. Hence, for any d periods, $\mathbf{T} = [T_1, T_2, \ldots, T_d]$, let the corresponding values of spectral acceleration at the sites be denoted by $S_a^j(T_i)$, where j is an index that denotes a given recording while T_i indicates a particular period. The mathematical procedures explained in this section can be used to test whether the random vectors of logarithmic spectral accelerations, $\{\ln[S_a(T_1)], \ln[S_a(T_2)], \ldots, \ln[S_a(T_d)]\}$, are jointly normal.

Testing for multivariate normality is much more complex than testing for univariate normality because there are many more properties in a multivariate distribution to be considered during the test. Among the many possible tests for multivariate normality of a given data set, eight are reviewed in detail by Mecklin and Mundfrom (2003). They examined the power of the eight tests using a Monte Carlo study for several data sets that had predetermined multivariate distributions. They recommend the use of the Henze–Zirkler test (Henze and Zirkler, 1990) as a formal test of multivariate normality, complemented by other test procedures such as the Mardia's skewness and kurtosis tests (Mardia, 1970). Multivariate normality can also be tested using the chi-square plot (also known as the gamma plot) (Johnson and Wichern, 2007), which is a multivariate equivalent of the normal

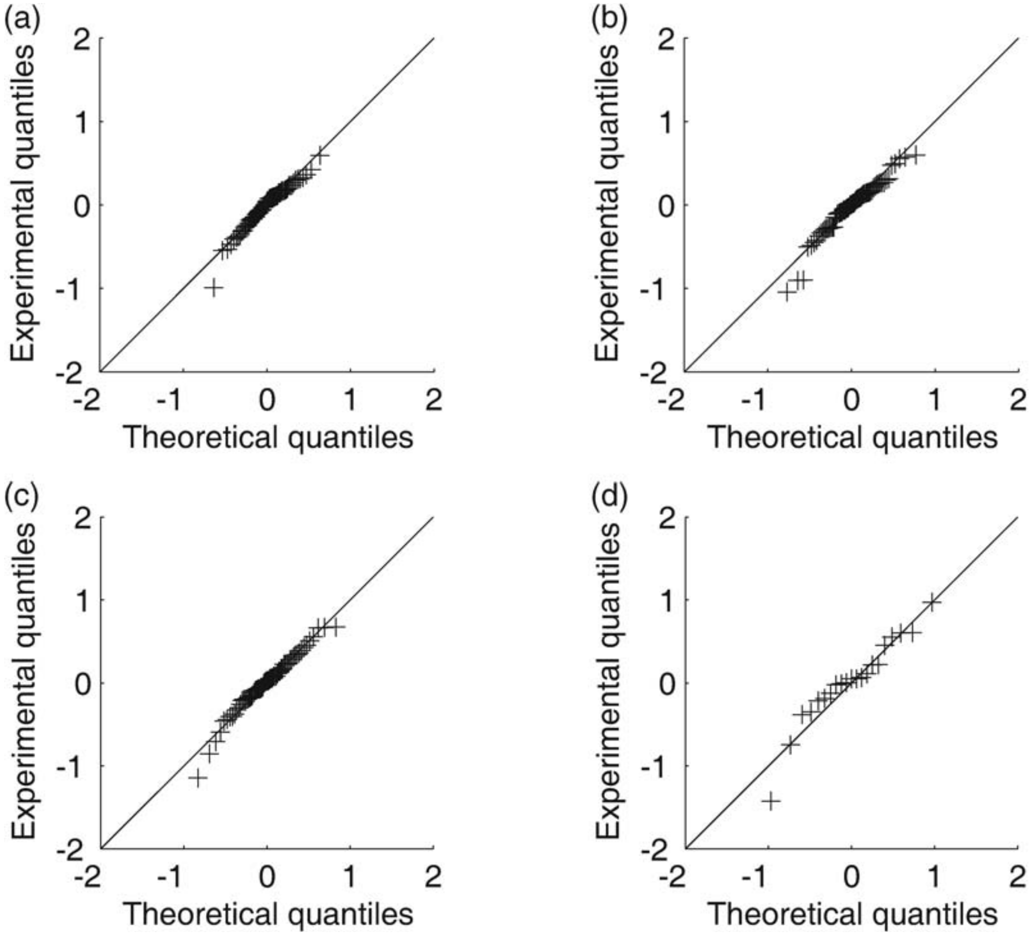

Figure 4. The normal *Q-Q* plots of interevent residuals at four different periods. (a) $T = 0.5$ sec (64 samples); (b) $T = 1.0$ sec (64 samples); (c) $T = 2.0$ sec (62 samples); and (d) $T = 10.0$ sec (21 samples).

Q-Q plot. The procedure to obtain the chi-square plot is similar to that used for a normal *Q-Q* plot except that squared Mahalanobis distances (Mardia *et al.*, 1979) of data samples are used in place of the data quantiles and a theoretical chi-square distribution is used in place of the theoretical normal distribution. A departure from linearity indicates departure from multivariate normality. In this work, however, only the three more quantitative tests, namely, the Henze–Zirkler test and Mardia's test of skewness and of kurtosis, are used. These three tests are described in the following paragraphs.

Henze–Zirkler Test

Henze and Zirkler (1990) proposed a class of invariant consistent tests for testing multivariate normality. The test procedure is based on the computation of a defined test statistic that is a function of the given data and whose asymptotic distribution is known if the data follows a multivariate normal distribution. The statistic can be compared to the asymptotic distribution to test whether the data set can be reasonably assumed to be normal. The Henze–Zirkler test statistic is defined as follows: let $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ be a set of n independent data samples (i.e., the $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ are obtained from n independent records) each of dimension d (i.e., $\mathbf{X}_i = \{X_{i1}, X_{i2}, \ldots, X_{id}\}$). It is to be noted that the variables $X_{i(j_1)}$ and $X_{i(j_2)}$ could be correlated.

$$T_{n,\beta} = \frac{1}{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\left[\exp\left(-\frac{\beta^2}{2}\|Y_j - Y_k\|^2\right)\right] - 2(1+\beta^2)^{-d/2}\sum_{j=1}^{n}\left[\exp\left(-\frac{\beta^2}{2(1+\beta^2)}\|Y_j\|^2\right)\right] + n(1+2\beta^2)^{-d/2}, \quad (3)$$

where

$$\beta = \frac{1}{\sqrt{2}}\left(\frac{2d+1}{4}\right)^{1/(d+4)} n^{1/(d+4)},$$
$$\|Y_j - Y_k\|^2 = (\mathbf{X}_j - \mathbf{X}_k)'\mathbf{S}^{-1}(\mathbf{X}_j - \mathbf{X}_k),$$
$$\|Y_j\|^2 = (\mathbf{X}_j - \bar{\mathbf{X}}_n)'\mathbf{S}^{-1}(\mathbf{X}_j - \bar{\mathbf{X}}_n),$$

where $T_{n,\beta}$ is the test statistic, $\bar{\mathbf{X}}_n$ is the sample mean vector of the n realizations $\mathbf{X}_1, \ldots, \mathbf{X}_n$, and $\mathbf{S}$ is the sample covariance matrix defined as $\mathbf{S} = \frac{1}{n}\sum_{j=1}^{n}(\mathbf{X}_j - \bar{\mathbf{X}}_n)(\mathbf{X}_j - \bar{\mathbf{X}}_n)'$.

Henze and Zirkler (1990) also approximated the limiting distribution of $T_{n,\beta}$ (given the multivariate normality of $\mathbf{X}$) with a lognormal distribution with the mean and the variance defined as follows:

$$E[T_\beta] = 1 - (1+2\beta^2)^{-d/2}\left[1 + \frac{d\beta^2}{1+2\beta^2} + \frac{d(d+2)\beta^4}{2(1+2\beta^2)^2}\right], \tag{4}$$

$$\begin{aligned}\mathrm{Var}[T_\beta] &= 2(1+4\beta^2)^{-d/2} \\ &\quad + 2(1+2\beta^2)^{-d}\left[1 + \frac{2d\beta^4}{(1+2\beta^2)^2} + \frac{3d(d+2)\beta^8}{4(1+2\beta^2)^4}\right] \\ &\quad - 4w(\beta)^{-d/2}\left[1 + \frac{3d\beta^4}{2w(\beta)} + \frac{d(d+2)\beta^8}{2w(\beta)^2}\right],\end{aligned} \tag{5}$$

where $w(\beta) = (1+\beta^2)(1+3\beta^2)$.

Based on the value of the statistic computed using the data and the asymptotic distribution of $T_{n,\beta}$, the p value of the test of multivariate normality can be calculated. The p value is the probability of obtaining a statistic value that is at least as extreme as the statistic computed from the data, if the null hypothesis of multivariate normality were true. The smaller the p value, the stronger the evidence against the null hypothesis. It is suggested that this test be used if the sample size n is at least 20 (Henze and Zirkler, 1990).

Mardia's Measures of Kurtosis and Skewness

Mardia (1970) extended the concepts of kurtosis and skewness from the univariate case to the multivariate case. Mardia (1970) also obtained the asymptotic distribution of the multivariate kurtosis and skewness parameters (which is needed to test the null hypothesis of multivariate normality).

Multivariate Kurtosis. Mardia (1970) defined the multivariate kurtosis coefficient as follows:

$$K = E[(\mathbf{X}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{X}-\boldsymbol{\mu})]^2, \tag{6}$$

where $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n]$ is the random vector whose distribution is tested, $\boldsymbol{\mu}$ is the mean vector of $\mathbf{X}$, $(\mathbf{X}-\boldsymbol{\mu})'$ refers to the transpose of $(\mathbf{X}-\boldsymbol{\mu})$, and $\boldsymbol{\Sigma}$ is the covariance matrix of $\mathbf{X}$. In practice, the value of multivariate kurtosis can be computed from the sample data as follows:

$$k = \frac{1}{n}\sum_{i=1}^{n}[(\mathbf{X}_i - \bar{\mathbf{X}}_n)'\mathbf{S}^{-1}(\mathbf{X}_i - \bar{\mathbf{X}}_n)]^2. \tag{7}$$

Mardia (1970) also showed that the asymptotic distribution of the previously defined multivariate kurtosis parameter (k) can be obtained from the following equation, if $\mathbf{X}$ follows the multivariate normal distribution:

$$\frac{k - [d(d+2)(n-1)/(n+1)]}{[8d(d+2)/n]^{0.5}} \Rightarrow N(0,1), \tag{8}$$

where $N(0,1)$ denotes the univariate standard normal distribution. The asymptotic distribution can be used to test if the sample data are from a multivariate normally distributed population, by allowing a p value to be computed.

Multivariate Skewness. Mardia (1970) and Mardia *et al.* (1979) defined the measure of multivariate skewness to be as follows:

$$S = E[(\mathbf{X_1}-\boldsymbol{\mu})'\,\boldsymbol{\Sigma}^{-1}(\mathbf{X_2}-\boldsymbol{\mu})]^3, \tag{9}$$

where $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n]$ is the random vector whose distribution is tested. This parameter can be computed from the sample data as follows:

$$s = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}[(\mathbf{X}_i - \bar{\mathbf{X}}_n)'\mathbf{S}^{-1}(\mathbf{X}_j - \bar{\mathbf{X}}_n)]^3. \tag{10}$$

The asymptotic distribution of the multivariate skewness parameter (s) can be obtained from the following equation:

$$\frac{ns}{6} \Rightarrow \chi^2_{d(d+1)(d+2)/6}, \tag{11}$$

where $\chi^2_{d(d+1)(d+2)/6}$ is the chi-square distribution with $d(d+1)(d+2)/6$ degrees of freedom. This asymptotic distribution can be used to test the null hypothesis of multivariate normality.

The preceding procedures can be used to test the multivariate normality of any random vector using a set of independent data samples. For instance, these tests can be used to verify the multivariate normality of intraevent residuals computed at multiple periods. In this case, in order to obtain a set of independent data samples, each random vector (composed of intraevent residuals computed at multiple periods) must be obtained from records that are independent of one another. A technique to obtain independent data samples is discussed in a subsequent section.

Results and Discussion

As mentioned earlier, multivariate normality tests need to be performed on intraevent and interevent residuals in order to verify multivariate normality of the logarithmic spectral accelerations. The intraevent residuals are normalized by the appropriate standard deviations before use, while the interevent residuals are used without normalization, for reasons mentioned previously.

Normalized Intraevent Residuals at Different Periods. Let $\tilde{\varepsilon}(\mathbf{T}) = [\tilde{\varepsilon}(T_1), \tilde{\varepsilon}(T_2), \ldots, \tilde{\epsilon}(T_d)]$ denote the random vector of normalized intraevent residuals computed at d different periods. During an earthquake, different sites experience different levels of ground motion based on their distance from the earthquake source, the local soil conditions, and other factors. These ground motions can be used to compute samples ($\tilde{e}_j(\mathbf{T})$) of the random vector $\tilde{\varepsilon}(\mathbf{T})$ at site j. This section uses the samples $\tilde{e}_j(\mathbf{T})$ obtained at various sites to test whether $\tilde{\varepsilon}(\mathbf{T})$ follows a multivariate normal distribution.

The results presented in this work are based on data from the 1994 Northridge earthquake and the 1999 Chi-Chi earthquake. The PEER NGA database (see the Data and Resources section) is used to obtain the data and contains 160 records from the Northridge earthquake and 421 records from the Chi-Chi earthquake (the aftershock data are not used). From these records, only those used by the authors of the Campbell and Bozorgnia (2007) ground-motion model are included in the analysis. Even this reduced data set cannot be used as such because the samples will not be independent of one another on account of the spatial correlation of the ground motion during a given earthquake. It is known, however, that the correlation between $\tilde{e}_i(T_p)$ and $\tilde{e}_j(T_p)$ decreases with increasing separation distance between the sites i and j, where T_p denotes any particular period. It is seen from the literature that the correlation coefficient drops close to zero (i.e., the $\tilde{\varepsilon}(T_p)$ are approximately uncorrelated) when the separation distance exceeds 10 km (Boore *et al.*, 2003). Moreover, it is shown subsequently in this manuscript that the $\tilde{\epsilon}(T_p)$ obtained at different sites from a single earthquake follow a multivariate normal distribution. Hence, approximately uncorrelated $\tilde{\varepsilon}(T_p)$ values are also approximately independent, and therefore, samples of random vectors obtained from recordings at mutually well-separated sites would be approximately independent and can be used in the tests described in the previous section. Therefore, in the current work, well-separated locations (with separation distances exceeding 20 km) are identified for the Northridge earthquake and the Chi-Chi earthquake, and the tests of normality are performed on the data set obtained by combining the Chi-Chi and Northridge earthquake data. There are several possible combinations of recordings that would satisfy the constraints on the minimum separation distance and the minimum sample size (as defined in the section Testing the Multivariate Normality Assumption for Random Vectors Using Independent Samples), and hence, the tests are carried out on the various allowable configurations. Though the test results vary slightly based on the configuration used, p values from only a single data set are reported in this manuscript. The combined data set has around 35 records at periods less than or equal to 2 sec and close to 30 records at periods below 7.5 sec, which are reasonable sample sizes for testing the hypothesis. At 10 sec, however, the number of independent samples available is 22, which barely exceeds the threshold of 20, mentioned in the section Testing the Multivariate Normality Assumption for Random Vectors Using Independent Samples. Hence, $\tilde{\varepsilon}$ values computed at 10 sec are not used often in the tests.

In order to strictly prove multivariate normality of $\tilde{\varepsilon}$, one must evaluate multivariate normality of normalized residuals having all possible period combinations (i.e., all pairs, triplets, etc.). For all practical purposes, however, it is sufficient to consider the joint distribution of $\tilde{\varepsilon}$ computed at five periods. Incidentally, if multivariate normality can be established for such a case, it can be inferred that the lower-order combinations (i.e., subsets of the five periods that are used) also follow a multivariate normal distribution and do not have to be tested explicitly. This is because all subsets of a random vector $\mathbf{X}$ are multivariate normal if $\mathbf{X}$ is multivariate normal (Johnson and Wichern, 2007).

Results from a set of hypothesis test results are shown in Table 1 and are explained in the following paragraphs. The table shows the set of periods at which the $\tilde{\varepsilon}$ values are computed and the p values are obtained based on the Henze–Zirkler test, Mardia's test of skewness, and Mardia's test of kurtosis. Case 1 shown in the table corresponds to the bivariate normality tests on the $\tilde{\varepsilon}$ obtained at 1 and 2 sec. The p values reported by all three tests are statistically insignificant at the 5% significance level typically used for testing. In case 2, five different periods ranging between 0.5 and 2 sec are chosen. The Henze–Zirkler test and the test of skewness report highly insignificant p values, and the test of kurtosis reports a p value of 0.05, which is insignificant as well. The normality tests are also performed considering long periods. In case 3, the periods are chosen over the 0.5–7.5 sec range, as shown in Table 1. The p values reported by all three tests are highly statistically insignificant. Finally, a test is carried out considering long periods exclusively (case 4); the p values obtained from all the tests are statistically insignificant. Overall, there seems to be not much evidence to reject the null hypothesis that $\tilde{\varepsilon}$ computed at different periods follows a multivariate normal distribution.

Interevent Residuals at Different Periods. This section discusses tests carried out on interevent residuals (η) at multiple periods. The number of interevent residuals available for the tests ranges from 64 at 0.5 sec to 40 at 7.5 sec. Only 21 records are available, however, at 10 sec.

Table 2 shows the hypothesis test results based on η values. In case 1, η values at two periods, 1 and 2 sec, are tested for bivariate normality. It can be seen that the p values reported by all three tests are highly insignificant. In case 2, five different periods are chosen ranging between

Table 1
Tests on Normalized Intraevent Residuals Computed at Different Periods

Case	Periods (sec)	P_{HZ}	P_{SK}	P_{KT}
1	$T = \{1.0, 2.0\}$	0.10	0.23	0.93
2	$T = \{0.5, 0.75, 1.0, 1.5, 2.0\}$	0.49	0.92	0.05
3	$T = \{0.5, 1.0, 2.0, 5.0, 7.5\}$	0.69	0.90	0.42
4	$T = \{5.0, 7.5, 10.0\}$	0.19	0.14	0.62

P_{HZ} indicates the p value obtained from Henze–Zirkler test, P_{SK} indicates the p value obtained from Mardia's test of skewness, and P_{KT} indicates the p value obtained from Mardia's test of kurtosis.

Table 2
Tests on Interevent Residuals Computed at Different Periods

Case	Periods (sec)	P_{HZ}	P_{SK}	P_{KT}
1	$T = \{1.0, 2.0\}$	0.85	0.20	0.35
2	$T = \{0.5, 0.75, 1.0, 1.5, 2.0\}$	0.00	0.01	0.01
3	$T = \{0.5, 0.75, 1.0, 1.5, 2.0; \text{Norm.}\}$	0.24	0.11	0.11
4	$T = \{0.5, 1.0, 2.0, 5.0, 7.5\}$	0.79	0.28	0.41
5	$T = \{5.0, 7.5, 10.0\}$	0.68	0.18	0.31

P_{HZ}, P_{SK}, and P_{KT} are defined as in Table 1; Norm. indicates the data transformed to the standard normal space.

0.5 and 2 sec. The table shows that the p values reported by all three tests are statistically significant. We believe, however, that this is a result of the deviations from marginal normality due to the small sample size being carried over to the higher-order distributions (i.e., even if the true marginal distribution is normal, a sample from the distribution will not be exactly normal). In order to verify this, the η values are again computed at the same set of periods as in case 2 and are transformed so that their marginal distributions are normal (in order to remove the deviations in the sample's univariate distribution from the normal distribution), using the normal score transform procedure described by Deutsch and Journel (1998). It is to be noted that the normal score transform (or any other monotonic transform) of the univariate distribution cannot change the basic nature of the bivariate and the other multivariate distributions. Further, the marginal distribution of η has been shown to be normal in the section Testing the Univariate Normality of Residuals, and hence, the transformation of the marginal distribution of the sampled data does not interfere with the tests for multivariate normality. This transformation procedure is described in Appendix A. The tests are performed on the transformed data (case 3) and the p values corresponding to all three tests are seen to increase significantly, indicating that the statistically significant p values in case 2 is probably a result of the deviation of the sample's marginal distribution from a normal distribution rather than an indicator of nonnormality in the joint distribution.

Case 4 involves testing η values at five periods ranging from 0.5 to 7.5 sec. The reported p values are, again, found to be insignificant. In case 5, η values at three long periods are tested for multivariate normality. The p values reported by the three tests are highly statistically insignificant. It can, hence, be concluded from the results that it is reasonable to assume that the η computed at different periods follow a multivariate normal distribution.

Because both the interevent and intraevent residuals computed at multiple periods follow multivariate normal distributions, it is concluded that the logarithmic spectral accelerations computed at different periods, at a given site during a given earthquake, follow a multivariate normal distribution.

Similar tests are used to approximately evaluate the bivariate normality of logarithmic spectral accelerations corresponding to two different orientations, and the results are shown in Appendix B. It is seen that pairs of residuals corresponding to the fault-normal and the fault-parallel data can be reasonably considered to be bivariate normal.

Testing the Multivariate Normality Assumption for Spatially Distributed Data

The tests that have been described so far are valid only for testing random vectors using independent samples. While testing spatially distributed data from a given earthquake, ground-motion recordings at closely separated sites should also be considered, and hence, it is not possible to obtain independent samples using the techniques described in the section Testing the Multivariate Normality Assumption for Random Vectors Using Independent Samples. Hence, certain other tests are needed for testing the multivariate normality assumption for ground-motion intensities distributed over space. Multivariate normality can be ascertained by verifying univariate normality, bivariate normality, trivariate normality, etc. Goovaerts (1997) and Deutsch and Journel (1998) described a procedure to test the assumption of bivariate normality of spatially distributed data whose marginal distribution is standard normal. This test procedure can be used to verify whether pairs of residuals computed at two different sites during a single earthquake follow a bivariate normal distribution. The test is described in the following subsection, followed by test results from recorded ground motions.

Check for Bivariate Normality

Let $X(u)$ denote the random variable (e.g., the residuals) in consideration at location u, and let $X(u+h)$ denote the random variable in consideration at location $u+h$ (h denotes the spatial separation between the two locations). The procedure to test bivariate normality (Goovaerts, 1997; Deutsch and Journel, 1998) involves the comparison of the indicator semivariogram of the data (the experimental indicator semivariogram) to the theoretical indicator semivariogram obtained by assuming that $[X(u), X(u+h)]$ follows a bivariate normal distribution.

An indicator semivariogram is a measure of spatial variability and is defined as follows:

$$\gamma_I(h; x_p) = \frac{1}{2} E(\{I[X(u+h); x_p] - I[X(u); x_p]\}^2), \quad (12)$$

where x_p denotes the p quantile of X and $I[X(u); x_p] = 1$ if $X(u) \le x_p$ and $I[X(u); x_p] = 0$ otherwise.

The experimental indicator semivariogram is a regression-based relationship between $\gamma_I(h; x_p)$ and h. In this study, an exponential model is assumed as the form of the regression. Based on an exponential model, the experimental indicator semivariogram can be defined as follows:

$$\gamma_I(h; x_p) = a_{x_p}[1 - \exp(-3h/b_{x_p})], \quad (13)$$

where a_{x_p} and b_{x_p} are the sill and range of the experimental indicator semivariogram, respectively. The sill of a semivariogram equals the variance of X, while the range of a semivariogram is defined as the separation distance h at which $\gamma_I(h; x_p)$ equals 0.95 times the sill (for the exponential model). The range and the sill can be computed using nonlinear least-squares regression based on observed values of $\gamma_I(h; x_p)$ and h. The values (observed) of $\gamma_I(h; x_p)$ for a given data set can be obtained as follows (based on equation 12):

$$\gamma_I(h; x_p) = \frac{1}{2N(h)} \sum_{\alpha=1}^{N(h)} \{I[X(u_\alpha + h); x_p] - I[X(u_\alpha); x_p]\}^2, \quad (14)$$

where $N(h)$ is the number of pairs of data points separated by h (within some tolerance) and $[X(u_\alpha + h), X(u_\alpha)]$ denotes the αth such pair.

Theoretically, if $X(u)$ and $X(u + h)$ follow a bivariate normal distribution, the indicator semivariogram is (Goovaerts, 1997)

$$\gamma_I(h; x_p) = p - \left[p^2 + \frac{1}{2\pi} \int_0^{\sin^{-1} C_x(h)} \exp\left(\frac{-x_p^2}{1 + \sin(\theta)}\right) d\theta\right], \quad (15)$$

where $C_x(h)$ denotes the covariance model of X, given as follows:

$$C_X(h) = \text{covariance}[X(u), X(u + h)]. \quad (16)$$

The null hypothesis that $X(u)$ and $X(u + h)$ follow a bivariate normal distribution is not rejected if the experimental indicator semivariogram compares well to the theoretical indicator semivariogram.

As mentioned earlier, univariate and bivariate normality are not sufficient conditions for multivariate normality. For realistic data sets, however, the tests for trivariate normality and normality at other higher dimensions are impractical. This is because, for example, the trivariate normality test requires many triplets of data points that have the same geometric configuration (in terms of the spatial orientation of the three points), which are usually not available. Hence, in practice, if the sample statistics do not show a violation of the univariate and bivariate normalities, a multivariate normal model can be assumed for X (Goovaerts, 1997).

Results and Discussion

If the spatially distributed normalized intraevent residuals ($\tilde{\varepsilon}$) follow a multivariate normal distribution, it can be seen from equation (2) that the logarithmic spectral accelerations conditioned on the predicted median spectral accelerations will be multivariate normal as well. This is because the interevent residuals at any particular period are constant across all sites, during any single earthquake. Hence, in this section, normality tests are carried out on the normalized intraevent residuals ($\tilde{\varepsilon}$) only.

It has been shown previously that the $\tilde{\varepsilon}$ values can be represented by a normal distribution marginally, and hence, only the bivariate normality test results are considered in this section. To prevent the deviations in the sample's univariate distribution from the normal distribution (which can arise even if the population actually follows a univariate normal distribution) from affecting the results of the bivariate normality test, the univariate distributions of $\tilde{\varepsilon}$ are transformed to the standard normal space using the normal score transform procedure described in Appendix A. As mentioned earlier, the normal score transform of the univariate distribution does not change the basic nature of the bivariate distributions and, hence, does not interfere with the test of bivariate normality.

The procedure to test the bivariate normality of spatially distributed data described by Goovaerts (1997) involves comparing the theoretical and the experimental indicator semivariograms obtained based on the $\tilde{\varepsilon}$ values computed at various periods and for all quantiles x_p (equations 13 and 15). However, such an exhaustive test is practically impossible and so a few sample periods and quantiles are tested here. Based on the symmetry of the bivariate normal distribution, only values of p in the interval [0, 0.5] are needed. The authors present results corresponding to $p = 0.1$, 0.25, and 0.5 so as to cover the entire range. The periods chosen for the illustrations vary over the range of periods for which the ground-motion models are usually valid.

Figure 5a–c shows comparisons of the theoretical and the experimental indicator semivariograms obtained using the Chi-Chi data set, with the $\tilde{\varepsilon}$ values computed at a period of 2 sec. It is to be noted that all records (that are usuable at the chosen period) can be part of the sample data used for obtaining the experimental indicator semivariograms (unlike in the section Testing the Multivariate Normality Assumption for Random Vectors Using Independent Samples where the sample data had to be independent of each other). The theoretical and experimental indicator semivariograms match reasonably well in all cases. Figure 5d shows the comparison of the theoretical and the experimental indicator semivariograms ($p = 0.25$) for the $\tilde{\varepsilon}$ values computed at $T = 2$ sec based on the Northridge earthquake data set, and a reasonable match can be seen there as well. Similar plots are obtained using the Northridge and the Chi-Chi earthquake data sets and are shown in Figure 6. In obtaining this figure, the value of p is kept constant at 0.25, while the value of T is varied from as low as 0.5 sec to as high as 5 sec. A reasonably good match between the theoretical and the experimental semivariograms can be seen in these figures as well. All these results suggest that bivariate normality can be safely assumed for spatially distributed $\tilde{\varepsilon}$. Incidentally, it can be seen from Figures 5 and 6 that the sill of the indicator semivariograms equals $p(1 - p)$, which is a consequence of the

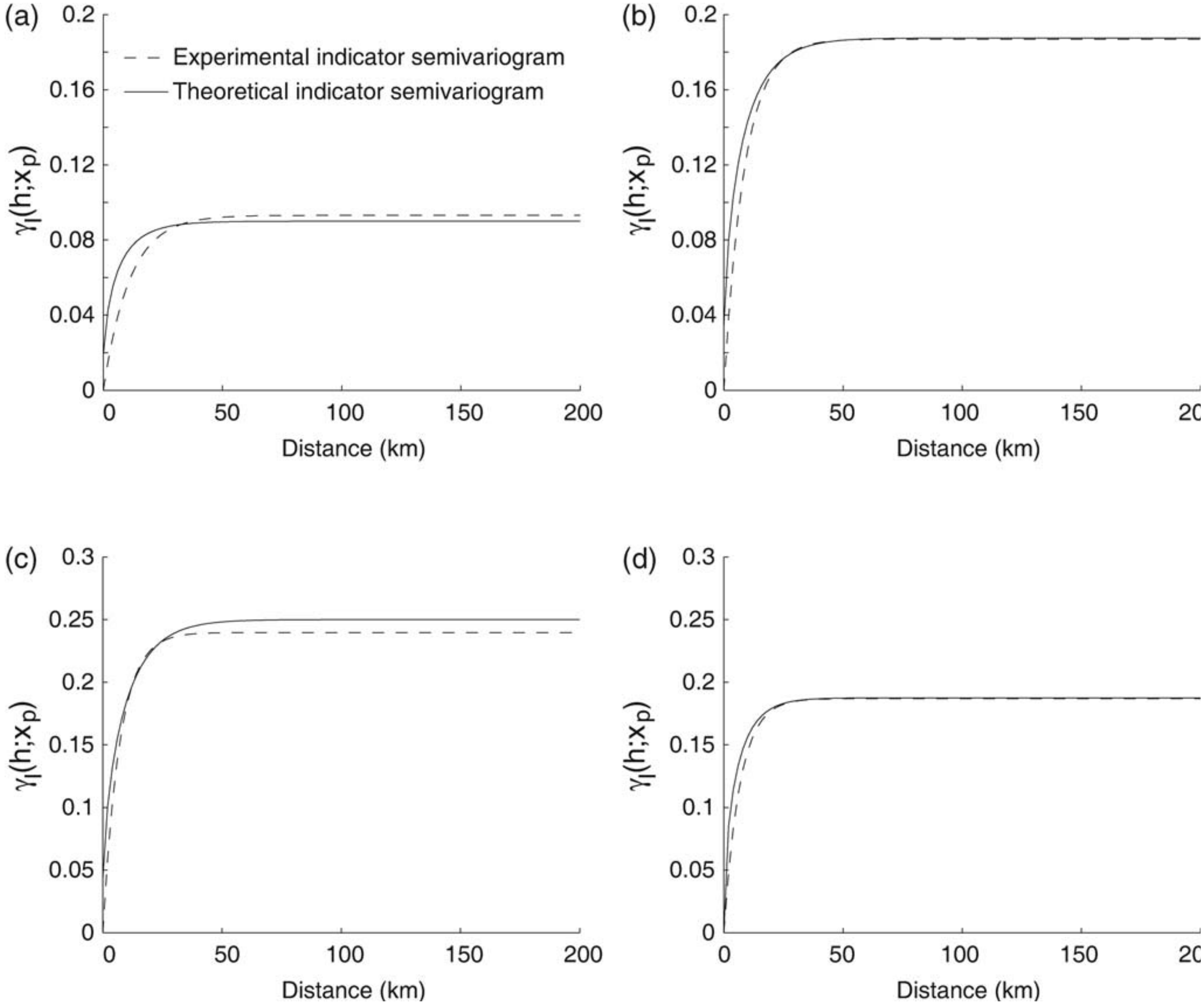

Figure 5. Theoretical and empirical semivariograms for residuals computed at 2 sec. (a) Results for the 0.1 quantile of the residuals from the Chi-Chi data; (b) results for the 0.25 quantile of the residuals from the Chi-Chi data; (c) results for the 0.5 quantile of the residuals based from the Chi-Chi data; and (d) results for the 0.25 quantile of the residuals from the Northridge data.

independence between well-separated intraevent residuals (Goovaerts, 1997).

Conclusions

Statistical tests have been used to test the assumption of joint normality of logarithmic spectral accelerations. Joint normality of logarithmic spectral accelerations was verified by testing the multivariate normality of interevent and intraevent residuals. Univariate normality of interevent and intraevent residuals was studied using normal *Q-Q* plots. The normal *Q-Q* plots showed strong linearity, indicating that the residuals are well represented by a normal distribution marginally. No evidence was found to support truncation of the marginal distribution of intraevent residuals as is sometimes done in PSHA. Using the Henze–Zirkler test, Mardia's test of skewness, and Mardia's test of kurtosis, it was shown that interevent and intraevent residuals at a site, computed at different periods, follow multivariate normal distributions. The normality test of Goovaerts was used to illustrate that pairs of spatially distributed intraevent residuals can be represented by the bivariate normal distribution. For a set of correlated spatially distributed data, it is practically impossible to ascertain the trivariate normality and the normality at higher dimensions, and hence, the presence of univariate and bivariate normalities is considered to indicate multivariate normality of the spatially distributed intraevent residuals (Goovaerts, 1997). The results reported in this study are based on the residuals computed using the ground-motion model of Campbell and Bozorgnia (2007), but similar results were obtained when using the Boore and Atkinson (2007) ground-motion model. This study provides a sound statistical basis for as-

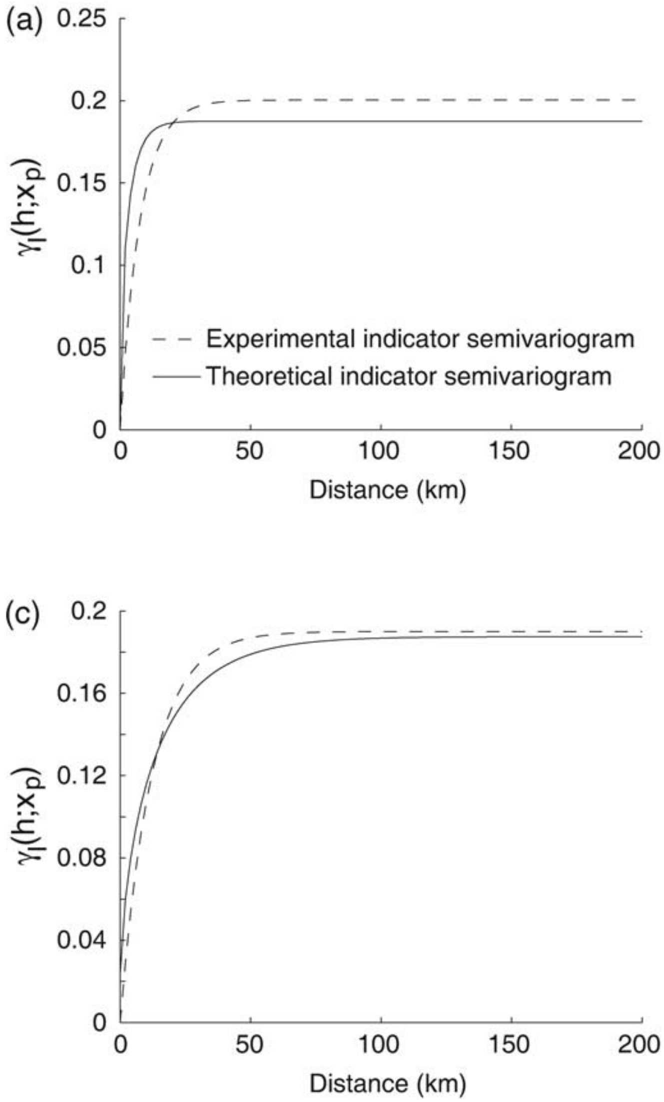

Figure 6. Theoretical and empirical semivariograms for the 0.25 quantile of the residuals. (a) results for the residuals computed at 0.5 sec from the Northridge data; (b) results for the residuals computed at 0.5 sec from the Chi-Chi data; (c) results for the residuals computed at 1 sec from the Chi-Chi earthquake data; and (d) results for the residuals computed at 5 sec from the Chi-Chi data.

sumptions regarding the marginal and joint distribution of ground-motion parameters that must be made for a variety of seismic hazard calculations.

Data and Resources

The data for all the ground motions studied here came from the PEER NGA database (http://peer.berkeley.edu/nga; last accessed May 2007).

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Appendix A

Normal Score Transform

The data sample can be transformed to have a standard normal distribution by a normal score transform. The transformation involves equating the various quantiles of the data to the corresponding quantiles of a standard normal distribution. Let z represent the given data set, and let the empirical cumulative distribution function of the data be denoted by $\hat{F}(z)$. The $\hat{F}(z)$ quantile of the standard normal distribution is given by $\mathbf{\Phi}^{-1}[\hat{F}(z)]$, where $\mathbf{\Phi}$ represents the standard normal cumulative distribution function. Hence, for a given z_k, the corresponding normal score value (y_k) is computed as follows:

$$y_k = \mathbf{\Phi}^{-1}[\hat{F}(z_k)]. \quad \text{(A1)}$$

Appendix B

Spectral Acceleration Values at Different Orientations

This appendix describes tests carried out to verify whether spectral acceleration values corresponding to two different orientations at a site follow a bivariate normal distribution. The test procedures are identical to those described in the section Testing the Multivariate Normality Assumption for Random Vectors Using Independent Samples, except that the random vector is now written as $[S_a^{H_1}(T_1), S_a^{H_2}(T_2)]$, where H_1 and H_2 refer to two orthogonal horizontal orientations (e.g., the fault-normal and the fault-parallel directions) and T_1 and T_2 denote the periods in consideration in the two orthogonal directions.

In order to verify bivariate normality of the spectral accelerations corresponding to two different orientations, normality tests should be carried out on the interevent and the intraevent residuals separately. The interevent residuals in the fault-normal and the fault-parallel directions, however, are not known. As a result, an approximate test for bivariate normality of spectral accelerations in different orientations is carried out by performing tests on normalized total residuals. Total residuals are computed based on the following alternate formulation of the ground-motion equations:

$$\ln(Y) = \ln(\bar{Y}) + \delta, \quad \text{(B1)}$$

where Y denotes the ground-motion parameter of interest, $\bar{Y}$ denotes the predicted median value of the ground-motion parameter, and δ refers to the total residual, which is a random variable that represents both the interevent and the intraevent residuals. From equations (1) and (B1), it can be inferred that δ has zero mean and standard deviation $\sqrt{\boldsymbol{\sigma}^2 + \tau^2}$. Hence, normalized total residuals ($\tilde{\delta}$) can be obtained as $\delta/\sqrt{\sigma^2 + \tau^2}$.

In this work, $\tilde{\delta}$ values are computed using the fault-normal and the fault-parallel time histories observed during the Chi-Chi and Northridge earthquakes (PEER NGA database, see the section Data and Resources). As mentioned earlier, the tests described in the section Testing the Multivariate Normality Assumption for Random Vectors Using Independent Samples require independent data samples, and hence, pairs of fault-normal and fault-parallel residuals are computed at well-separated sites (separation distances exceeding 20 km).

Table B1 shows a sample of the multivariate normality test results obtained when $\tilde{\delta}$ values are computed at different orientations (fault-normal and fault-parallel) and/or different periods. In case 1, the $\tilde{\delta}$ values corresponding to the fault-normal direction and to the fault-parallel direction are computed at the same period (2 sec). The three tests of multivariate normality report insignificant p values in this case. In case 2, the $\tilde{\delta}$ values corresponding to the fault-normal and fault-parallel directions are computed at two different periods. All three tests report insignificant p values in case 2 as well. Finally, it is intended to check if a larger separation in the periods affects the bivariate distributional properties. Hence, in case 3, the fault-normal $\tilde{\delta}$ values are computed at 0.5 sec, while the fault-parallel $\tilde{\delta}$ values are computed at 10 sec. It can be seen from the table that the p values are highly insignificant in this case as well.

Table B1
Tests on Residuals Corresponding to Two Orthogonal (Fault-Normal and Fault-Parallel) Directions

Case	Periods (sec)	P_{HZ}	P_{SK}	P_{KT}
1	$T_1 = 2$; $T_2 = 2$	0.14	0.13	0.41
2	$T_1 = 1$; $T_2 = 2$	0.17	0.34	0.96
3	$T_1 = 0.5$; $T_2 = 10$	0.94	0.80	0.22

P_{HZ}, P_{SK}, and P_{KT} are defined as in Table 1.

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