Reliability-based calibration of design seismic response spectra and structural acceptance criteria

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ABSTRACT: Earthquake engineering design requires an evaluation of the structure's reliability over future seismic loads. The structure's reliability can itself be formulated as an "implicit" performance goal, such as a specified annual probability of collapse, or a target probability of exceedance of a given level of Engineering Demand Parameter (EDP). Design procedures typically evaluate achievement of this goal through some alternate "explicit" check, which commonly specifies a design spectrum and associated structural response acceptance criteria. Much discussion exists as to which design spectrum is the most appropriate for such analysis. In this work, we show that the use of a specific Conditional Mean Spectrum (CMS) is a natural choice to perform the explicit check of a target reliability goal. This determination is based on structural reliability techniques that consider spectral accelerations at multiple modal periods as basic random variables predicting the EDP response. The design point obtained with Inverse FORM methods is chosen as conditioning point to compute the full design spectrum. In the case where there are multiple structural response parameters of interest, or it is unknown what excitation frequencies are important to the structural response, it may be necessary to perform analyses using multiple conditional mean spectra. It is shown that this approach is suitable for response spectrum methods where the structural response is evaluated as a combination of single mode participations, as well as for the more complex case of nonlinear dynamic analyses which additionally take into account ground motion uncertainty (record-to-record variability). The definition of these design spectra fits naturally with explicit structural performance assessments, and is thus relevant for the future refinement of seismic design codes.

1 INTRODUCTION

1.1 *Motivation*

Earthquake engineering design aims at achieving safe structures based on the formulation of implicit performance goals, which can be defined as a specified annual probability of collapse, or a target probability of exceedance p_f of a given level edp_f of Engineering Demand Parameter (EDP):

$$P(EDP > edp_f \text{ in } N \text{ years}) = p_f \tag{1}$$

This is a "*time specific*" performance goal, meaning that we look at the performance of a structure over a given time span in the future. Such terminology applies for the particular case of collapse rate assessment (Ibarra and Krawinkler 2005, Liel et al. 2009), where one is interested in the probability of a structural collapse in a specific period of time. Even though the intensity of a future earthquake may vary widely, the earthquake loading is generally defined in building codes via a single response spectrum. If the building behaves acceptably when analyzed under that design spectrum, it is deemed to be safe in terms of anticipated future performance under earthquake loads. This analysis is an explicit design check, and can be compared to the classical non-seismic approach where demands from specific load combinations are verified to be less than factored resistances (AISC 1999). The main challenge we address in this research is the definition of an appropriate design spectrum to use in the seismic design check, given that the response spectra of future earthquakes will vary.

1.2 *Current standards*

Much discussion exists as to which design spectrum is the most appropriate for structural performance evaluation. For instance, the Uniform Hazard Spectrum (UHS), representing a consistent probability of exceedance of spectral acceleration amplitudes over all periods, has been the main choice in the past. More recently, Luco et al. (2007) have proposed to target a uniform probability of collapse rather than a uniform hazard, which has lead to the definition of the Uniform Risk Spectrum (URS). These two spectra are

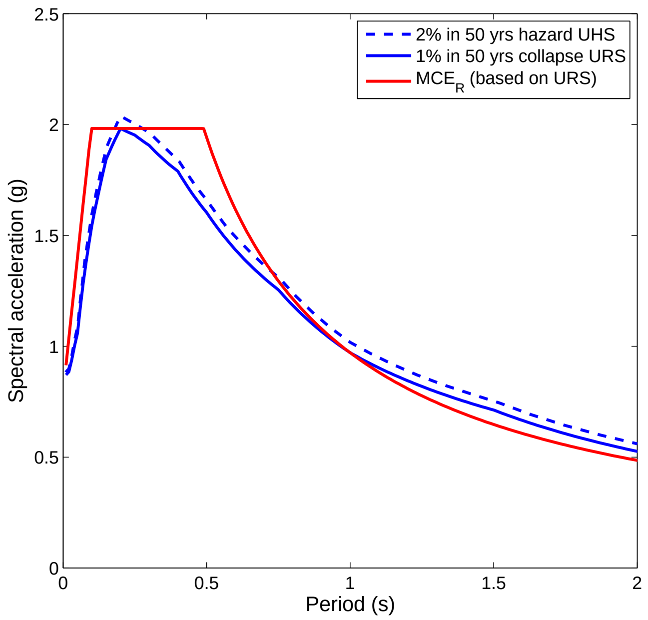

Figure 1: Target response spectra for a site subject to one earthquake per 50 years on a strike-slip fault, with $M = 7$, $R = 10\ km$, $V_{s30} = 400\ m/s$, using the Boore and Atkinson (2008) ground motion prediction equation.

shown on Figure 1, along with the piecewise linear approximation used in one building code (SEI/ASCE 2010). In this research, we study these spectra from a reliability perspective and present a more appropriate alternative consisting of using less conservative Conditional Mean Spectra (CMS).

1.3 *Scope of this research*

The examples presented here make the assumption that the EDP of interest involved in Equation 1 is a function of spectral accelerations at multiple periods:

$$EDP = f_1([Sa(T_1), ..., Sa(T_n)]) \tag{2}$$

where $Sa(T_i)$ denotes the spectral acceleration at period T_i and $T_1, ..., T_n$ are spectral acceleration periods relevant to the structural response. This situation is observed in the response spectrum method, where the demand parameter is exactly determined by values of spectral accelerations at modal periods (Der Kiureghian 1981). A refinement of Equation 2 is considered in Equation 3 to account for ground motion variability (the value of the demand could still be random even with fixed values of the spectral accelerations of interest), relevant to nonlinear dynamic analysis:

$$EDP = f_2([Sa(T_1), ..., Sa(T_n)], \delta) \tag{3}$$

where δ is a random variable that represents the demand variability not explained by the spectral accelerations. From this formulation, we may also define a "*loading specific*" performance goal following the time specific goal format from Equation 1:

$$P(EDP > edp_f \mid [\widetilde{Sa}(T_1), ..., \widetilde{Sa}(T_n)]) = p_d \tag{4}$$

In Equation 4, the probability of exceedance of the demand level edp_f is not measured over a specific time span but rather under a given ground motion level specified by fixed spectral values $[\widetilde{Sa}(T_1), ..., \widetilde{Sa}(T_n)]$. Examples of this type of performance goal can be found in ASCE 7-10 (SEI/ASCE 2010) where the objective is a probability $p_d = 10\%$ of collapse under the Maximum Considered Earthquake (MCE), as well as in ASCE 43-05 (SEI/ASCE 2005), which proposes two loading specific performance goals ($p_{d1} = 1\%$ probability of unacceptable performance under the Design Based Earthquake (DBE), $p_{d2} = 10\%$ probability of unacceptable performance under 1.5 times the DBE).

In this paper, we describe the above problem in the framework of a structural reliability assessment (Madsen et al. 2006). Viewed in this light, the design point associated with a reliability assessment is a rational choice for deriving a design spectrum. To illustrate how this approach would be used in a seismic design context, the case of a building analyzed using the response spectrum method is considered. In this example, the spectral acceleration level from the design point is seen to match a particular conditional mean spectrum.

This paper will show that in the case of Equation 2, the appropriate spectrum to use is a Conditional Mean Spectrum conditioned on a UHS amplitude at a single period. In the case of Equation 3, the appropriate spectrum will also be a CMS, but conditioned on a URS amplitude at a single period.

However, in both cases, when it is unknown what excitation frequencies are important to the structural response, it may be necessary to perform analyses using multiple conditional mean spectra. This will require more effort than a single analysis performed using a uniform hazard spectrum, but the avoidance of the conservatism associated with a single UHS or URS will make the extra effort worthwhile in some cases. These results provide a theoretical justification for the use of the CMS in design checks. This approach has been intuitively identified as reasonable, but not previously justified in depth.

Finally, it should be noted that while we present a non-straightforward derivation of the target spectra, the user would not need to reproduce any part of it and could simply use the end results from the derivation.

2 PRESENTATION OF THE PROBLEM

2.1 *Current standards*

One of the key inputs in performance assessment is the hazard analysis of various intensity measures acting on the structure. Probabilistic Seismic Hazard Analysis (PSHA) (McGuire 2004) is often conducted to evaluate the level of spectral acceleration $Sa(T)$ that is exceeded with a given annual probability p. This analysis can be repeated independently for a number of periods, and the result is a Uniform Hazard Spectrum (UHS) associated with the targeted p. The use of the UHS is prescribed in several design guidelines (e.g. LATBSDC 2008, SFDBI 2010). However, this approach is problematic for two main reasons:

- It is unlikely to observe a ground motion having such high level of spectral accelerations at all periods, because the probability of observing a ground motion exceeding the whole UHS spectrum is far less than the probability of observing a ground motion exceeding a particular value of that spectrum at a single period;

- In general, the choice of the targeted annual probability p associated with the design UHS cannot be directly related to any of the probabilities p_f or p_d defined in our implicit performance goals (Equations 1, 4). This spectrum is only dependent on the seismic hazard at the site of interest and is not linked with any structural performance goal.

Luco et al. (2007) have proposed to address the second issue by adjusting the UHS to target a specific collapse probability, leading to the definition of the Uniform Risk Spectrum (URS), which has become the current standard in ASCE 7-10 (SEI/ASCE 2010). This particular URS aims at achieving a uniform collapse probability $p_f = 1\%$ in 50 years for sites that are not nearby an active fault (time specific goal), while ensuring a probability of collapse $p_d = 10\%$ under that spectrum (loading specific goal). However, the URS is built independently for each spectral period (the collapse fragility depends on a single spectral acceleration period), even though the structural performance may depend on multiple spectral accelerations, and therefore retains the first drawback mentioned for the UHS.

The present contribution addresses this problem by showing the derivation of a set of design spectra directly associated with performance goals following Equations 1 and 4 with an EDP depending on multiple spectral acceleration periods such as in Equations 2 and 3. The results can be considered as a multi-period generalization of the Luco et al. approach.

2.2 *Incorporation of joint hazard information*

Since our implicit goals are defined in terms of the EDP hazard rather than each individual intensity measure hazard (Equations 1, 4), we need to know the joint hazard of $[Sa(T_1), ..., Sa(T_n)]$ at our site of interest. In general, this can be achieved with vector PSHA (Bazzurro & Cornell 2002), which provides the mean occurrence rate of a vector of spectral accelerations at different periods. This result is similar to a joint probability density function, an example of which is shown in Figure 2a. An important input for the calculations will be the associated joint probability contours depicted in Figure 2b. These contours are merely a 2D representation of the joint probability density function. Each contour is a set of spectral acceleration values having the same probability density, the center of the ellipses being the most probable outcome. The angle of the ellipses is due to the correlation of spectral accelerations at multiple periods.

In the present paper, we will illustrate results based on the use of a single earthquake scenario (with $M = 7$, $R = 10\ km$, $V_{s30} = 400\ m/s$, strike-slip fault) occurring over a fixed time span of 50 years, using the Boore & Atkinson (2008) ground motion prediction equation. This simplifies the example calculations as the joint distribution of spectral accelerations from a single earthquake was shown to be a multivariate lognormal distribution (Jayaram & Baker 2008).

3 DESIGN SPECTRA DERIVATION FROM STRUCTURAL RELIABILITY THEORY

3.1 *Relevance of the design point*

Our approach involves techniques from structural reliability theory, which have been used previously in the context of earthquake engineering (Van De Lindt & Niedzwecki 2000). We assume that structural performance is related to spectral accelerations at multiple periods (Equations 2 and 3). For each combination of Sa's, we can determine whether performance is acceptable or unacceptable ("safe" or "failed"), implying that we can find a failure boundary or limit state as drawn on Figure 2b. The design point represents the most likely values $Sa^*(T_1), ..., Sa^*(T_n)$ that cause failure to the structure, and can be computed using First-Order Reliability Method (FORM). For this reason, the design point is intuitively a relevant set of spectral accelerations to be used in structural analysis in order to ensure that the implicit goals are verified. Section 4 further confirms this proposal with an application of the response spectrum method. The following will present how to compute this design point, and detail the derivation of a full target spectrum based on the design point values.

3.2 *First-Order Reliability Method (FORM)*

We first address the determination of a target spectrum for the time specific goal mentioned in Equation 1 by using a reliability problem involving a set of random variables $\mathbf{X} = [Sa(T_1), ..., Sa(T_n)]$ that defines a failure function $g(\mathbf{X})$ as:

$$g(\mathbf{X}) = edp_f - EDP \tag{5}$$

The failure domain is defined as the set of $\mathbf{X}$ such that $g(\mathbf{X}) < 0$, $g(\mathbf{X}) > 0$ corresponds to the safe domain, and $g(\mathbf{X}) = 0$ is the boundary between the two domains. Coherently with the formulation of the time

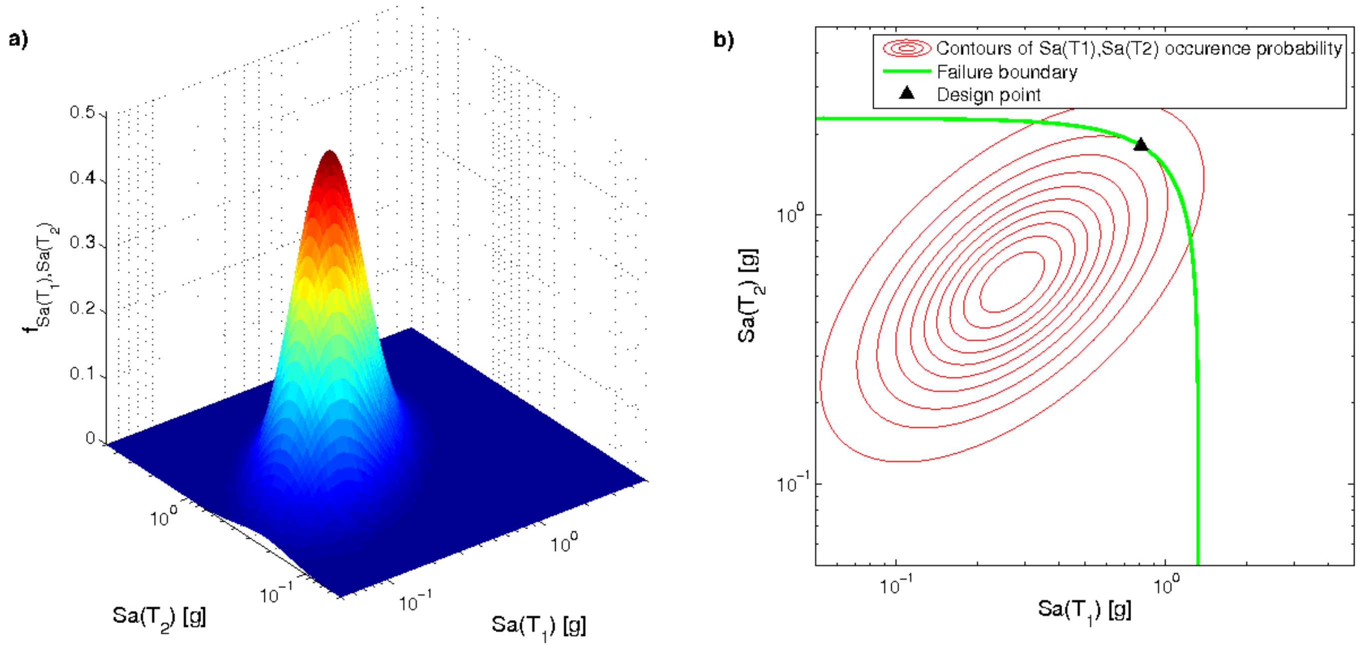

Figure 2: Joint distribution (left) and corresponding joint contour (right) of spectral accelerations at $T_1 = 1\ s$ and $T_2 = 0.3\ s$, for a single scenario earthquake occuring in 50 years on a strike-slip fault with $M = 7$, $R = 10\ km$, $V_{s30} = 400\ m/s$ using the Boore and Atkinson (2008) ground motion prediction equation, along with an example of a failure function.

specific performance goal, the failure probability can then be quantified with:

$$p_f = P(g(\mathbf{X}) \leq 0) = \int_{g(\mathbf{X}) \leq 0} f_{\mathbf{X}}(\mathbf{x})\mathbf{dx} \qquad (6)$$

with $f_{\mathbf{X}}(\mathbf{x})$ the joint probability density function associated with the occurrence of $\mathbf{x}$ over N years (see 2.2).

The First-Order Reliability method (FORM) evaluates the design point $\mathbf{x}^*$ with an iterative algorithm working toward an estimation of the probability of failure p_f based on Equation 6. It uses a simplification of the integrand by mapping the variables into a standard normal space ($\mathbf{X} \rightarrow \mathbf{U}$), in order to obtain a standard normal probability density function $f_{\mathbf{U}}(\mathbf{u})$ and a new formulation of the failure function $h(\mathbf{U}) = g(\mathbf{X})$. The main assumption is a linearization of the integration boundary $g(\mathbf{X}) = 0$ in the standard normal space (at $h(\mathbf{U}) = 0$). The algorithm iterates to minimize $\|\mathbf{u}\|$ subject to $h(\mathbf{u}) = 0$. The solution obtained from this optimization problem is denoted $\mathbf{u}^*$. Once $\mathbf{u}^*$ is found[1], a back-transformation into the original space provides the desired design point $\mathbf{x}^*$.

3.3 *Inverse FORM*

In our case, the time specific goal suggests that the value of p_f is imposed, whereas the value of edp_f included in the failure function $g(\mathbf{X})$ (Equation 5) is unknown. We will therefore refer to the *inverse problem*, which is the evaluation of edp_f associated with a known probability of exceedance p_f, satisfying Equation 6. This is called "Inverse FORM" procedure (IFORM), or "Environmental Contours" (Haver & Winterstein 2008). It should be noted that in addition to the demand threshold edp_f (which could be estimated with a simple Monte-Carlo simulation of EDP), IFORM also provides with the coordinates of the design point $\mathbf{x}^* = [Sa^*(T_1), ..., Sa^*(T_n)]$ corresponding to the limit state defined by Equation 5.

3.4 *Conditional Mean Spectra*

An interesting result occurs in the case of a random variable not explicitly included in the formulation of the failure function. Suppose that $\mathbf{X} = [Sa(T_1), Sa(T_2), Sa(T_3)]$ with a failure function defined as $g(\mathbf{X}) = 1 - Sa(T_1) - Sa(T_2)$. In this case, $Sa(T_3)$ is a component of the input random vector $\mathbf{X}$ but does not appear in the failure function. It can be shown that the design point value $Sa^*(T_3)$ associated with the spectral acceleration not included in the failure function is the mean value of $Sa(T_3)$ conditioned on the design values of the two other spectral accelerations $[Sa^*(T_1), Sa^*(T_2)]$.

This observation motivates the use of the Conditional Mean Spectrum (Baker 2011) to compute a full response spectrum. The Conditional Mean Spectrum (CMS) associates at each period T the mean value of the log spectral acceleration $\ln Sa(T)$ conditioned on the design point values $\mathbf{x}^* = [Sa^*(T_1), ..., Sa^*(T_n)]$. This is done by using the multivariate normality property of the residuals of $\ln Sa(T)$. The value of the log

[1] An interesting quantity that is extensively used in structural reliability is the norm of the $\mathbf{u}^*$ vector, also called the reliability index β. This index informs on how safe the structure is: the higher β is, the lower the probability of failure.

spectral acceleration at any period T (including the ones present in the design point) is evaluated by:

$$\mu_{\ln Sa(T)|\mathbf{x}^*} = \mu_{\ln Sa(T)}(M,R,T) + \mathbf{\Sigma}_{12}\mathbf{\Sigma}_{22}^{-1}\mathbf{e} \qquad (7)$$

where $\mu_{\ln Sa(T)}(M,R,T)$ is the mean of $\ln Sa$ at period T from the ground motion model with magnitude M and distance R, $\mathbf{\Sigma}_{12}$ the covariance matrix between $\ln Sa(T)$ and the vector $[\ln Sa(T_1),...,\ln Sa(T_n)]$, $\mathbf{\Sigma}_{22}$ the covariance matrix of $[\ln Sa(T_1),...,\ln Sa(T_n)]$, and $\mathbf{e}$ the vector of residuals of non-standardized design values ($e_i = \ln Sa^*(T_i) - \mu_{\ln Sa}(M,R,T_i)$).

Section 4 presents an application of the design point determination and resulting spectrum in the case of response spectrum method.

4 RESPONSE SPECTRUM METHOD

4.1 *General formulation*

The response spectrum method estimates the seismic demand with a rule, such as SRSS (Square Root of the Sum of the Squares), for combining spectral values at modal periods:

$$EDP = \sqrt{\alpha_1 Sa(T_1)^2 + ... + \alpha_n Sa(T_n)^2} \qquad (8)$$

where the periods T_i's and coefficients α_i's are determined by modal analysis. This can be seen as a particular case of Equation 2 applied to modal periods with f_1 as the SRSS function.

4.2 *Single-degree-of-freedom example*

It is interesting to look at the simple case where only one spectral acceleration period is included in the equation:

$$EDP = \sqrt{\alpha_1 Sa(T_1)^2} = \sqrt{\alpha_1} Sa(T_1) \qquad (9)$$

Finding the design point in this reduced case is straightforward from the hazard curve of $Sa(T_1)$, since we will only need to find the spectral value $Sa^*(T_1)$ that will produce the response edp_f:

$$\begin{aligned} p_f &= P(EDP > edp_f) \\ &= P(\sqrt{\alpha_1} Sa(T_1) > \sqrt{\alpha_1} Sa^*(T_1)) \qquad (10) \\ &= P(Sa(T_1) > Sa^*(T_1)) \end{aligned}$$

In this particular case, $Sa^*(T_1)$ is therefore the Sa with a probability of exceedance p_f. The FORM calculation using the demand from Equation 9 would yield the same result, along with conditional mean values for the spectral accelerations at other periods conditioned on the value of $Sa^*(T_1) = Sa_{UHS}(T_1)$, as explained in Section 3.4. This motivates the use of a CMS conditioned on the UHS spectral acceleration at the first mode period.

Table 1: α_i coefficients to compute the roof and second story accelerations according to Equation 8

	Roof	2^{nd} story
α_1	1.567	0.467
α_2	0.131	0.155
α_3	0.025	0.004
α_4	0.004	0.008
α_5	0.000	0.002

Table 2: Design spectral accelerations in g's. The italicized values are conditional means.

	Roof design point	2^{nd} st. design point	UHS	CMS 1^{st} mode	CMS 2^{nd} mode
$Sa(T_1)$	0.659	0.434	**0.677**	**0.677**	*0.364*
$Sa(T_2)$	1.133	1.616	**1.644**	*0.932*	**1.644**
$Sa(T_3)$	1.187	1.654	2.090	*0.988*	*1.653*
$Sa(T_4)$	1.140	1.585	2.259	*0.961*	*1.574*
$Sa(T_5)$	1.103	1.524	2.326	*0.936*	*1.516*

4.3 *Multiple-degree-of-freedom example*

We now consider a uniform 5-story shear building where all 5 modes have significant contribution to the seismic demand, with constant lumped mass $m = 100\ kips/g$ and stiffness $k = 31.54\ kips/in$ for each floor (see section 12.8 of Chopra 2011). We perform a full reliability analysis to compute the roof acceleration under the joint hazard from the scenario earthquake detailed in Section 2.2, using all modal contributions and targeting a probability of failure $p_f = 1\%$. The α_i's to be used in the SRSS formula from Equation 8 can be found in Table 1.

This yields a design point $[Sa^*_{roof}(T_1),...,Sa^*_{roof}(T_5)]$, which is shown in the first column of Table 2. As can be seen in Table 1, the roof acceleration is first mode dominated, which is why we also consider the situation described in Section 4.2 where one estimates the seismic demand with the first mode CMS. In this case, the design point will be the CMS conditioned on $Sa^*(T_1)(= Sa_{UHS}(T_1))$ (fourth column of Table 2).

The analysis is repeated for the calculation of the second story acceleration (the corresponding α_i's are shown in the last column of Table 1), first using the full 5 modes (yielding the design point given in the second column of Table 2), and then the CMS conditioned on $Sa^*(T_2) = Sa_{UHS}(T_2)$ (last column of Table 2), since this demand is second mode dominated. Table 3 shows the estimated roof and second story accelerations for all discussed spectra and a comparison with the UHS is also proposed. We observe that the UHS significantly overestimates the true value of the 5-mode demand. The first mode CMS provides an adequate result for the roof acceleration (0.928g vs 0.943g) while the second mode CMS is better for the second story acceleration (0.719g vs 0.727g).

Table 3: EDP values from each spectrum described in Table 2

	Roof	2^{nd} story
5 modes	**0.943**	**0.727**
UHS	1.099	0.839
1^{st} mode CMS	0.928	0.601
2^{nd} mode CMS	0.800	0.719
max CMS	**0.928**	**0.719**

4.4 *Recommended design spectra and acceptance criteria*

The two single mode CMS are shown on Figure 3, along with the two 5-mode CMS for the roof and second story accelerations. As can be seen in Table 3, using the first mode CMS to compute the second story acceleration yields a significant underestimation of the true demand, and the similar conclusion is observed when using the second mode CMS to obtain the roof acceleration. Therefore, when one is unsure of the relative participation of each mode in the final demand calculation, the joint use of the two CMS should be included in the evaluation of the demand. This means that a design check will require two different spectra instead of a single UHS but the avoidance of the conservatism of the UHS may make this extra effort worthwhile. It should be noted that the use of multiple conditional mean spectra has already been suggested by Baker & Cornell (2006) and mentioned in the Tall Building Initiative (PEER 2010), but not carefully justified.

In practicality, using two spectra means that we would compute a first demand edp_{CMS1} based on the first mode CMS conditioned on $Sa_{UHS}(T_1)$, and a second demand edp_{CMS2} from the second mode CMS (conditioned on $Sa_{UHS}(T_2)$). The final demand would just be:

$$edp = \max\left(edp_{CMS1}, edp_{CMS2}\right) \tag{11}$$

This edp is then compared to admissible values in the acceptance criteria.

5 INCORPORATION OF GROUND MOTION VARIABILITY

5.1 *Example*

Estimating the seismic demand from nonlinear dynamic analysis is less straightforward than with the response spectrum method, because the demand is not only a function of the response spectrum but also of the inherent record-to-record variability. This variability can be modeled as an additional random variable δ following the notations from Equation 3. Denoting the random vector $\mathbf{X} = [Sa(T_1), ...Sa(T_n)]$, the previous method can first be applied to solve the reliability problem associated with the failure function:

$$g(\mathbf{X}, \delta) = edp_f - f_2(\mathbf{X}, \delta) \tag{12}$$

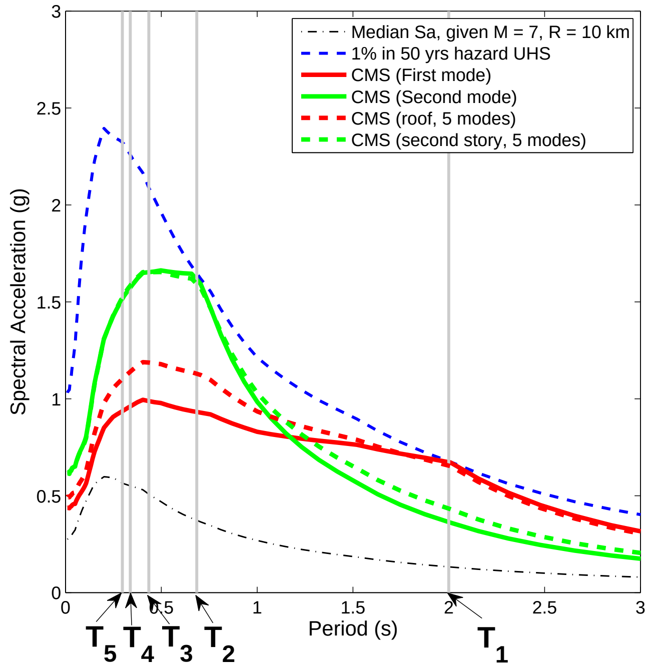

Figure 3: Single mode and 5-mode CMS for a time specific performance goal.

resulting in a design point $[Sa^*(T_1), ..., Sa^*(T_n), \delta^*]$ verifying the time specific performance goal of Equation 1. This is an inverse FORM problem, since we choose the value of p_f, thus solving for the value of $edp_f = f_2([Sa^*(T_1), ..., Sa^*(T_n)], \delta^*)$. However, there is no direct way to incorporate the value of δ^* in the resulting spectrum.

To account for this variability, a loading specific performance goal as described in Equation 4 is involved, and can be written as:

$$\begin{aligned} p_d &= P(EDP > edp_f | [\widetilde{Sa}(T_1), ..., \widetilde{Sa}(T_n)]) \\ &= P(f_2([\widetilde{Sa}(T_1), ..., \widetilde{Sa}(T_n)], \delta) > edp_f) \end{aligned} \tag{13}$$

where $[\widetilde{Sa}(T_1), ..., \widetilde{Sa}(T_n)]$ are the targeted design spectral values to be determined. Studies of ground motion variability (e.g., Cornell et al. 2002) have often shown that δ can be modeled as an independent "error" term, meaning there exists a positive function $\widetilde{f}_1$ such that:

$$f_2(\mathbf{X}, \delta) = \widetilde{f}_1(\mathbf{X}) e^{\delta} \tag{14}$$

with δ normally distributed with mean zero and standard deviation σ_δ, independent from $\mathbf{X}$. The formulation of the loading specific performance goal from Equation 13 becomes:

$$p_d = P(\widetilde{f}_1([\widetilde{Sa}(T_1), ..., \widetilde{Sa}(T_n)]) e^{\delta} > edp_f) \tag{15}$$

which implies:

$$\widetilde{f}_1([\widetilde{Sa}(T_1), ..., \widetilde{Sa}(T_n)]) = edp_f \, e^{\sigma_\delta \Phi^{-1}(p_d)} \tag{16}$$

with Φ^{-1} the standard normal inverse cumulative distribution function. Consequently, finding the design spectral values $[\widetilde{Sa}(T_1), ..., \widetilde{Sa}(T_n)]$ is equivalent to solving the reliability problem associated with the failure function:

$$\widetilde{g}(\mathbf{X}) = \widetilde{edp}_f - \widetilde{f}_1(\mathbf{X}) \tag{17}$$

with $\widetilde{edp}_f = edp_f \; e^{\sigma_\delta \Phi^{-1}(p_d)}$, where the resulting design point will correspond to the target spectral values $[\widetilde{Sa}(T_1), ..., \widetilde{Sa}(T_n)]$. Note that this is a (direct) FORM problem, since the value of $\widetilde{edp}_f$ is fixed [2].

5.2 *Single mode responses*

Similar to section 4.2 in the case of the response spectrum method, solving the above problem for a single mode EDP provides some useful conclusions. Without loss of generality (see Equation 10), let us assume that $EDP = \widetilde{f}_1(Sa(T_1))e^{\delta} = Sa(T_1)e^{\delta}$. Following the procedure of section 5.1, we first need to solve the reliability problem from Equation 12 defined with the failure function:

$$g(Sa(T_1), \delta) = edp_f - Sa(T_1)e^{\delta} \tag{18}$$

For a given p_f, inverse FORM will provide a design point $[Sa^*(T_1), \delta^*]$ and $edp_f = Sa^*(T_1)e^{\delta^*}$. But the targeted spectral value $\widetilde{Sa}(T_1)$ is different from $Sa^*(T_1)$, and rather determined through the definition of the loading specific performance goal, which reduces to a FORM problem (Equation 17):

$$\widetilde{g}(Sa(T_1)) = edp_f \; e^{\sigma_\delta \Phi^{-1}(p_d)} - Sa(T_1) \tag{19}$$

Since there is only one spectral acceleration involved, the solution can be found by setting $\widetilde{g} = 0$ and solving for $Sa(T_1)$, which gives:

$$\widetilde{Sa}(T_1) = edp_f \; e^{\sigma_\delta \Phi^{-1}(p_d)} \tag{20}$$

This $\widetilde{Sa}(T_1)$ can be seen as a particular Uniform Risk Spectrum (URS) value. Indeed, the definition of a URS typically involves both types of performance goals, for instance in ASCE 7-10 (SEI/ASCE 2010):

- the time specific goal is often a $p_f = 1\%$ collapse probability in 50 years,
- the loading specific goal can be a $p_d = 10\%$ collapse probability under the URS.

In our particular situation, the considered risk or performance measure (collapse in the above case) is the event $EDP > edp_f$.

[2] If f_2 is a linear function and if the random vector $[Sa(T_1), ..., Sa(T_n), \delta]$ follows a multivariate normal distribution, an omission sensitivity factor (Madsen 1988) may be used instead of the combination of Inverse FORM and FORM. In our case, the method would consist in removing the random variable δ from the calculations by a modification of the FORM search algorithm targeting a higher reliability index, which is equivalent to decreasing the final probability of failure. However, there is no closed-form solution associated with a nonlinear f_2.

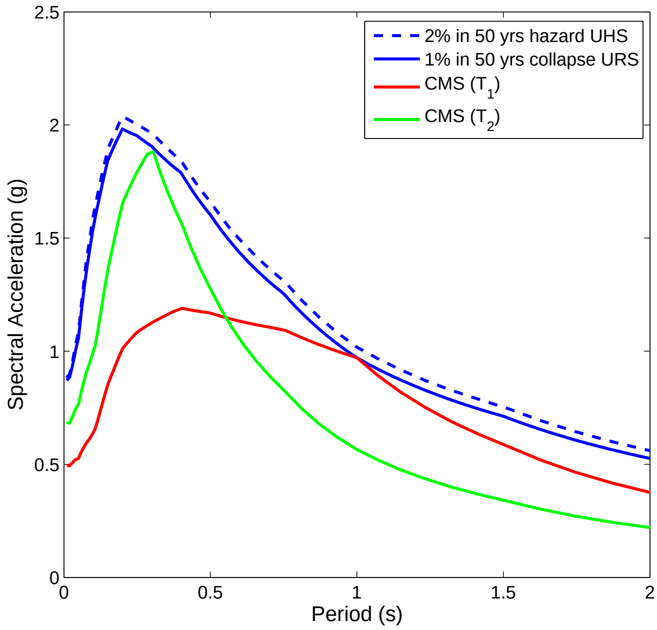

Figure 4: Single mode CMS for time specific and loading specific performance goals.

5.3 *Recommended design spectra and acceptance criteria*

Similar to the response spectrum method example, we propose to use multiple Conditional Mean Spectra conditioned on the appropriate URS amplitudes. Each CMS will be conditioned on the URS value at a spectral period of interest (participating in the EDP). An example of such CMS are plotted on Figure 4 assuming periods of interest of $T_1 = 1\ s$ and $T_2 = 0.3\ s$. This is less straightforward than the response spectrum method as there could be no explicit formula for the EDP as a function of spectral accelerations. For instance, higher periods than the first mode T_1 may be used as conditional values to account for nonlinear effects (period elongation).

Acceptance criteria are also more complicated to formulate than in the case of response spectrum method, where only a time specific goal was considered, and the evaluated demand could simply be compared to some acceptable value, implying a p_f probability of failure over N years. In nonlinear dynamic analysis, the choice of p_d from the loading specific goal should also be taken into account in the acceptance criteria. The lower the value of p_d, the higher the design spectral values will be, thus preventing any meaningful comparison of the demand to some unrelated fixed capacity.

The acceptable capacity $edp_{capacity}$ that would normally be considered with the time specific goal is still useful, but instead of comparing $edp_{demand} < edp_{capacity}$, one should verify that the ratio $edp_{capacity}/edp_{demand}$ is greater than some acceptable value, function of the chosen p_d. This kind of acceptance criteria has been formulated in FEMA P695 (2009) as a “Collapse Margin Ratio” (CMR) of the spectral acceleration at median collapse level over the MCE spectral acceleration.

6 CONCLUSIONS

Low probability of failure under earthquake loading is the main objective of seismic design, and can be seen as an "implicit performance goal." Since current building codes consider that the structure is safe if it has an acceptable behavior under analysis performed using a single design spectrum (e.g. the Uniform Hazard Spectrum or UHS, which targets a given level of seismic hazard) the variability of the ground motion shaking is ignored. The present research has demonstrated how this problem could be termed within a structural reliability framework, where the design spectra are determined based on a pre-specified implicit performance goal. A new set of design spectra (CMS conditioned on an appropriate UHS amplitude at specific periods) are first proposed in the case of a performance goal associated with the exceedance of an EDP of interest over some time period (time specific performance goal). Examples of the derivation and use of these CMS were shown in the context of response spectrum analyses, where the EDP depends on several spectral acceleration periods, and the influence of high mode contributions on the target spectra were examined. It should be noted that all of the examples provided here assumed a joint lognormal distribution of spectral accelerations at multiple periods (based on a single earthquake scenario with fixed magnitude and distance), while the true distribution of those contours should be determined with vector PSHA. The proposed methodology was then extended to consider implications for the more complex case of nonlinear dynamic analysis, where ground motion variability has to be accounted for. Similar conclusions were drawn as multiple CMS are proposed, each conditioned on an appropriate Uniform Risk Spectrum (URS) amplitude at a specific period. This generalizes the simpler risk-targeted design approach to consider design spectra associated with multiple correlated Sa's and a specified target probability of failure under a given ground motion level (loading specific performance goal).

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