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CALIBRATED RESPONSE SPECTRA FOR COLLAPSE ASSESSMENT UNDER MULTIVARIATE HAZARD AND STRUCTURAL RESPONSE UNCERTAINTIES

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ABSTRACT

Earthquake engineering design requires an evaluation of the structure's reliability over future seismic loads. The structure's reliability can itself be quantified with performance goals, such as a specified annual rate of collapse, or a probability of collapse under a particular level of ground motion. Design procedures typically evaluate achievement of the target reliability using structural analyses based on design spectra and their associated structural response acceptance criteria. Previous research based on structural reliability theory applied to a vector Intensity Measure (IM) has shown that Conditional Mean Spectrum (CMS) targets provide more accurate seismic demand values than those obtained with current design spectra, which approximate uniform hazard spectra (UHS) or uniform risk spectra (URS). In this work, we look at the case of collapse assessment, and propose a methodology to assess collapse performance using a set of particular CMS and their associated acceptance criteria. To illustrate the use of this approach, nonlinear time history analyses of two degrading Single-Degree-Of-Freedom systems are considered. Assuming a known seismic hazard, we compute CMS, select ground motions, perform analyses and evaluate performance. A comparison of the proposed approach with FEMA P695 guidelines is developed.

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Calibrated response spectra for collapse assessment under multivariate hazard and structural response uncertainties

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Earthquake engineering design requires an evaluation of the structure's reliability over future seismic loads. The structure's reliability can itself be quantified with performance goals, such as a specified annual rate of collapse, or a probability of collapse under a particular level of ground motion. Design procedures typically evaluate achievement of the target reliability using structural analyses based on design spectra and their associated structural response acceptance criteria. Previous research based on structural reliability theory applied to a vector Intensity Measure (IM) has shown that Conditional Mean Spectrum (CMS) targets provide more accurate seismic demand values than those obtained with current design spectra, which approximate uniform hazard spectra (UHS) or uniform risk spectra (URS). In this work, we look at the case of collapse assessment, and propose a methodology to assess collapse performance using a set of particular CMS and their associated acceptance criteria. To illustrate the use of this approach, nonlinear time history analyses of two degrading Single-Degree-Of-Freedom systems are considered. Assuming a known seismic hazard, we compute CMS, select ground motions, perform analyses and evaluate performance. A comparison of the proposed approach with FEMA P695 guidelines is developed.

Introduction

The objective of seismic design is to ensure that structures will sustain future earthquake shaking with a low probability of failure. Evaluating this probability is difficult as significant variation in the potential future ground shaking exists. Despite this variability, most current building codes propose to use a single response spectrum, as no alternative has been proven superior. In this work, we present the calibration and use of structural-reliability-based response spectra for structural collapse assessment. The idea is to propose simple design checks to verify a target reliability (i.e. implicit performance goal), quantified here as a rate of collapse (for instance, 1% collapse probability in 50 years). The result will be the use of multiple Conditional Mean Spectra (CMS), conditioned at periods of interest and obtained in a similar manner as for the Response Spectrum Method example justified in the authors' previous research [1]. Based on the proposed spectra, we then select adequate ground motion records to be used in nonlinear time history analysis. Finally, acceptance criteria are formulated with respect to the chosen performance

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goals.

We illustrate the validity of the proposed procedure by analyzing two Single-Degree-Of-Freedom (SDOF) structures with a known target reliability and use the proposed approach to verify that this reliability is achieved. Finally, a comparison of the proposed procedure with the single structure collapse assessment of FEMA P695 [2] is shown.

Description of the problem

We first consider a structure located at a known site, for which we can quantify earthquake hazard via the occurrence of some levels of Intensity Measures (IM). Given this hazard, we would like to determine whether our structure satisfies a target reliability goal or implicit performance goal. To ensure that the desired performance is achieved, we submit a model of the structure to an appropriate seismic load. Acceptable structural behavior under this load will be equivalent to meeting the target reliability goal.

Quantification of the seismic loading

In the present case, this seismic load will be characterized by sets of ground motions having target spectra, which may be used to conduct nonlinear time history analyses and evaluate the structure's behavior. The derivation of the appropriate design spectra, examined in the next section of this paper, will take into account both earthquake hazard (quantified with a hazard curve from Probabilistic Seismic Hazard Analysis (PSHA) [3]) and structural response uncertainties (e.g., uncertainty in the collapse capacity), and will provide simple ready-to-use spectra for the engineer.

Performance goals

Performance goals quantify the reliability targets that the structure should achieve. We distinguish two types of performance goals. The first is an implicit performance goal, which relates to future performance under uncertain hazard and structural behavior. An example of such a goal from ASCE 7-10 [4] is an annual rate of collapse:

$$\nu_{\text{target}}(\text{collapse}) = 0.0002 \text{ yr}^{-1} \quad (1)$$

which corresponds to a 1% probability of occurrence in 50 years. Collapse performance itself has been well studied in past research [e.g., 5,6], as collapse safety is a critical objective of seismic design. In order to determine a specific ground motion level to be used in a design check, we consider an explicit goal corresponding to performance of the structure under a specific value of the considered seismic loading. For instance, such goal may be formulated as a probability of collapse under a given spectrum:

$$P(\text{collapse} \mid \text{design spectrum}) = p_d = 50\% \quad (2)$$

As we will illustrate in the last section of this paper, FEMA P695 uses a similar explicit goal

when assessing the collapse safety of a single structure (Appendix F). The explicit goal is the performance level we choose to consider when conducting the design check, while the implicit goal represents the main reliability objective that we intend to verify.

Structural analyses and acceptance criteria

Once a target spectrum has been selected, nonlinear dynamic analysis requires subjecting the structure to ground motion time histories "representative" of that target spectrum and verifying that the structure exhibits acceptable behavior. Statistics on the results from these analyses are computed (e.g., means or quantiles), and used as inputs for some acceptance criteria (also referred to as design check), consisting of the verification of a simple condition equivalent to meeting the performance goals.

Definition of the design spectra

This section addresses the determination of design spectra aimed at the verification of the performance goals defined in Eqs. 1-2. We first summarize the definition of risk-targeted spectral acceleration, and then use this result to derive a set of more appropriate target spectra to be included in the proposed procedure.

Risk-targeted spectral acceleration

As described in [7], the risk-targeted spectral acceleration at period T is based on a target value for the collapse rate, which can be seen as the implicit performance goal that we are trying to verify:

$$\nu(\text{collapse}) = \int_0^{+\infty} P(\text{collapse} \mid Sa(T) = u) MRD_{Sa(T)}(u) du \tag{3}$$

where $MRD_{Sa(T)}(u)$ is the mean rate density of $Sa(T)$ in the neighborhood of u, obtained by differentiating the hazard curve from PSHA [8]. If we assume the fragility function $P(\text{collapse}|\ Sa(T))$ to be lognormally distributed as a function of $Sa(T)$, with a fixed standard deviation β (for instance β=0.4), setting this $\nu(\text{collapse})$ equal to $\nu_{\text{target}}(\text{collapse})$ produces a unique solution for the fragility function. The risk-targeted spectral acceleration at period T, $Sa_{\text{RT}}(T)$, is then based on the explicit performance goal and determined as the p_d -quantile of the collapse fragility, such that:

$$P(\text{collapse} \mid Sa(T) = Sa_{\text{RT}}(T)) = p_d \tag{4}$$

By repeating this analysis for each period T independently, a complete target spectrum may be constructed, referred to as uniform risk spectrum (URS). However, analytical modeling of collapse [e.g., 9] shows that collapse is likely to exhibit a joint dependence on spectral accelerations at multiple periods. To account for this joint dependence, we now introduce the definition of a set of more appropriate target spectra for collapse assessment.

Computation of Conditional Mean Spectrum

The Conditional Mean Spectrum computes at each period T the mean value of the log spectral acceleration ln$Sa(T)$ conditioned on a log spectral acceleration value at a given period T^*, ln$Sa(T^*)$. This is done by using the multivariate normality property of the residuals of ln$Sa(T)$. The value of the CMS log spectral acceleration at any period T is evaluated by:

$$\mu_{\ln Sa(T)|\ln Sa(T^*)} = \mu_{\ln Sa}(M,R,T) + \rho(T,T^*)\sigma_{\ln Sa}(T)\left(\frac{\ln Sa(T^*) - \mu_{\ln Sa}(M,R,T^*)}{\sigma_{\ln Sa}(T^*)}\right) \quad (5)$$

where $\mu_{\ln Sa}(M,R,T)$ (resp. $\sigma_{\ln Sa}(T)$) is the mean (resp. standard deviation) of lnSa at period T from the ground motion model with moment magnitude M and distance R, and $\rho(T,T^*)$ is the correlation coefficient between the logarithmic spectral accelerations at T and T^*, estimated with the empirical formula from, for example, Baker and Jayaram [10].

Previous research by the authors [1] using structural reliability theory has shown that in such case where performance is a function of correlated spectral accelerations at multiple periods, the CMS [11] is a more appropriate target than a uniformly derived spectrum. The current proposal also relies on the use of CMS as design spectra, but conditioned on risk-targeted spectral acceleration amplitudes at some periods of interest. It should be noted that the use of multiple CMS in this manner has been suggested by Baker and Cornell [12] and mentioned in the PEER Tall Building Initiative Guidelines [13].

Proposed procedure for collapse assessment

For a given structure at a particular site, the proposed performance assessment is based on the two types of performance goals described in Eqs. 1-2. The procedure consists of four main steps: 1) determine conditioning periods; 2) compute a CMS for each period; 3) select ground motion records for each CMS and conduct nonlinear time history analyses; 4) verify that the results are consistent with the explicit goal. We now provide more details for each step of the assessment.

1) We first determine which periods are relevant for collapse behavior. One of the conditioning periods should be the first mode period T_1, which can be a good predictor of first order elastic effects. Other periods may consist of higher modes, and/or larger periods to include softening responses. As we illustrate in the next section, such inelastic period may correspond to a tangent stiffness estimated from an available backbone curve or a pushover analysis.

2) The magnitude M and distance R representative of the seismic hazard are identified (either from seismic hazard deaggregation or directly from a scenario earthquake), and used to compute the CMS according to Eq. 5, conditioned on risk-targeted spectral accelerations $Sa_{RT}(T)$ (Eqs. 3-4) at each of the determined periods.

3) For each CMS, we independently select a set of ground motions using the algorithm proposed by Jayaram et al. [14], which allows us to find scaled ground motion records with a response spectrum closely matching the CMS. We then conduct nonlinear time history analyses

with the scaled records.

4) The structure is considered to satisfy the implicit goal if, for each CMS, the fraction of the ground motion records that cause collapse is less than the p_d value of the explicit goal. For instance, for p_d=50%, we will check that less than half of the ground motions for each CMS cause collapse. It should be noted that the number of needed ground motion records to obtain a robust criterion will depend on the various uncertainties involved in the analysis, and will increase as p_d gets further away from 50%.

Example application to single-degree-of-freedom structures

We next illustrate the procedure with two bilinear Single-Degree-Of-Freedom structures (denoted SDOF A and SDOF B), defined with the backbone curves shown in Figure 1. Both SDOF's have a fundamental period of T_1=1s, but SDOF A is less ductile than SDOF B.

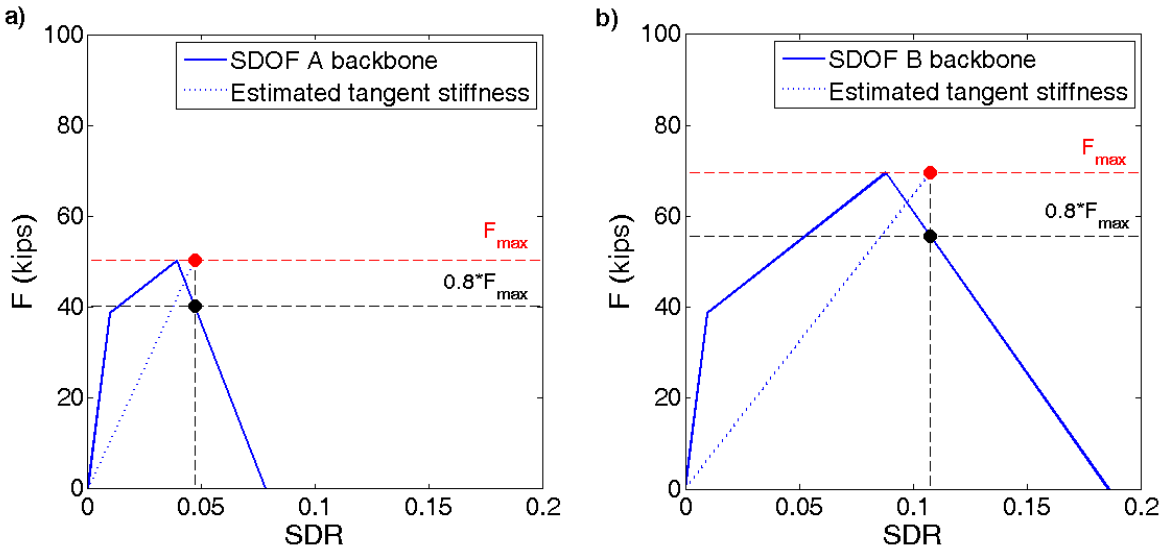

Figure 1. Backbone curves of the two considered SDOF systems, with the tangent stiffness from which the elongated period T_2 is derived: a) SDOF A; b) SDOF B.

The structures will be subjected to a single earthquake scenario of magnitude M=7 at distance R=10km, occurring with a rate ν_0, with ground motions modeled using the Boore and Atkinson 2008 ground motion prediction equation for a site with shear wave velocity V_{s30}=400m/s [15]. A value for ν_0 was determined such that the structure achieves a known target collapse reliability of ν_{target}(collapse)=0.0002 yr^{-1}. Each SDOF will have a distinct ν_0 value, since their collapse capacities differ (ν_0=0.002 yr^{-1} for SDOF A, 0.011 yr^{-1} for SDOF B). While the appendix describes this calculation in more detail, it is merely a way to set up our example problems with the known reliability from the implicit goal (note that in a real application of our procedure, no such calculation of earthquake rates would be needed, since the hazard information will be known). As a consequence, we expect both structures to pass the collapse assessment check under this particular earthquake hazard. The explicit goal used in the

determination of the design spectra will follow Eq. 2 with a p_d value of 50%.

Design spectra and selected ground motions

For both considered SDOFs, we use the elastic period T_1=1s and an elongated period of T_2=2s as the conditioning periods. This second period was chosen based on the tangent stiffnesses described in Figure 1 (1.92s for SDOF A, 2.15s for SDOF B). Thus, for each SDOF, we obtain two CMS denoted CMS1 (conditioned on $Sa_{RT}(T_1)$) and CMS2 (conditioned on $Sa_{RT}(T_2)$), using the same ground motion prediction equation used to compute the hazard. Values for $Sa_{RT}(T_1)$ and $Sa_{RT}(T_2)$ are summarized in Table 1 for both structures. The CMS and response spectra of 40 selected ground motion records for SDOF A are plotted in Figure 2.

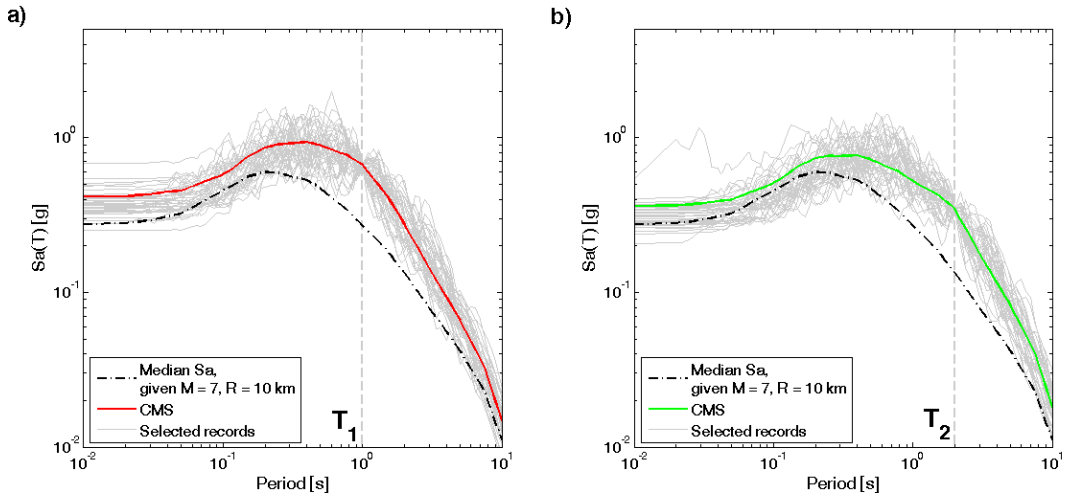

Figure 2. CMS and response spectra of 40 corresponding selected ground motion records for SDOF A: a) CMS conditioned at T_1; b) CMS conditioned at T_2.

Results and acceptance criteria

Results of the analyses are summarized in Table 1. For each SDOF, both CMS's yield a fraction of collapses smaller than the explicit goal value of p_d=50%, therefore the two SDOF's pass the design check as expected, and the requirements from the performance goals from Eqs 1-2 are verified for both structures. A closer look at the results shows that for SDOF A, the highest fraction of collapses is obtained with CMS1 (40%), while for SDOF B, the highest fraction of collapses is obtained with CMS2 (48%). This can be explained by the fact that the collapse performance of SDOF B is more dependent on the inelastic period T_2 due to a higher ductility.

Table 1. Risk-targeted spectral accelerations and collapse probabilities obtained from the proposed procedure for the two SDOF structures.

	CMS1		CMS2	
	$Sa_{RT}(T_1)$ (g)	P(collapse|CMS1)	$Sa_{RT}(T_2)$ (g)	P(collapse|CMS2)
SDOF A	0.66	**0.40**	0.35	0.33

SDOF B	1.30	0.18	0.71	**0.48**

While the proposed procedure provides satisfactory assessments of the collapse performance, common practice still often prescribes the use of ground motion records without consideration of spectral shape, scaled to a particular $Sa(T)$ value. If we were to apply such approach with records consistent with the median spectrum of the scenario earthquake (Figure 3a) and scaled to the $Sa_{\mathrm{RT}}(T_1)$ values from Table 1, we would obtain 73% collapses for SDOF A and 63% collapses for SDOF B, thus incorrectly assessing that both SDOF's fail to meet the design requirements. For this reason, FEMA P695 has proposed a modified scaling method to account for these spectral shape effects. The next section will compare the FEMA P695 approach with our proposed procedure.

Comparison with FEMA P695 Appendix F

While the main objective of FEMA P695 is to propose seismic performance factors for a variety of structural design archetypes, its Appendix F details a procedure to assess the collapse performance of a single structure. Ground motions from a provided set (Near- or Far-Field) are collectively scaled up using the following scale factor:

$$SF = \frac{ACMR}{SSF}\frac{Sa_{\mathrm{MCE}}(T_1)}{Sa_{\mathrm{NRT}}(T_1)} \tag{6}$$

where *ACMR* quantifies the acceptable value of Adjusted Collapse Margin Ratio (corresponding to the first mode spectral acceleration at collapse to the first mode MCE spectral acceleration $Sa_{\mathrm{MCE}}(T_1)$ based on the total system uncertainty), $Sa_{\mathrm{NRT}}(T_1)$ is the median first mode spectral acceleration of the chosen unscaled ground motion set, and *SSF* is a spectral shape factor, introduced in order to adjust the collapse capacity due to the unrealistic spectral shape of the 2% in 50 years MCE spectrum. The acceptance criteria consists in checking that less than half of the scaled ground motions have lead to collapse.

The FEMA P695 procedure bears interesting similarities with our proposed approach. Specifically, its spectral shape factor is a function of the structure's ductility ratio, which is a proxy for period elongation, while the use of our proposed CMS already accounts for the dependence on elongated periods by considering a CMS conditioned at T_2. Even though our proposed approach requires to conduct more analyses (using 2 CMS instead of a single MCE spectrum), its main advantage lies in the fact that spectral shape is inherently accounted for in the CMS rather than with a scale factor adjustment, and ground motions are directly selected to match the CMS spectral shape.

Conclusions

Safety against collapse is of primary importance in any seismic design. This type of structural performance assessment involves the definition of performance goals, which may be implicit (e.g., an annual rate of collapse) or explicit (e.g., collapse probability under a particular ground

motion intensity level). Based on the joint specification of these two performance goals, a collapse assessment procedure was established using Conditional Mean Spectra conditioned on risk-targeted spectral acceleration amplitudes at structural periods of interest. Example applications were shown using two SDOF structures having varying ductility levels, and results indicated that the procedure provides an accurate assessment of the structural collapse performance. A comparison with FEMA P695 guidelines showed that the use of CMS, while requiring to conduct more analyses, has the significant advantage of intrinsically accounting for spectral shape, whereas FEMA P695 involves a less straightforward adjustment in the ground motion scaling to compensate for spectral shape effects.

Future work will apply the methodology to Multiple-Degree-Of-Freedom structures, for which additional conditioning periods (such as higher mode periods) may be required. The methodology should also be extended to account for other structural performance measures such as exceedence of some relevant Engineering Demand Parameters (*EDP*).

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Appendix: computation of ν_0 using an analytical modeling of collapse

In this appendix, we describe a method to compute the rate of occurrence of a single earthquake scenario such that a structure has a specified target reliability $\nu_{\text{target}}(\text{collapse})$. The formula for ν_0 is:

$$\nu_0 = \frac{\nu_{\text{target}}(\text{collapse})}{\int_0^{+\infty} P(\text{collapse} \mid \mathbf{IM}=\mathbf{im}) f_{\mathbf{IM}}(\mathbf{im})\mathbf{dim}} \quad (7)$$

where **IM** is a vector intensity measure predicting the structural behavior, and $f_{\mathbf{IM}}(\mathbf{im})\mathbf{dim}$ corresponds to the joint lognormal probability density function of the vector **IM** being in the neighborhood of the vector-valued **im** given the occurrence of the considered earthquake scenario (the medians and standard deviations of each component of **IM** are obtained from the ground motion prediction equation, whereas correlation coefficients between two components of **IM** are estimated from an empirical equation (e.g., [10])). $P(\text{collapse}|\mathbf{IM}=\mathbf{im})$ is obtained using Incremental Dynamic Analyses (IDA) [16]. In IDA, ground motion records are selected to match the scenario earthquake spectrum (Figure 3a), incrementally scaled to increasing values of $Sa(T_1)$, and nonlinear time history analysis is conducted at each step until the structure collapses. The $Sa_{\text{collapse},i}(T_1)$ value corresponding to the first mode spectral acceleration at collapse is computed from each ground motion record i. We then consider an additional structural period $T_2 > T_1$. The ratio $r = Sa(T_2)/Sa(T_1)$ remains constant for each ground motion record regardless of the scale factor and may be used as an additional predictor for collapse.

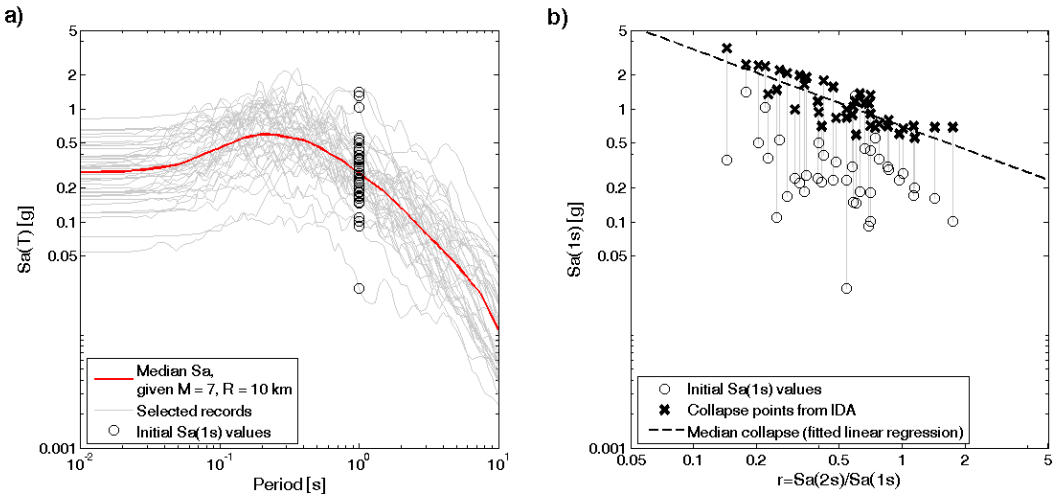

Figure 3. a) Median *Sa* for scenario earthquake and corresponding selected ground motion records; b) Collapse points from IDA and fitted linear regression (SDOF B).

The $Sa_{\text{collapse}}(T_1)$ and r values corresponding to each record and can then be used to determine a collapse capacity by first fitting a regression line (Figure 3b) and obtaining the coefficients of the following linear combination:

$$\ln Sa_{\text{collapse}}(T_1) = a_1 \ln r + a_2 + e \tag{8}$$

where e is the random variable corresponding to the residuals from the linear regression. A bivariate fragility function can then be computed by mapping standardized residuals of the linear regression from Eq. 8 into collapse probabilities:

$$P(\text{collapse} \mid Sa(T_1), Sa(T_2)) = \Phi\left(\frac{\ln Sa(T_1) - (a_1 \ln r + a_2)}{\sigma_e}\right) \tag{9}$$

where σ_e is the standard deviation of e and Φ is the standard normal cumulative distribution function.

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