Rational Design Spectra for Structural Reliability Assessment Using the Response Spectrum Method

Christophe Loth[a] and Jack W. Baker,[a] M.EERI

Current design spectra, which approximate uniform hazard or risk spectra, are known to have shortcomings, but no alternative has been proven superior for the purposes of design checks. In this work, we use response spectrum method analysis to show that the "design point" associated with a structural reliability assessment is a rational choice for a design spectrum. When the response parameter of interest is sensitive to excitation at a particular period, the design point corresponds to a conditional mean spectrum (CMS) conditioned on that period. In the case where there are multiple structural response parameters of interest, or it is unknown what excitation periods are important to the structural response, the CMS can be used by considering multiple conditioning periods and taking the maximum structural response from any of the spectra for design checks. This observation is used to justify the CMS as a target response spectrum for design checks. [DOI: 10.1193/041314EQS053M]

INTRODUCTION

The objective of seismic design is to ensure that structures will sustain future earthquake shaking with a low probability of failure. Evaluating this probability is difficult, as significant uncertainty in the potential future ground shaking exists. The present research will address this objective in the framework of a structural reliability assessment, considering the analysis of a building using the response spectrum method (Chopra 2011).

Most building codes require a response spectrum analysis to evaluate the behavior of structures that are sensitive to multiple-mode excitations (e.g., section 12.9 in ASCE/SEI 2010). Individual mode responses are calculated using a design spectrum, and then superimposed using a combination rule such as square root of the sum of the squares (SRSS) or complete quadratic combination (CQC; Rosenblueth 1951, Der Kiureghian 1981). This design spectrum is commonly based on a uniform hazard spectrum, which is developed by computing the spectral amplitude at each period that has a specified probability of exceedance. This spectrum has been shown to produce conservative structural responses, because occurrence of an extreme $Sa(T)$ (spectral acceleration at period T) amplitude at a single period does not imply occurrence of equally extreme levels at all periods (Baker and Cornell 2006).

Here we present a new approach to define a design spectrum for structural performance assessment. Our objective is to formulate an explicit design check that verifies an implicit

[a] Dept. of Civil and Environmental Engineering, Stanford University, Stanford, CA 94305-4020

performance goal. As an example of this terminology, the non-seismic load and resistance factor design, or LRFD (e.g., AISC 1999), defines an implicit performance goal for structural elements to have a low annual rate of failure. For instance, when considering member design, such implicit goal can be:

$$\nu(\text{member failure}) \leq 10^{-3}\,\text{yr}^{-1} \tag{1}$$

where $\nu(\,)$ is used to indicate the annual rate of the specified event occurring. The associated LRFD explicit design check compares factored resistances and loads:

$$\gamma L \leq \phi R \tag{2}$$

where L is the nominal value for the load effect on the member, γ is the load factor, R is the nominal value for the resistance of the member, and ϕ is the resistance factor. The factors γ and ϕ are calibrated to account for the inherent uncertainties in the load and resistance effects. An example of such a design check derivation can be found in Fisher et al. (1978) for the case of principal fastening elements. The implicit performance goal of Equation 1 is anticipated to be achieved if the inequality of Equation 2 is verified for a given member. In the following, we formulate our general implicit performance goal as a low annual rate, ν_f, of exceeding a given level of Engineering Demand Parameter (EDP) response, $edp_{allowable}$:

$$\nu(EDP > edp_{allowable}) \leq \nu_f \tag{3}$$

Evaluating this rate involves computing the rate of all possible levels of ground motion intensity and the probability of exceeding $edp_{allowable}$ given each of those intensity values (Jalayer and Cornell 2004). Rather than directly evaluating Equation 3, explicit design checks are commonly conducted at specific intensity levels (ATC 2011, Bradley 2013). Similar to Equation 2, the explicit design check can take the general form of an inequality:

$$edp_{demand} \leq edp_{allowable} \tag{4}$$

where edp_{demand} is the edp value obtained from a structural analysis using a particular design spectrum. While the present work will focus on elastic response multimodal $EDPs$, a related well-studied application of the problem is the assessment of the probability of structural failure using nonlinear response history analysis (e.g., Ibarra and Krawinkler 2005, Liel et al. 2009).

Our problem can be stated as follows: when defining an explicit design check (Equation 4) for a structure's multimodal response, how should we choose the response spectrum to use in order for the implicit performance goal from Equation 3 to be verified? We will present a structural reliability–based technique to obtain the so-called design point and associated target spectrum corresponding to our implicit goal. We will show that a uniform hazard spectrum is typically a conservative target when multiple modes participate in the response EDP, and that conditional mean spectra can overcome this conservatism.

QUANTIFICATION OF GROUND MOTION HAZARD

A key input in seismic reliability assessment is ground motion hazard analysis. Probabilistic seismic hazard analysis (PSHA; e.g., McGuire 2004) is conducted to evaluate the level of a spectral acceleration $Sa(T)$ that is exceeded with a given annual rate. This analysis can be repeated independently for a number of periods, and the set of associated spectral acceleration values forms the uniform hazard spectrum (UHS) associated with a targeted annual exceedance rate ν_{UHS}. The response spectrum method is sometimes used with this UHS specifying each individual modal response. However, this use of the UHS is problematic for two main reasons:

- The probability of observing a ground motion whose spectrum exceeds the whole UHS spectrum is typically far less than the probability of observing a ground motion whose spectrum exceeds a particular value of that spectrum at a single period;
- The targeted annual exceedance rate ν_{UHS} associated with the UHS cannot be generally related to the exceedance rate ν_f of a response level as defined in the implicit performance goal of Equation 3. This spectrum is only dependent on the ground motion hazard at the site of interest and is not linked with any structural performance goal.

Both issues are also found in other types of structural analysis, such as response history analysis. As an example of the second issue, several design guidelines (LATBSDC 2008, SFDBI 2010) prescribe to conduct an explicit design check at two-thirds of a maximum considered earthquake (MCE) level in order to achieve an implicit goal of a "low" annual rate of collapse. While this MCE amplitude was defined as the minimum of a deterministic spectrum (150% of the largest median shaking from characteristic earthquakes on all active faults) and a UHS with $\nu_{UHS} = 0.0004\,\text{yr}^{-1}$ (equivalent to a probability of exceedance of 2% in 50 years), the value of ν_{UHS} was not established based on any desired value of annual rate of collapse. Luco et al. (2007) have proposed to adjust this MCE spectrum to target a specific collapse rate, leading to the definition of the uniform risk spectrum (URS), resulting in a risk-targeted MCE_R now used in ASCE 7-10 (ASCE/SEI 2010). This adjustment is done for each structural period independently, and so still does not account for the fact that structural performance may be related to spectral accelerations at multiple periods. The present research addresses this problem by showing the derivation of a set of design spectra directly associated with a more general implicit performance goal described in Equation 3.

Our work involves techniques from structural reliability theory, which have been used previously in the context of earthquake engineering (Bazzurro et al. 1996, Der Kiureghian 1996, Der Kiureghian and Dakessian 1998, Van de Lindt and Niedzwecki 2000, Ellingwood 2001) Since our implicit goal is defined in terms of the rate of exceeding some EDP level rather than a spectral acceleration level (Equation 3), we need to know the mean occurrence rate $\lambda_{Sa(T_1)=x_1,\ldots,Sa(T_n)=x_n}$ of a vector of spectral accelerations at different periods $[Sa(T_1),\ldots,Sa(T_n)]$ being in the neighborhood of the values $[x_1,\ldots,x_n]$, which can be determined using vector PSHA (Bazzurro and Cornell 2002). This mean occurrence rate may be characterized by its mean rate density (MRD); for instance in the case of $n = 2$ periods, the mean occurrence rate $\lambda_{Sa(T_1)\in[b_{11},b_{12}],Sa(T_2)\in[b_{21},b_{22}]}$ of events where $b_{11} \le Sa(T_1) \le b_{12}$ and $b_{21} \le Sa(T_2) \le b_{22}$ can be determined as:

$$\lambda_{Sa(T_1)\in[b_{11},b_{12}],Sa(T_2)\in[b_{21},b_{22}]} = \int_{b_{21}}^{b_{22}}\int_{b_{11}}^{b_{12}} MRD_{Sa(T_1),Sa(T_2)}(x_1,x_2)dx_1dx_2 \tag{5}$$

The *MRD* can be seen as a particular probability distribution function $f_{Sa(T_1),\ldots,Sa(T_n)}$ multiplied by a constant $\nu_0 = \lambda_{Sa(T_1)\geq 0,\ldots,Sa(T_n)\geq 0}$ corresponding to the rate of occurrence of non-zero spectral acceleration values:

$$MRD_{Sa(T_1),\ldots,Sa(T_n)}(x_1,\ldots,x_n) = \nu_0 f_{Sa(T_1),\ldots,Sa(T_n)}(x_1,\ldots,x_n) \tag{6}$$

In the present work, we assume the example building's hazard comes from a strike-slip fault at a distance of 10 km, producing only magnitude $M = 7$ earthquakes at a mean rate of $\nu_0 = 1/50\,\mathrm{yr}^{-1}$. The building is assumed to be on soil with an average shear wave velocity in the top 30 meters of $V_{S30} = 400\,\mathrm{m/s}$. By assuming only a single earthquake scenario, the PSHA calculations are much simplified, though the numerical results are still illustrative of seismic hazard for cases of buildings located near an active fault (cases involving the considerations of multiple earthquake sources can be found in Chapter 3 of Loth 2014). In this single-earthquake-scenario case, the spectral accelerations at multiple periods follow a joint lognormal distribution (Jayaram and Baker 2008), and so only means, standard deviations, and pairwise correlation coefficients are needed for the $\ln Sa$ values of interest. We obtain the median Sa's and lognormal standard deviations $\sigma_{\ln Sa}$ at each period of interest from the Boore and Atkinson (2008) ground motion model, and correlation coefficients between periods from Baker and Jayaram (2008). Figure 1a shows the distribution corresponding to the joint lognormal probability density function of the vector $[Sa(1\,\mathrm{s}), Sa(0.3\,\mathrm{s})]$. An important input for later calculations is the set of associated joint probability contours depicted in Figure 1b. These contours are a two-dimensional (2-D) representation of the joint probability density function. Each contour is a set of spectral acceleration values having the same

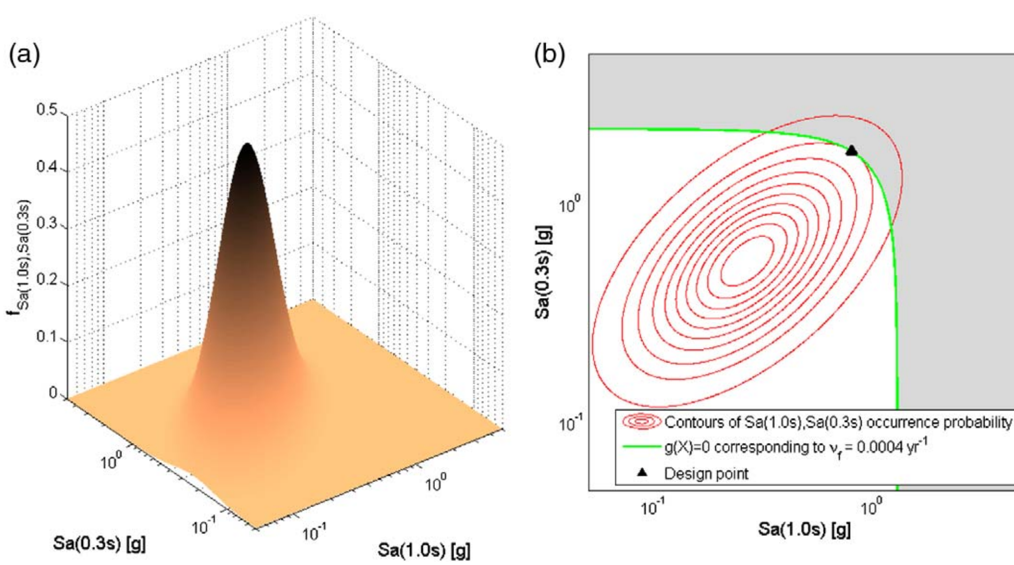

Figure 1. (a) Joint distribution and (b) corresponding joint contour of spectral accelerations given occurrence of the scenario earthquake ($M = 7$, $R = 10\,\mathrm{km}$ and $V_{S30} = 400\,\mathrm{m/s}$) using the Boore and Atkinson (2008) ground motion prediction equation. An example failure function is also shown in (b), where the failure region is shaded.

probability density, the center of the ellipses being the most probable outcome. The angle of the ellipses is due to the correlation of spectral accelerations at multiple periods.

STRUCTURAL RELIABILITY THEORY AND DESIGN SPECTRA

FIRST-ORDER RELIABILITY METHOD (FORM)

We consider a reliability problem (Madsen et al. 2006) involving a set of random variables $\mathbf{X} = [Sa(T_1), \ldots, Sa(T_n)]$ that defines a failure function g as:

$$g(\mathbf{X}) = edp_f - EDP(X) \tag{7}$$

where edp_f is a fixed constant and $EDP(\mathbf{X})$ is a function that computes the structural response given values of the input spectral accelerations (e.g., a modal combination rule when using the response spectrum method). The failure domain is the set of $\mathbf{X}$ such that $g(\mathbf{X}) < 0$ (i.e., $\mathbf{X}$ for which the structural demand is greater than edp_f, as illustrated by the shaded region in Figure 1b), $g(\mathbf{X}) > 0$ is the safe domain, and $g(\mathbf{X}) = 0$ is the boundary between the two domains. Based on Equation 7, the failure rate can then be quantified with:

$$\nu_f = \nu(EDP(\mathbf{X}) > edp_f) = \nu(g(\mathbf{X}) < 0) = \int_{g(\mathbf{X})<0} MRD_{\mathbf{X}}(\mathbf{x})\mathbf{dx} \tag{8}$$

with $MRD_{\mathbf{X}}(\mathbf{x})$ the mean rate density function associated with $\mathbf{X}$.

Using the first-order reliability method (FORM) to evaluate Equation 8, it is possible to find a so-called "design point" $\mathbf{x}^* = [Sa^*(T_1), \ldots, Sa^*(T_n)]$, which represents the most likely set of spectral acceleration values that will cause failure of the structure (Ditlevsen and Madsen 1996). For this reason, we propose this design point to be the place where the explicit design check is conducted.

FORM evaluates $\mathbf{x}^*$ with an iterative algorithm working toward an estimation of the failure rate ν_f based on Equation 8. It uses a simplification of the integral by mapping the variables into a standard normal space ($\mathbf{X} \rightarrow \mathbf{U}$), in order to obtain a standard normal probability density function $f_{\mathbf{U}}(\mathbf{u})$ and a new formulation of the failure function $h(\mathbf{U}) = g(\mathbf{X})$. The algorithm iterates to find the minimization value of $\|\mathbf{u}\|$, the norm of $\mathbf{u}$, given $h(\mathbf{u}) = 0$. The solution is denoted $\mathbf{u}^*$. The main approximation used in this approach is a linearization of the failure function in the standard normal space at the design point (i.e., at $h(\mathbf{u}^*) = 0$). The norm of the $\mathbf{u}^*$ vector, also called the reliability index $\beta = \|\mathbf{u}^*\|$, indicates how safe the structure is: the higher β, the lower the corresponding failure rate. Once $\mathbf{u}^*$ is found, a back-transformation into the original space provides the desired design point $\mathbf{x}^*$. Figure 2 illustrates the steps involved in the FORM algorithm.

INVERSE FORM

In order for the design check in Equation 4 to be equivalent to verifying Equation 3, the theoretical target value for edp_{demand} is chosen as $edp_{demand} = edp_f$ where edp_f is the EDP level exceeded with rate ν_f:

$$\nu(EDP > edp_f) = \nu_f \tag{9}$$

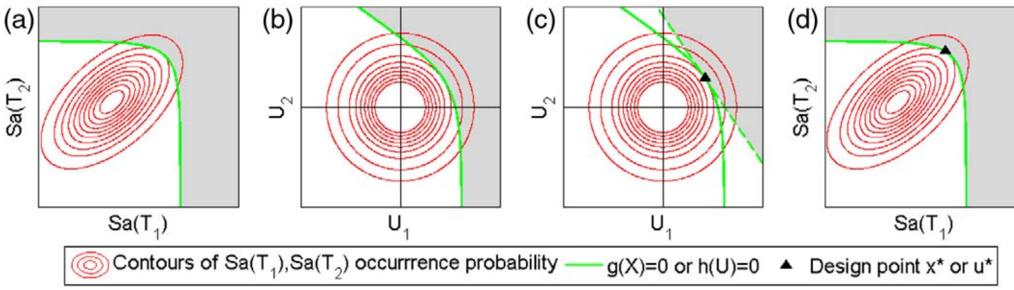

Figure 2. Graphic illustration of the first-order reliability method: (a) joint contour of spectral accelerations with failure function; (b) mapping of the variables into standard normal space - $\mathbf{X} \to \mathbf{U}$, $g(\mathbf{X}) \to h(\mathbf{U})$; (c) $\mathbf{u}^*$ obtained by minimizing $\|\mathbf{u}\|$ such that $h(\mathbf{u}) = 0$, and linearization of the failure function; (d): design point $\mathbf{x}^*$ obtained by transforming $\mathbf{u}^*$ back to the original space.

Referring to Equation 8, when edp_f is specified, the above section briefly outlines the use of FORM to compute ν_f. Conversely, if ν_f is specified and a corresponding edp_f is the quantity of interest (i.e., "What is the demand level exceeded with rate ν_f?"), the problem can be solved using so-called "inverse FORM" (IFORM), or "environmental contours" (Haver and Winterstein 2009). In addition to the demand threshold edp_f, IFORM also provides the coordinates of the design point $\mathbf{x}^* = [Sa^*(T_1), \ldots, Sa^*(T_n)]$ corresponding to the limit state defined by $g(\mathbf{X}) = 0$. Inverse FORM will be used below when evaluating the ability of various candidate response spectra to estimate an *edp* level exceeded with a given probability.

LINK TO THE CONDITIONAL MEAN SPECTRUM

An interesting result occurs in the case of a random variable not explicitly included in the failure function. Suppose for example that $\mathbf{X} = [Sa(T_1), Sa(T_2), Sa(T_3)]$, with a failure function defined as $g(\mathbf{X}) = 1 - Sa(T_1) - Sa(T_2)$. In this case, $Sa(T_3)$ is a component of the input random vector $\mathbf{X}$ but does not appear in the failure function. We can still find the design point for the three spectral accelerations $\mathbf{x}^* = [Sa^*(T_1), Sa^*(T_2), Sa^*(T_3)]$, and it can be shown (Chapter 3, Loth 2014) that the design point value $Sa^*(T_3)$ is the mean value of $Sa(T_3)$ conditioned on the design point values of the two other spectral accelerations $[Sa^*(T_1), Sa^*(T_2)]$.

This observation motivates the use of the conditional mean spectrum (Baker 2011) to compute a full response spectrum. The conditional mean spectrum (CMS) associates at each period T the mean value of the log spectral acceleration, $\ln Sa(T)$, conditioned on a particular design point value at a specific period $Sa^*(T_0)$. This is done by using the multivariate normality property of the residuals of $\ln Sa(T)$. The value of the log spectral acceleration at any period T (including the ones present in the design point) is evaluated by:

$$\mu_{\ln Sa(T)|\ln Sa^*(T_0)} = \mu_{\ln Sa}(M, R, T) + \rho(T, T_0)\sigma_{\ln Sa}(T)\left(\frac{\ln Sa^*(T_0) - \mu_{\ln Sa}(M, R, T_0)}{\sigma_{\ln Sa}(T_0)}\right) \tag{10}$$

where $\mu_{\ln Sa}(M, R, T)$ (resp. $\sigma_{\ln Sa}(T)$) is the mean (resp. standard deviation) of $\ln Sa$ at period T from a ground motion prediction equation for an earthquake with moment magnitude M and distance R, and $\rho(T, T_0)$ is the correlation coefficient between the logarithmic spectral accelerations at T and T_0. Figure 3 shows an example of the CMS conditioned at $T_0 = 1$ s on a spectral acceleration value of $Sa^*(1\text{ s}) = 1.02$ g, for an earthquake with magnitude $M = 7$, distance $R = 10$ km and recorded at a site with $V_{S30} = 400$ m/s.

This spectrum is consistent with the structural reliability framework, because each CMS ordinate corresponds to the design point value $Sa^*(T)$ of a spectral acceleration at a period not included in the failure function. This suggests that if the structure's true failure function is dependent on Sa at only a single period (or can be approximated as such), then the structure's design point is the CMS conditioned on that period. The general computation of this single period CMS is convenient as it does not require the use of vector PSHA, but simply a magnitude and distance available from deaggregation results (in the examples developed in this paper, we simply use M and R from the chosen scenario earthquake). Furthermore, it should be noted that the CMS calculation detailed in Equation 10 may be extended to compute a "generalized CMS" conditioned on Sa at multiple periods (Jayaram et al. 2011; Appendix A, Loth 2014). However, the CMS recommended in this paper will strictly be based on the single period conditioning of Equation 10 (and unless otherwise specified, "CMS" will thus refer to a single period conditioning). Demands estimated from the generalized CMS will only be shown to measure the accuracy of the single period CMS. In the following sections, we evaluate the use of a CMS conditioned at a modal period as a target response spectrum.

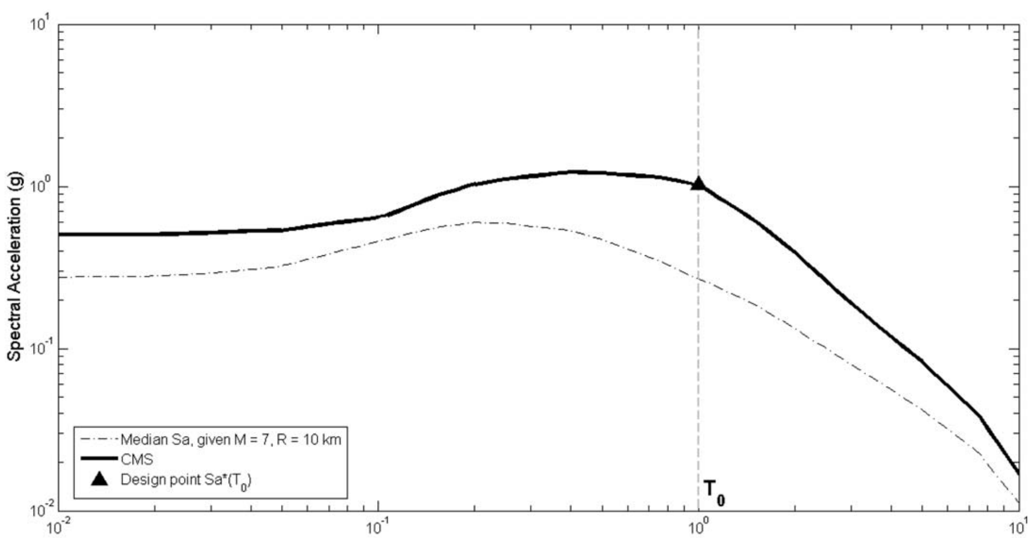

Figure 3. CMS conditioned at $T_0 = 1$ s. The design point values $Sa^*(1\text{ s})$ is represented by the triangle. The rest of the spectrum is obtained computing the mean conditioned on this value, based on the same earthquake scenario from Figure 1.

EXAMPLE ANALYSIS FOR A TWO-MODE EDP

In this section we consider example calculations using the response spectrum method, to illustrate the link between implicit and explicit objectives, the information provided by the design point, and the reasonableness of using the CMS as an approximate representation of the design point.

We assume a response *EDP* equation that takes the form of an SRSS modal combination rule from a response spectrum analysis:

$$EDP = \sqrt{\alpha_1 Sa(T_1)^2 + \alpha_2 Sa(T_2)^2} \tag{11}$$

with T_1 and T_2 the first and second mode periods, α_1 and α_2 positive constants. For this example, we assume $T_1 = 1s$, $T_2 = 0.3s$, $\alpha_1 = 0.75$, and $\alpha_2 = 0.25$. Following Equation 7, the failure function is:

$$g(\mathbf{X}) = g\left(\begin{bmatrix} Sa(1s) \\ Sa(0.3s) \end{bmatrix}\right) = edp_f - \sqrt{0.75\, Sa(1s)^2 + 0.25\, Sa(0.3s)^2} \tag{12}$$

where edp_f is the structural response amplitude to be specified based on Equation 9. With the joint distribution of $[Sa(1s), Sa(0.3s)]$ from Figure 1 and IFORM, we can find the design point and the edp_f value corresponding to a given ν_f.

For a target exceedance rate $\nu_f = 0.0004\,\mathrm{yr}^{-1}$ (equivalent to a probability of exceedance $p_f = 2\%$ in 50 years), Inverse FORM yields the design point $Sa^*(T_1) = 0.81\,\mathrm{g}$, $Sa^*(T_2) = 1.81\,\mathrm{g}$, and setting $g(\mathbf{X}) = 0$ and substituting into Equation 12 gives:

$$edp_f = \sqrt{\alpha_1 Sa^*(T_1)^2 + \alpha_2 Sa^*(T_2)^2} = 1.14 \tag{13}$$

Given the idealized nature of $g(\mathbf{X})$, this value is not physically interpretable, but is used for later comparisons to approximate results. Figure 1b shows the plot of the failure function along with the associated design point for this example.

While still considering the *EDP* to follow the functional form of Equation 11, we examine the case of a failure function with single mode participation. The failure function g_1 only involves the spectral acceleration at the first mode period:

$$g_1(\mathbf{X}) = edp_{f1} - \sqrt{\alpha_1 Sa(T_1)^2} = edp_{f1} - \sqrt{\alpha_1} Sa(T_1) \tag{14}$$

It is straightforward to notice that in this case, the actual value of α_1 does not influence the value of the design point for a given ν_f, since:

$$\nu_f = \nu(\sqrt{\alpha_1} Sa(T_1) > edp_{f1}) = \nu(\sqrt{\alpha_1} Sa(T_1) > \sqrt{\alpha_1} Sa_1^*(T_1)) = \nu(Sa(T_1) > Sa_1^*(T_1)) \tag{15}$$

where $Sa_1^*(T1)$ is the value of the design point at the first mode period associated with the failure function g_1. This value is simple to obtain from a standard hazard analysis by finding the $Sa(T_1)$ exceeded with rate ν_f (note that this is the $Sa(T_1)$ of the UHS with exceedance rate ν_f). Here, $Sa_1^*(T_1) = 1.02\,g$. $Sa(T_2)$ is not present in Equation 14, so the design point value at the second mode period $Sa_1^*(T_2)$ comes from the CMS conditioned on $Sa_1^*(T_1)$. This CMS, denoted CMS1, can be computed using Equation 10. We obtain $Sa_1^*(T_2) = 1.15\,\text{g}$.

With the simpler failure function g_1, no structural reliability calculation is necessary to find the design point (we only used the hazard curve for $Sa(T_1)$ and computed a conditional mean of $Sa(T_2)$). In cases where $\alpha_1 \gg \alpha_2$ (which is the case for deformation-based *EDP*s such as story drift ratios or roof displacement), g_1 will be a good approximation of the original failure function g. They both would lead to approximately the same design points with $edp_f \approx edp$ (CMS1) (where edp(CMS1)) refers to the *EDP* demand evaluated with CMS1) and we can obtain this response value without any FORM calculations. Using the design point from Equation 14, and returning to the failure function of Equation 11, the estimate of the *EDP* exceeded with 2% probability in 50 years is $edp(\text{CMS1}) = 1.05$.

Similarly, we may define a second mode failure function g_2 as:

$$g_2(\mathbf{X}) = edp_{f2} - \sqrt{\alpha_2 Sa(T_2)^2} = edp_{f2} - \sqrt{\alpha_2}Sa(T_2) \tag{16}$$

Following the same reasoning, we obtain the design point $Sa_2^*(T_1) = 0.58g$, $Sa_2^*(T_2) = 1.97\,\text{g}$. This time, $Sa_2^*(T_2)$ comes from the UHS with the exceedance rate ν_f, and $Sa_2^*(T_1)$ is the conditional mean of $Sa(T_1)$ conditioned on $Sa_2^*(T_2)$. The resulting *EDP* response is $edp(\text{CMS2}) = 1.11$, where CMS2 refers to the CMS conditioned on $Sa_2^*(T_2)$. Figure 4 shows the FORM results for the two single mode responses associated with the failure functions g_1 and g_2.

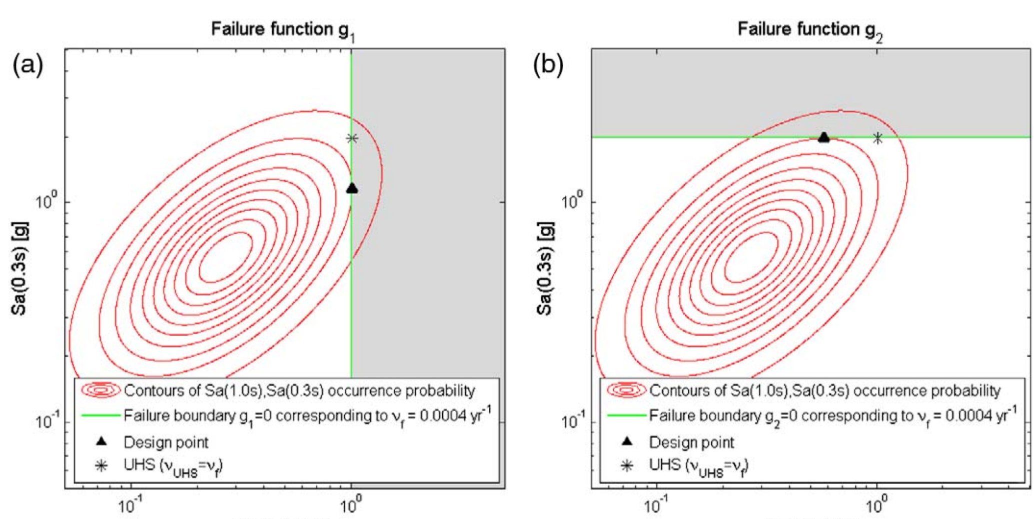

Figure 4. Limit state functions and design points for (a) the first-mode limit-state function of Equation 14 and (b) the second-mode limit state function of Equation 16.

For each failure function and associated design point, a target spectrum (CMS or generalized CMS) can be computed as detailed in the previous section. Figure 5 shows the target spectra from the three failure functions considered above (Equations 12, 14, and 16), as well as the UHS associated with the same exceedance rate ν_f. The *EDP* demand obtained from using the UHS is $edp(\mathrm{UHS}) = 1.32$. In all cases, we observe that the design point spectra lead to smaller spectral accelerations and thus smaller *EDP* demands than the UHS.

The above calculations point to several alternatives for the choice of a target response spectrum. If one is willing to conduct the full reliability analysis with the true failure function that is dependent on *Sa* at multiple periods, one should use the inverse FORM design point and corresponding generalized CMS. For this example, the generalized CMS will yield a single *edp* value of 1.14 (Equation 13), to be checked against the acceptable capacity $edp_{allowable}$ (acceptance criterion in Equation 4). However, if one prefers not to do any reliability calculations, two other alternatives may be considered. The first is to use a UHS with the target exceedance rate, ν_f, but this will result in a considerable overestimation of the true demand value ($edp(\mathrm{UHS}) = 1.32 > 1.14$). The second, which is our proposal, is to compute (single period) CMS spectra conditioned upon the two periods of interest, compute the corresponding *EDP* responses $edp(\mathrm{CMS1})$ and $edp(\mathrm{CMS2})$, and take the maximum of the two as the demand value to be checked in the acceptance criterion. In this example, the single mode approximation gives a reasonable estimate of the true response:

$$edp_{demand} = \underbrace{edp_f}_{1.14} \approx \max\left(\underbrace{edp(\mathrm{CMS1})}_{1.05}, \underbrace{edp(\mathrm{CMS2})}_{1.11}\right) \tag{17}$$

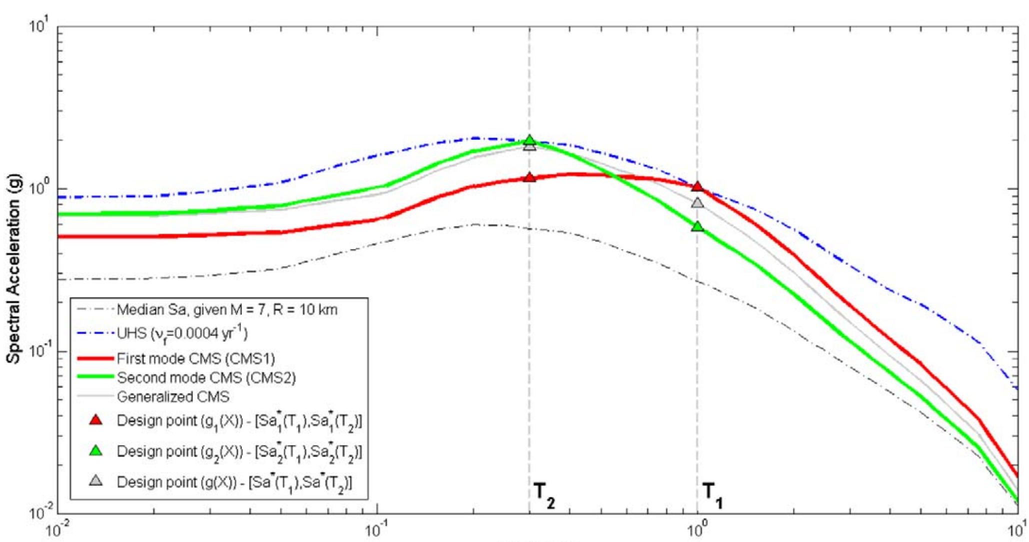

Figure 5. Response spectra associated with the FORM calculations for the two-period *EDP* example.

It should be noted that the use of multiple conditional mean spectra in this manner has been suggested by Baker and Cornell (2006) and is allowed in the Tall Buildings Initiative Guidelines (PEER 2010). Even though this use of multiple CMS is becoming more common in practice (e.g., Almufti et al. 2015), it has not previously been justified using reliability theory. An additional benefit from using the CMS in a response spectrum method framework is observed when considering that the CMS also aims at representing the spectrum from a specific ground motion (as opposed to the UHS which is an envelope of spectral accelerations exceeded from multiple ground motions).

Another approach to obtain a spectrum close to the generalized CMS with no reliability calculation would consist in increasing the value of the single period CMS at periods different than the conditioning period. As can be seen in Figure 5, by increasing the value of the second mode CMS at the other period T_1, we could obtain a spectrum closer to the generalized CMS computed with g. This spectral "broadening" has been suggested by past research (Carlton and Abrahamson 2014), but is not explicitly addressed in this paper.

EXAMPLE ANALYSIS FOR A FIVE-STORY FRAME STRUCTURE

PROBLEM DESCRIPTION

In this example, we will show the results of a response spectrum analysis on a five-story frame, and compare *EDP* predictions from a uniform hazard spectrum and our reliability-based design spectrum. This structure is drawn from Section 12.8 of Chopra (2011) and has been designed to have more higher-mode participation than a real five-story building (it has a lumped mass 100 kips/g and stiffness equal to 31.54 kips/in for each floor).

MODAL ANALYSIS

The mode shapes and modal periods for this structure can be obtained by solving the eigenvalue problem:

$$\mathbf{K} - \omega^2\mathbf{M} = \mathbf{0} \tag{18}$$

where $\mathbf{K}$ is the stiffness matrix of the structure, $\mathbf{M}$ is the mass matrix, $\omega = 2\pi/T$ is a circular frequency. Here we obtain:

$$\begin{cases} \omega_1 = 3.14\,\mathrm{rad/s} \\ \omega_2 = 9.17\,\mathrm{rad/s} \\ \omega_3 = 14.46\,\mathrm{rad/s} \\ \omega_4 = 18.57\,\mathrm{rad/s} \\ \omega_5 = 21.18\,\mathrm{rad/s} \end{cases} \Rightarrow \begin{cases} T_1 = 2.00\,\mathrm{s} \\ T_2 = 0.69\,\mathrm{s} \\ T_3 = 0.43\,\mathrm{s} \\ T_4 = 0.34\,\mathrm{s} \\ T_5 = 0.30\,\mathrm{s} \end{cases} \tag{19}$$

The associated mode shapes are:

$$\phi_1 = \begin{bmatrix} 0.170 \\ 0.326 \\ 0.456 \\ 0.549 \\ 0.597 \end{bmatrix}, \quad \phi_2 = \begin{bmatrix} -0.456 \\ -0.597 \\ -0.326 \\ 0.170 \\ 0.549 \end{bmatrix}, \quad \phi_3 = \begin{bmatrix} 0.597 \\ 0.170 \\ -0.549 \\ -0.326 \\ 0.456 \end{bmatrix}, \quad \phi_4 = \begin{bmatrix} 0.549 \\ -0.456 \\ -0.170 \\ 0.597 \\ -0.326 \end{bmatrix},$$
$$\phi_5 = \begin{bmatrix} -0.326 \\ 0.549 \\ -0.597 \\ 0.456 \\ -0.167 \end{bmatrix} \tag{20}$$

where the j^{th} component of each vector corresponds to the j^{th} story of the structure. Using Equation 20, the participation factors can be determined with:

$$\Gamma_n = \frac{\phi_n^T \mathbf{M}\{1\}}{\phi_n^T \mathbf{M}\phi_n} = \frac{\sum_{j=1}^{N} m_j \phi_{jn}}{\sum_{j=1}^{N} m_j \phi_{jn}^2} \Rightarrow \begin{cases} \Gamma_1 = 2.0971 \\ \Gamma_2 = -0.6602 \\ \Gamma_3 = 0.3480 \\ \Gamma_4 = 0.1938 \\ \Gamma_5 = -0.0885 \end{cases} \tag{21}$$

In the following, we calculate story forces. For each mode, the story forces are given by:

$$F_{jn} = m_j \Gamma_n \phi_{jn} Sa(T_n) \tag{22}$$

where j is the story, n is the mode, and $Sa(T_n)$ is the spectral acceleration corresponding to the n^{th} modal period. The total story force[1] is then determined using a combination rule such as SRSS, which gives:

$$F_{story\,j,SRSS} = \sqrt{\sum_{n=1}^{5} F_{jn}^2} = \sqrt{\sum_{n=1}^{5} \alpha_{jn} Sa(T_n)^2} \tag{23}$$

where $\alpha_{jn} = (m_j \Gamma_n \phi_{jn})^2$. We will consider these story forces as our *EDPs* of interest. Equation 23 is similar in form to the simpler Equation 11.

In this example, we will examine the relative contribution of the higher modes by comparing the response quantities obtained with the design point calculations from two types of failure functions:

$$g_{j,5\,\text{modes}}\left([Sa(T_i)]_{i=1\ldots5}\right) = F_{f,5\,\text{modes}} - \sqrt{\sum_{n=1}^{5} \alpha_{jn} Sa(T_n)^2} \tag{24}$$

[1] It should be noted that this computation of story forces is approximate, as based on a modal analysis that does not predict actual floor accelerations. These floor accelerations may be more accurately estimated using methods presented by Taghavi-Ardakan and Miranda (2006).

$$g_{j,2\,\text{modes}}([Sa(T_1), Sa(T_2)]) = F_{f,2\,\text{modes}} - \sqrt{\alpha_{j1} Sa(T_1)^2 + \alpha_{j2} Sa(T_2)^2} \tag{25}$$

with $F_{f,\cdot}$ the force exceeded with a rate $\nu_f = 0.0004\,\text{yr}^{-1}$ particular to each failure function and determined by the inverse FORM algorithm. Each of these failure functions will yield a different generalized CMS to be used in the calculation of the final demand following Equation 23. Calculations of the roof force ($j = 5$, first-mode dominated) as well as the second story force ($j = 2$, second-mode dominated) are presented for the failure functions defined in Equations 24 and 25. We may also avoid reliability calculations by computing the simpler single period CMS for the first mode (CMS1) and the second mode (CMS2) as presented in the previous sections. Results are summarized in Table 1 and discussed further below.

It can be noted in Table 1 (column corresponding to five-mode FORM for the roof) that $|F_{51}| > |F_{52}|$ which confirms that the roof force is first mode dominated, while $|F_{21}| < |F_{22}|$ shows that the second story force has a higher second mode participation (column corresponding to the five-mode FORM for the second story).

Table 1. Estimates of roof and second story forces exceeded with rate $\nu_f = 0.0004\,\text{yr}^{-1}$. The true story force values from the full five-mode failure functions are shown in the bottom row (79.5 kips for the roof, 61.7 kips for the second story) based on Equation 23. The first five rows ($Sa(T_i)$'s) are spectral acceleration values (in g); the next five rows (F_{jn}'s) simply compute the contribution of each mode n to the story j's force based on Equation 22 (in *kips*). The FORM design points are presented using a failure function only including all five modes (Equation 24) or the first two modes (Equation 25). The spectral acceleration values from the two single period CMS and the UHS are shown and do not depend on the *EDP* of interest. Bold values are design point values associated with spectral accelerations included in the failure function; italicized values represent conditional mean values equivalent to design point values associated with accelerations *not* included in the failure function.

	Roof ($j = 5$)					2nd story ($j = 2$)				
	FORM (*All* 5 *modes*)	FORM (*First* 2 *modes*)	CMS1	CMS2	UHS	FORM (*All* 5 *modes*)	FORM (*First* 2 *modes*)	CMS1	CMS2	UHS
$Sa(T_1)$	**0.541**	**0.549**	**0.560**	*0.323*	0.560	**0.382**	**0.391**			
$Sa(T_2)$	**1.022**	**0.981**	*0.837*	**1.383**	1.383	**1.357**	**1.352**	(Same	(Same	(Same
$Sa(T_3)$	**1.105**	*1.045*	*0.914*	*1.439*	1.774	**1.439**	*1.396*	as	as	as
$Sa(T_4)$	**1.075**	*1.019*	*0.900*	*1.391*	1.916	**1.399**	*1.344*	roof)	roof)	roof)
$Sa(T_5)$	**1.046**	*0.993*	*0.882*	*1.351*	1.967	**1.357**	*1.302*			
F_{j1}	**67.8**	**68.7**	**70.1**	*40.4*	70.1	**26.1**	**26.8**	**38.3**	*22.1*	38.3
F_{j2}	**−37.0**	**−35.5**	*−30.3*	**−50.1**	−50.1	**53.5**	**53.3**	*33.0*	**54.6**	54.6
F_{j3}	**17.5**	*16.6*	*14.5*	*22.8*	28.1	**8.9**	*8.3*	*5.4*	*8.5*	10.5
F_{j4}	**−6.8**	*−6.4*	*−5.7*	*−8.8*	−12.1	**−12.4**	*−11.9*	*−8.0*	*−12.3*	−16.9
F_{j5}	**1.6**	*1.5*	*1.3*	*2.0*	3.0	**−6.6**	*−6.3*	*−4.3*	*−6.6*	−9.6
$F_{story\,j,SRSS}$	79.5	79.4	77.9	68.9	91.5	61.7	61.7	51.6	61.0	70.2

COMPARISONS OF DESIGN SPECTRA AND RESPONSE QUANTITIES

Figure 6 shows the three main spectra associated with the roof force (the two generalized CMS conditioned on the design points obtained from $g_{5,5\,\text{modes}}$ and $g_{5,2\,\text{modes}}$, and CMS1). They essentially match the UHS at the first mode period, and are lower than the UHS at all other periods. Similarly, Figure 7 shows the spectra corresponding to the second story force (the two generalized CMS conditioned on the design points obtained from $g_{2,5\,\text{modes}}$ and $g_{2,2\,\text{modes}}$, and CMS2), matching the UHS at the second mode period and also lower elsewhere.

The "true" value of the seismic demands (edp_f) is determined by using the full reliability analyses from both five-mode failure functions ($g_{j,5\,\text{modes}}$), with one generalized CMS needed for each *EDP* (*j* value). We obtain $F_{roof} = 79.5kips$ and $F_{story2} = 61.7kips$. The forces obtained using $g_{5,2\,\text{modes}}$ and $g_{2,2\,\text{modes}}$ are almost equal to the ones taking into account all five modes. The single mode approximations are also very close to the five-mode answers, when considering the first mode for the roof force and the second mode for the second story force. Therefore, as suggested in the previous sections, we can use the maximum of the demands obtained from both single period CMS to estimate each design demand:

$$\begin{cases} F_{roof} = 79.5 \approx \max(F_{roof}(\text{CMS1}), F_{roof}(\text{CMS2})) = \max(77.9, 68.9) = 77.9 \\ F_{story2} = 61.7 \approx \max(F_{story2}(\text{CMS1}), F_{story2}(\text{CMS2})) = \max(51.6, 61.0) = 61.0 \end{cases} \tag{26}$$

where $F_{roof}(\text{CMS})$ (resp. $F_{story2}(\text{CMS})$) denotes the calculation of the roof (resp. second story) force with Equation 23 using this particular CMS. The errors in *EDPs* estimated from the single period CMS, relative to the true values, are 1% to 2%. However, as

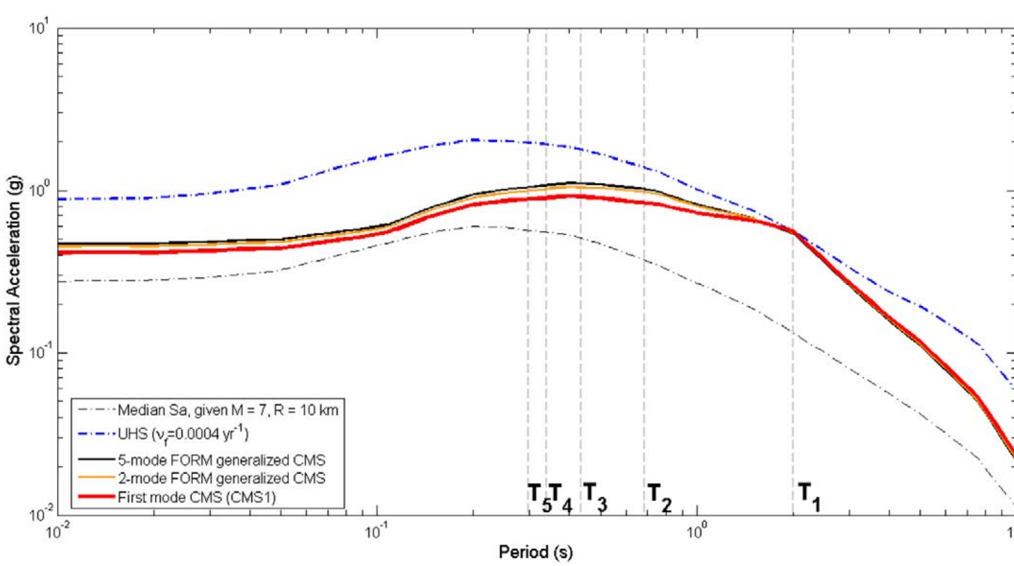

Figure 6. Target spectra associated with the roof force demand.

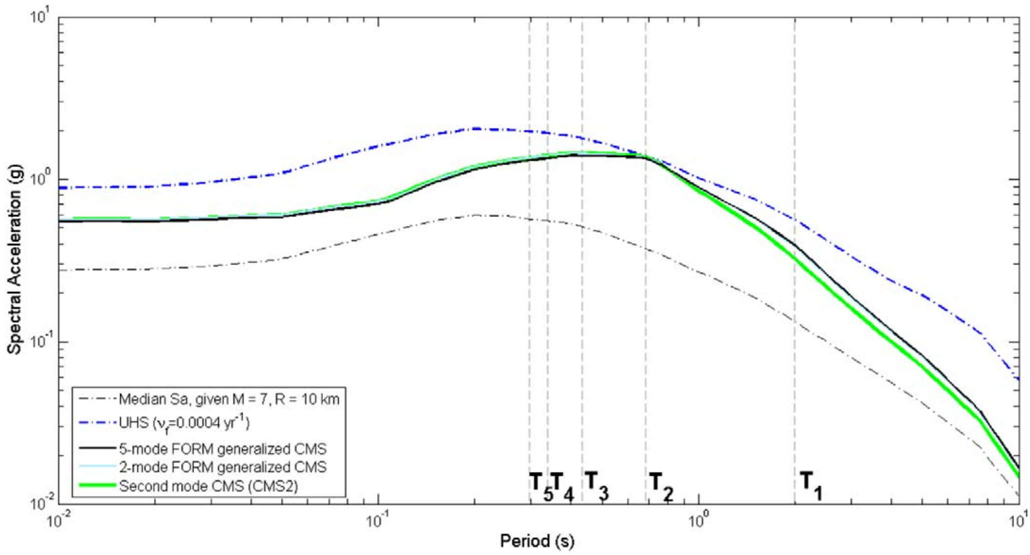

Figure 7. Target spectra associated with the second story force demand.

expected, the second mode (resp. first mode) alone is quite inaccurate for the roof (resp. second story) force.

It can be anticipated from Figure 6 and Figure 7 that the CMS will also yield lower *EDP* demands than the UHS. For example, in the case of the five-mode response, considerable overestimation (around 15%) of the seismic demand is observed by using the UHS instead of the calibrated generalized CMS, for both roof and second story forces.

IMPACT OF USING OTHER CONDITIONING PERIODS ON EDP ESTIMATION

In this simple response spectrum method case, the participation of known modal periods are deterministically quantified. In other situations, such as nonlinear dynamic analysis, relevant periods of interest for an *EDP* calculation may not be known or accurately estimated by the user. In this final section, we show the limited impact of different choices of conditioning periods T_{Cond1}, T_{Cond2}, to compute the single period CMS and the corresponding seismic demands for the same example structure discussed above.

Figure 8a shows a contour plot of the relative reduction of the roof force, when estimated as the maximum of the demands from the single period CMS associated with varying T_{Cond1} and T_{Cond2}, with respect to the true value (from the generalized CMS using the five modal periods), for a target exceedance rate $\nu_f = 0.0004\,\mathrm{yr}^{-1}$. The single period CMS are optimal when one of the periods is equal to the first mode period—this case has a 2% error relative to the *edp* estimated from a full reliability analysis—but the resulting error is still less than 5% if one of the conditioning periods is in the interval from 1.8 s to 2.1 s. Similarly, Figure 8b is a similar plot for second story force, and shows that a CMS conditioned at the second mode period is optimal and provides a 1% error relative to the full reliability analysis estimate, but choosing one of the periods in the range from 0.65 s to 0.75 s leads to an error of 5% or less.

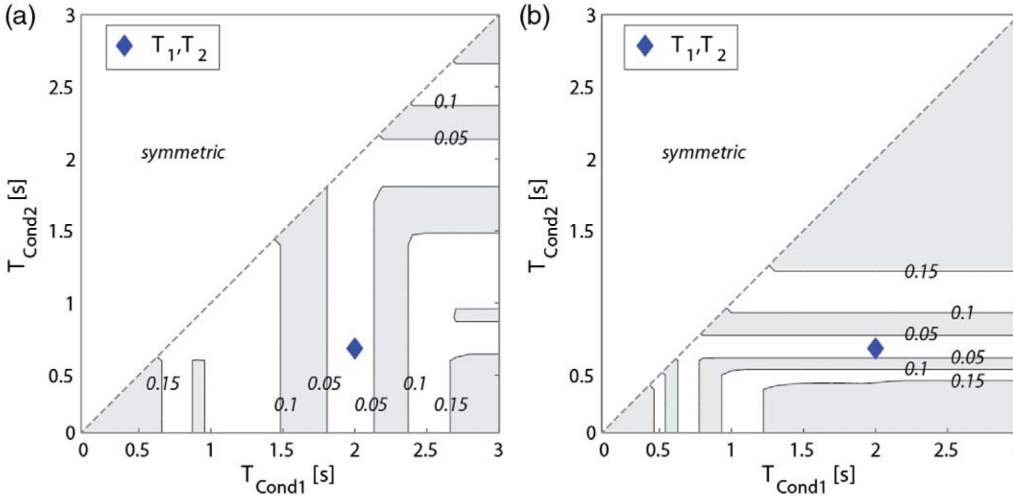

Figure 8. Relative reduction in *EDP* estimation for the example structure using two single period CMS conditioned on T_{Cond1} and T_{Cond2}, with respect to the true value, for a probability of failure of 2% in 50 years; (a): roof force error; (b): second story force error. The blue diamond represents the two modal periods.

Similar calculations were run for a higher failure rate ($\nu_f = 0.002\,\text{yr}^{-1}$), equivalent to a more common ground motion level (1.2 standard deviations above the median *Sa* at the conditioning period), and the resulting period intervals to maintain less than 5% error were broadened somewhat: 1.7 s to 2.2 s and 0.6 s to 0.9 s for estimation of roof and second-floor forces, respectively. These calculations are informative for the case of nonlinear response history analysis, where the choice of optimal periods may not be clear a priori, because they indicate that there is a relatively large range around the optimal periods that will produce comparable results when estimating *EDPs* from conditional mean spectra.

RECOMMENDATIONS

The proposed study has shown the sufficiency of the use of CMS conditioned at modal periods to accurately evaluate a multimodal *EDP* demand. In the particular example of story accelerations, the joint use of first and second mode CMS provided satisfactory estimates of the true multimodal values. However, in a more general case, one might for instance be interested in *EDPs*, which may have significant contributions from even higher modes. For that reason, we recommend to use multiple single period CMS conditioned at all modal periods, as this will not require excessive additional effort once the modal analysis is done. The estimated demand will then be:

$$edp_{demand} = \max\{edp(\text{CMS1}), edp(\text{CMS2}), \ldots, edp(\text{CMS}n)\} \tag{27}$$

where $edp(\text{CMS}i)$ is the demand computed using a CMS conditioned at the i^{th} modal period.

Finally, in order to ensure the initial implicit performance goal $\nu(EDP > edp_{allowable}) \leq \nu_f$ (Equation 3), the explicit design check will consist in checking that the computed edp_{demand} in Equation 27 is less than $edp_{allowable}$ (Equation 4).

CONCLUSIONS

Low probability of failure under earthquake loading is the main objective of seismic design, and is referred to here as the "implicit performance goal." Since most building codes judge that a structure is safe if it has an acceptable behavior under an "explicit design check" performed using a single design spectrum, (which typically approximates a uniform hazard spectrum at present), the variability of the ground motion shaking is not treated consistently with the goal. This paper has addressed this problem within a structural reliability framework. Such analysis involves the characterization of the ground motion hazard by a joint occurrence rate of spectral accelerations at multiple periods, the calculation of demands associated with all plausible spectral acceleration (*Sa*) amplitudes, and the computation of a "design point" indicating the *Sa* amplitudes most likely to cause failure of the system. This design point is shown to be a natural fit with a single explicit design check to verify the structure's performance.

Unfortunately, exact computation of the design point requires a structural reliability and vector probabilistic seismic hazard analysis calculations, which negates the benefit of using a simple design check. This work thus identified that a simpler conditional mean spectrum (CMS) is a close approximation of the design point, is easily computed, and does not require any non-standard seismic hazard information. The approximating CMS is the one conditioned upon a spectral period closely correlated with the engineering demand parameter (*EDP*) of interest. Because multiple *EDP*s will in general be correlated with differing spectral periods, more than one CMS may be needed as response spectra at which to perform design checks. The explicit design check resulting from this approach consists in checking that the seismic demands from each CMS are all less than the tolerable level. Examples of the derivation and use of these CMS were shown here in the context of response spectrum analyses, where the *EDP* of interest depends on several spectral accelerations at modal periods, and the influence of multiple-mode contributions on the target spectra was examined. In the example analysis of a five-story frame, less than a 2% error was observed using these multiple CMS when estimating specific story forces, while the use of the UHS yielded a 15% overestimation of the demand. For the particular case of response spectrum method analysis, we thus recommend the use of multiple CMS conditioned at all modal periods, where the conditioning spectral amplitude of each CMS corresponds to the spectral acceleration value exceeded with the target demand exceedance rate.

These concepts may also be extended to the more complex case of nonlinear response history analysis, where there is no explicit equation linking spectral acceleration amplitudes with *EDP* levels, and thus no explicit spectral periods to use in computing conditional mean spectra. However, the concept of using multiple CMS as design spectra, computing *EDP*s from time histories matching each spectrum, and checking that responses associated with each spectrum are tolerable, is still a rational approach and likely a good approximation of the design point concept. Remaining challenges with application of this approach to response history analysis are the consideration of record-to-record variability and modeling uncertainty, but both of these are tractable using the design point approach and are currently under further study.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation under NSF grant number CMMI 0952402. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation. We also thank the Shah Family Fellowship for providing additional financial support for this work. Thanks to Professors Greg Deierlein and Eduardo Miranda for their feedback, as well Damian Grant and an anonymous reviewer for helpful review comments.

REFERENCES

American Institute of Steel Construction (AISC), 1999. *LRFD Specification for Structural Steel Buildings*, Chicago, IL.

Almufti, I., Motamed, R., Grant, D. N., and Willford, M., 2015. Incorporation of velocity pulses in design ground motions for response history analysis using a probabilistic framework, *Earthquake Spectra* **31**, 1647–1666.

American Society of Civil Engineers (ASCE/SEI), 2010. *Minimum Design Loads for Buildings and Other Structures, ASCE 7-10*, Reston, VA.

Applied Technology Council (ATC), 2011. *Guidelines for Seismic Performance Assessment of Buildings, ATC-58*, Technical Report, Redwood City, CA.

Baker, J. W., 2011. Conditional mean spectrum: Tool for ground-motion selection, *Journal of Structural Engineering* **137**, 322–331.

Baker, J. W., and Cornell, C. A., 2006. Spectral shape, epsilon and record selection, *Earthquake Engineering & Structural Dynamics* **35**, 1077–1095.

Baker, J. W., and Jayaram, N., 2008. Correlation of spectral acceleration values from NGA ground motion models, *Earthquake Spectra* **24**, 299–317.

Bazzurro, P., and Cornell, C. A., 2002. Vector-valued probabilistic seismic hazard analysis (VPSHA), *Proceedings of the 7th US National Conference on Earthquake Engineering*, 21–25.

Bazzurro, P., Winterstein, S. R., Ude, T. C., and Cornell, C. A., 1996. Magnitude-distance contours for probabilistic seismic hazard analysis, in *Proc. of the ASCE Specialty Conference on Probabilistic Mech. And Struct. Reliability*, Worcester, MA, available at http://www.rms-group.org/RMS_Papers/ps/paolo/worcester.ps.

Boore, D. M., and Atkinson, G. M., 2008. Ground-motion prediction equations for the average horizontal component of PGA, PGV, and 5%-damped PSA at spectral periods between 0.01 S and 10.0 S, *Earthquake Spectra* **24**, 99–138.

Bradley, B. A., 2013. A comparison of intensity-based demand distributions and the seismic demand hazard for seismic performance assessment, *Earthquake Engineering & Structural Dynamics* **42**, 2197–2367, doi:10.1002/eqe.2322.

Carlton, B., and Abrahamson, N., 2014. Issues and approaches for implementing conditional mean spectra in practice, *Bulletin of the Seismological Society of America*, doi:10.1785/0120130129.

Chopra, A. K., 2011. *Dynamics of Structures*, 4th Edition, Prentice Hall, Upper Saddle River, NJ.

Der Kiureghian, A., 1981. A response spectrum method for random vibration analysis of MDF systems, *Earthquake Engineering & Structural Dynamics* **9**, 419–435.

Der Kiureghian, A., 1996. Structural Reliability Methods for Seismic Safety Assessment: A Review, *Engineering Structures* **18**, 412–424.

Der Kiureghian, A., and Dakessian, T., 1998. Multiple design points in first and second-order reliability, *Structural Safety* **20**, 37–49.

Ditlevsen, O., and Madsen, H. O., 1996. *Structural Reliability Methods*, First Edition, John Wiley & Sons, New York.

Ellingwood, B. R., 2001. Earthquake risk assessment of building structures, *Reliability Engineering & System Safety* **74**, 251–262.

Fisher, J. W., Ravindra, M. K., Kulak, G. L., and Galambos, T. V., 1978. Load and resistance factor design criteria for connectors, *Journal of the Structural Division* **104**, 1427–1441.

Haver, S., and Winterstein, S. R., 2009. Environmental contour lines: A method for estimating long term extremes by a short term analysis, *Transactions, Society of Naval Architects and Marine Engineers* **116**, 116–127.

Ibarra, L. F., and Krawinkler, H., 2005. *Global Collapse of Frame Structures under Seismic Excitations*, Pacific Earthquake Engineering Research Center, University of California, Berkeley.

Jalayer, F., and Cornell, C. A., 2004. *A Technical Framework for Probability-Based Demand and Capacity Factor Design (DCFD) Seismic Formats*, Pacific Earthquake Engineering Research Center, University of California, Berkeley.

Jayaram, N., and Baker, J. W., 2008. Statistical tests of the joint distribution of spectral acceleration values, *Bulletin of the Seismological Society of America* **98**, 2231–2243.

Jayaram, N., Lin, T., and Baker, J. W., 2011. A computationally efficient ground-motion selection algorithm for matching a target response spectrum mean and variance, *Earthquake Spectra* **27**, 797–815.

LA Tall Buildings Structural Design Council (LATBSDC), 2008. *An Alternative Procedure for Seismic Analysis and Design of Tall Buildings Located in the Los Angeles Region*, Los Angeles, CA.

Liel, A. B., Haselton, C. B., Deierlein, G. G., and Baker, J. W., 2009. Incorporating modeling uncertainties in the assessment of seismic collapse risk of buildings, *Structural Safety* **31**, 197–211.

Loth, C., 2014. *Multivariate Ground Motion Intensity Measure Models, and Implications for Structural Reliability Assessment*, Dept. of Civil and Environmental Engineering, Stanford University, Stanford, CA.

Luco, N., Ellingwood, B. R., Hamburger, R. O., Hooper, J. D., Kimball, J. K., and Kircher, C. A., 2007. risk-targeted versus current seismic design maps for the conterminous United States, in *SEAOC 2007 Conference Proceedings*, 26–29 September 2007, Squaw Creek, CA, Structural Engineers of California.

Madsen, H. O., Krenk, S., Lind, N. C., and Engineering, 2006. *Methods of Structural Safety*, First Edition, Dover Publications, New York.

McGuire, R. K., 2004. *Seismic Hazard and Risk Analysis*, Earthquake Engineering Research Institute, Oakland, CA.

Pacific Earthquake Engineering Research Center (PEER), 2010. *Guidelines for Performance-Based Seismic Design of Tall Buildings*, Report 2010/05, Prepared by the TBI Guidelines Working Group, Berkeley, CA.

Rosenblueth, E., 1951. A Basis for Aseismic Design, Ph.D. Thesis, University of Illinois, Champaign–Urbana.

San Francisco Department of Building Inspection (SFDBI), 2010. *Requirements and Guidelines for the Seismic Design of New Tall Buildings Using Non-Prescriptive Seismic Design Procedures*, Administrative Bulletin AB-083, San Francisco, CA.

Taghavi-Ardakan, S., and Miranda, E., 2006. *Probabilistic Seismic Assessment of Floor Acceleration Demands in Multi-Story Buildings*, John A. Blume Earthquake Engineering Center Technical Report, Stanford University, Stanford, CA.

Van de Lindt, J. W., and Niedzwecki, J. M., 2000. Environmental contour analysis in earthquake engineering, *Engineering Structures* **22**, 1661–1676.

(Received 13 April 2014; accepted 13 September 2014)