

Environmental Contours for Determination of Seismic Design Response Spectra

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ABSTRACT: This paper presents a procedure for using environmental contours and structural reliability design points for the purpose of deriving seismic design response spectra for use in structural engineering design checks. The proposed approach utilizes a vector-valued probabilistic seismic hazard analysis to characterize the multivariate distribution of spectral accelerations at multiple periods that may be seen at a given location, and a limit state function to predict failure of the considered system under a given level of shaking. A reliability assessment is then performed to identify the design point - the response spectrum with the highest probability of causing structural failure. This is proposed as the spectrum for which engineering design checks can be performed to evaluate performance of a given structure. While the full structural reliability analysis would not be performed in any practical application, this analysis does provide three major insights into appropriate response spectra to use in engineering evaluations. First, when the structure's limit state function is dependent on a spectral acceleration at a single period, this approach produces risk-targeted spectral accelerations consistent with those recently adopted in several building codes. Second, the design point spectrum can be approximated by a Conditional Mean Spectrum (conditioned at the spectral period most closely related to the structure's failure). This motivates recent proposals to use the Conditional Mean Spectrum for engineering design checks. Third, the design point spectrum will vary depending upon the structural limit state of interest, meaning that multiple Conditional Mean Spectra will be needed in practical analysis cases where multiple engineering checks are performed (though this can be avoided, at the expense of conservatism, by using a uniform risk spectrum). With the above three observations, this work thus adds theoretical support for several recent advances in seismic hazard characterization.

This paper develops a procedure for using environmental contours and structural reliability design points to formulate improved design spectra for use in assessing the performance of buildings under earthquakes. Most seismic building codes and design guidelines are based on implicit performance goals that structures should achieve. Despite the significant uncertainty in future ground motion occurrence, building codes commonly check a structure's behavior under a single level of earthquake loading, quantified with a design spectrum. However, this explicit design check is often not defined with respect to the performance goals.

The objective of this paper is to provide the link

between the explicit design check and the implicit performance goals. Using structural reliability approaches, with environmental contours of spectral accelerations at multiple periods, a justification of the use of multiple conditional mean spectra (CMS) (Baker, 2011) for design checks is detailed. More specifically, it is shown that exceedance levels of particular engineering demand parameters (*EDP*) can be estimated with a small number of structural analyses based on the calibrated CMS.

1. DESCRIPTION OF THE PROBLEM

For a structure located a given site, we intend to estimate the value edp_f , the level of *EDP* exceeded

with a given annual rate v_f . This is an important quantity in the context of structural performance assessments, where we might want structures to achieve performance goals such as an annual exceedance rate of a given $edp_{allowable}$ being less than v_f . In this case, we would simply need to verify that edp_f is less than $edp_{allowable}$, this inequality being seen as an explicit design check of the performance goal. The estimation of edp_f may be achieved by conducting Probabilistic Seismic Demand Analysis (PSDA) (Shome and Cornell, 1999), which provides the exceedance rates of all *EDP* levels based on a large number of structural analyses at varying ground motion intensity levels. However, this approach may require too much computational effort for the current purpose. Alternatively, we want to determine one target response spectrum that can be used in a structural analysis to more directly determine the value edp_f .

We quantify the ground motion at a given site with a set of spectral accelerations Sa at various periods T_1, \dots, T_n , and denote the vector $\mathbf{X} = [Sa(T_1), \dots, Sa(T_n)]$. The *EDP* of interest corresponds here to an implicit function of \mathbf{X} , with additional variability corresponding to the structural response uncertainty given \mathbf{X} . Section 2 will first examine how to quantify the joint distribution of \mathbf{X} and how to use this information to estimate *EDP* exceedance levels. While this first approach will initially assume *EDP* to be an explicit function of \mathbf{X} , two key simplifications will be introduced in section 3 to consider the general case of *EDP* as an unknown function of \mathbf{X} . A brief explanation of the incorporation of structural response uncertainty is shown in section 4, and a performance assessment of a tall structure using the calibrated CMS is described in section 5.

2. ENVIRONMENTAL CONTOURS WITH VECTOR-VALUED SEISMIC HAZARD

2.1. Quantification of the seismic hazard

When considering a single spectral acceleration $Sa(T)$ at period T , accounting for the aggregation of various possible earthquake scenarios is classically achieved by conducting Probabilistic Seismic Hazard Analysis (PSHA) (McGuire, 2004). The main result of such an approach is a hazard

curve, which provides $MRE_{Sa(T)}(x)$, the mean rate of $Sa(T)$ exceeding the value x . Using the *MRE*, we may also define a mean rate density (*MRD*) by differentiating the *MRE* and taking the absolute value. Either the *MRD* or the *MRE* can be used to quantify the rate of occurrence of spectral acceleration values within a specified interval.

In this paper, we are interested in using the joint hazard associated with a vector of spectral accelerations at multiple periods. The joint distribution of a vector of spectral accelerations $\mathbf{X} = [Sa(T_1), \dots, Sa(T_n)]$ can be obtained using Vector-valued PSHA (VPSHA) (Bazzurro and Cornell, 2002). Similar to scalar PSHA, we can characterize this joint distribution with a multivariate *MRD* of the vector \mathbf{X} . For instance, in the case of $n = 2$ periods, the mean occurrence rate $v(Sa(T_1) \in [b_{11}, b_{12}], Sa(T_2) \in [b_{21}, b_{22}])$ of events where both $b_{11} \leq Sa(T_1) \leq b_{12}$ and $b_{21} \leq Sa(T_2) \leq b_{22}$ can be determined as:

$$\begin{aligned} &v(Sa(T_1) \in [b_{11}, b_{12}], Sa(T_2) \in [b_{21}, b_{22}]) \\ &= \int_{b_{21}}^{b_{22}} \int_{b_{11}}^{b_{12}} MRD_{Sa(T_1), Sa(T_2)}(x_1, x_2) dx_1 dx_2 \quad (1) \end{aligned}$$

An exact computation of a multivariate *MRD* may not be tractable when considering many spectral acceleration periods (typically, for $n \geq 4$), or when there are many possible earthquake sources. Therefore, a simpler method has been proposed and referred to as an indirect approach to VPSHA (Bazzurro et al., 2010), where the results from a single *Sa* deaggregation (Bazzurro and Cornell, 1999) are jointly used with a marginal hazard curve in order to obtain the desired *MRD* with a reduced computational effort. For example, a two-period *MRD* will be given as:

$$\begin{aligned} &MRD_{Sa(T_1), Sa(T_2)}(x_1, x_2) \\ &\approx MRD_{Sa(T_1)}(x_1) \iint f_{Sa(T_2)|Sa(T_1), M, R}(x_2|x_1, m, r) \times \\ &\quad f_{M, R|Sa(T_1)}(m, r|x_1) dm dr \quad (2) \end{aligned}$$

where $f_{M, R|Sa(T_1)}(m, r|x_1)$ is the joint probability density function (pdf) of magnitude and distance given occurrence of $Sa(T_1) = x_1$, obtained from deaggregation, and $f_{Sa(T_2)|Sa(T_1), M, R}(x_2|x_1, m, r)$ is

the conditional pdf of $Sa(T_2)$ given $Sa(T_1)$, m and r . This pdf can be evaluated using the joint log-normality of $[Sa(T_1), Sa(T_2)]$ for a given magnitude and distance.

It should be noted that this formula is not symmetric in $Sa(T_1)$ and $Sa(T_2)$, unlike the exact computation from the direct approach. In Equation 2, $Sa(T_1)$ is the *conditioning* variable, while $Sa(T_2)$ is the *conditioned* variable. This indirect approach formulation introduces small errors in the marginal distribution of the conditioned variable, but the marginal distribution of the conditioning variable is preserved.

Example of 2-IM MRD calculation

In this section, we show an example of VPSHA results carried out at a site in Berkeley (latitude: 37.87° , longitude: -122.29° , average shear wave velocity in the top 30 meters $V_{s30} = 300$ m/s), using the USGS model for seismic sources (Petersen et al., 2008) and the Chiou and Youngs (2008) ground motion prediction equation. Events with moment magnitudes ranging from 4.5 to 8.5 and source-to-site distances from 0 to 200 km were considered. VPSHA code from Barbosa (2011) has been used to compute the joint mean rate density of spectral accelerations at periods $T_1 = 1$ s and $T_2 = 0.3$ s. Figure 1 shows the contours of the obtained joint MRD with $Sa(1s)$ as the conditioning variable.

2.2. Incorporation of VPSHA distributions in a structural reliability framework

Given this hazard information, we now consider a structural response function of spectral accelerations at periods $T_1 = 1$ s and $T_2 = 0.3$ s:

$$EDP = \sqrt{\alpha_1 Sa(T_1)^2 + \alpha_2 Sa(T_2)^2} \quad (3)$$

Note that this *EDP* functional form was previously used as an example in Loth and Baker (2014). Here, we retain the values $\alpha_1 = 0.75$ and $\alpha_2 = 0.25$.

For a given v_f , we seek the value of edp_f (the *EDP* level exceeded with rate v_f), using a calibrated response spectrum, which will here be characterized by a particular vector value for \mathbf{X} , $[Sa^*(T_1), Sa^*(T_2)]$, also referred to as the “design

point”. In particular, from a structural reliability perspective, this design point should correspond to the most likely realization of \mathbf{X} yielding the desired *EDP* value. Using the joint mean rate density of $[Sa(T_1), Sa(T_2)]$ from VPSHA, we can directly quantify *EDP* exceedance rates with numerical integration. More precisely, we define the associated failure function as:

$$g(\mathbf{X}) = edp_f - \sqrt{\alpha_1 Sa(T_1)^2 + \alpha_2 Sa(T_2)^2} \quad (4)$$

with $\mathbf{X} = [Sa(T_1), Sa(T_2)]$. Note that by definition, the function $g(\mathbf{X})$ must verify:

$$v_f = \iint_{g(\mathbf{X}) \leq 0} MRD_{Sa(T_1), Sa(T_2)}(x_1, x_2) dx_1 dx_2 \quad (5)$$

This equation is used to find the value for edp_f , with a trial and error approach.

Using the obtained value for edp_f , we can also compute the failure contour defined as the set of $[Sa(T_1), Sa(T_2)]$ values producing $g(\mathbf{X}) = 0$. The associated design point $[Sa^*(T_1), Sa^*(T_2)]$, by definition the most likely set of spectral acceleration values causing “failure”, is the point having the highest mean rate density on the failure contour:

$$[Sa^*(T_1), Sa^*(T_2)] = \arg \max_{g(\mathbf{X})=0} MRD_{Sa(T_1), Sa(T_2)}(x_1, x_2) \quad (6)$$

This solution can be determined using a simple trial and error approach, by searching all $g(\mathbf{X}) = 0$ values. This type of optimization problem is a particular example of environmental contours (Haver and Winterstein, 2009), which provide reliability-based demand exceedance levels.

Figure 1 shows the failure function and design point for the example problem for a v_f value of 0.0004 yr⁻¹ (equivalent to an occurrence probability of 2% in 50 years). The shaded region represents the area where $\sqrt{\alpha_1 Sa(T_1)^2 + \alpha_2 Sa(T_2)^2} > edp_f$. The obtained value for edp_f is 1.691, and the corresponding design point is $[Sa^*(T_1), Sa^*(T_2)] = [1.760g, 1.464g]$.

While this approach can be used for any type of *EDP* functional form, an analytical equation will generally not be known in practice (i.e., it will result from structural analysis software). The next section

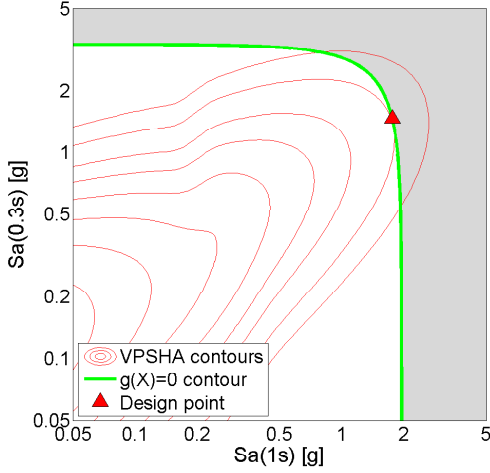


Figure 1: Two-period failure function $g(\mathbf{X})$ and associated design point $[Sa^*(T_1), Sa^*(T_2)]$

will show how to remediate to this issue by a simplification of the failure function. Another simplification of the joint distribution of \mathbf{X} will also be detailed.

3. PROPOSED SIMPLIFICATIONS

In the previous section, we illustrated how to use environmental contours with a joint distribution of spectral accelerations to estimate a seismic demand level exceeded with a particular annual rate. This approach is limited to cases where EDP is an explicit function of \mathbf{X} . The first simplification we propose will allow us to address cases where the EDP functional form is unknown (only spectral acceleration periods of interest are known). The second simplification will discuss a simplified model for the joint spectral acceleration hazard, corresponding to a simplified version of VPSHA that identifies multiple single event scenarios. The use of these single event scenarios should provide comparable seismic demands as the ones obtained using the VPSHA joint spectral acceleration hazard.

3.1. Simplification 1: Single period failure function

The first simplification we propose is to replace g with single period failure functions g_i , $i \in \{1, \dots, n\}$, which depend only on $Sa(T_i)$. In the context of the two-period EDP example from Equa-

tion 3, we define:

$$\begin{cases} g_1(\mathbf{X}) = x_{1f} - Sa(T_1) \\ g_2(\mathbf{X}) = x_{2f} - Sa(T_2) \end{cases} \quad (7)$$

where x_{1f} and x_{2f} are constants obtained by targeting the same rate $v(g_i \leq 0) = v_f$ for $i \in \{1, 2\}$. The corresponding design points are denoted:

$$[Sa_i^*(T_1), Sa_i^*(T_2)] = \arg \max_{g_i(\mathbf{X})=0} MRD_{Sa(T_1), Sa(T_2)}(x_1, x_2) \quad (8)$$

As detailed in Loth and Baker (2014), we may use g_1 and g_2 to approximate the initial failure function g . We then evaluate seismic demands based on Equation 3 with the obtained single period design points:

$$edp_i = \sqrt{\alpha_1 Sa_i^*(T_1)^2 + \alpha_2 Sa_i^*(T_2)^2} \quad (9)$$

and obtain the following approximation:

$$edp_f \approx \max\{edp_1, edp_2\} \quad (10)$$

Figure 2 depicts the single period examples.

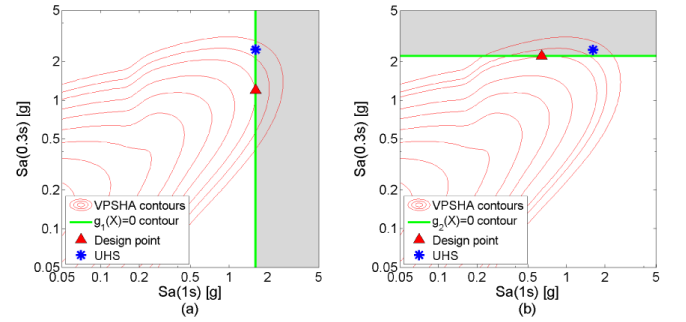


Figure 2: Simplification 1: single period failure functions and associated design points; (a): g_1 , $[Sa_1^*(T_1), Sa_1^*(T_2)]$; (b): g_2 , $[Sa_2^*(T_1), Sa_2^*(T_2)]$

This approximation is critical as the problem can now be solved with no knowledge of the actual EDP functional form, by using scalar PSHA. For instance, $g_1(\mathbf{X}) = 0$ corresponds to $Sa(T_1) = x_{1f}$ where x_{1f} is the spectral acceleration value exceeded with rate v_f , obtained with the hazard curve for $Sa(T_1)$ (i.e., x_{1f} will satisfy $MRE_{Sa(T_1)}(x_{1f}) = v_f$). Similarly, using $MRE_{Sa(T_2)}$, $g_2(\mathbf{X}) = 0$ can theoretically be determined as $Sa(T_2) = x_{2f}$ where $MRE_{Sa(T_2)}(x_{2f}) = v_f$. The point $[x_{1f}, x_{2f}]$, which

can be seen as a UHS point associated with the exceedance rate ν_f , is shown in Figure 2. We observe that while the UHS point lies precisely on the $g_1(\mathbf{X}) = 0$ contour (i.e., $Sa_1^*(T_1) = x_{1f}$, Figure 2a), it is slightly above the $g_2(\mathbf{X}) = 0$ contour (i.e., $Sa_2^*(T_2) < x_{2f}$, Figure 2b). This error originates from the fact that in the indirect approach (Equation 2) used here, $Sa(T_1)$ is the *conditioning* variable, while $Sa(T_2)$ is the *conditioned* variable. Therefore, the marginal distribution of $Sa(T_2)$ is approximate whereas the marginal distribution of $Sa(T_1)$ is preserved. It should be noted that these discrepancies are also due to the binning of the various random variables (Sa , M , R). The quantification of these various sources of error has been studied by Bazurro et al. (2010).

3.2. Simplification 2: Single event scenarios

The second simplification complements the first by considering an approximation of the *MRD* from VPSHA with multiple joint lognormal distributions, corresponding to *MRD*'s from single event scenarios.

3.2.1. Objective

While using numerical integration with environmental contours provides fairly accurate results, it is rather complicated to carry out. Ideally, we would like to be able to conduct reliability calculations with an analytical representation of the joint *MRD* from VPSHA, such as the approach detailed in Loth and Baker (2014). A first potential solution would be to transform the full joint *MRD* from VPSHA to the standard normal space (see Appendix B.2. in Melchers, 1999). This approach is rather complex and may generate significant approximation in the computation of the transformed joint distribution. However, for the present applications, the joint distribution is only needed at high amplitude Sa values associated with structural failures. For such Sa values, we might be able to fit a joint lognormal distribution to the VPSHA contours, which will further simplify the estimation of seismic demands. In this section, we show how to approximate the joint *MRD* from VPSHA by multiple single event *MRD*'s.

3.2.2. Single event approximations based on deaggregation

The proposed fitting approach is based on deaggregation results. First, we obtain the $Sa(T_1)$ value x_{1f} exceeded with a given rate ν_f , and conduct deaggregation at this exceedance level to obtain the mean magnitude M_1 and distance R_1 . Let us denote “event 1” the earthquake scenario with magnitude M_1 and distance R_1 , associated with a rate of occurrence ν_1 . The value for ν_1 is determined such that the single event distribution associated with event 1 provides the same exceedance rate ν_f for x_{1f} . This can be achieved by using the integration method described in the previous section to the single event *MRD* of event 1. This results in fixing ν_1 as the ratio of ν_f divided by the probability of $Sa(T_1)$ exceeding x_{1f} given the occurrence of event 1, which can be computed from the chosen ground motion prediction equation. We obtain $M_1 = 6.98$, $R_1 = 4$ km, $\nu_1 = 0.0103 \text{ yr}^{-1}$. Similarly, we define “event 2”, based on the deaggregation results for $Sa(T_2)$ at the amplitude x_{2f} . Event 2's characteristics are $M_2 = 6.64$, $R_2 = 6$ km, and $\nu_2 = 0.0389 \text{ yr}^{-1}$. These events represent an approximation of the full earthquake hazard (accounting for all sources) at the site of interest. Figure 3 shows contours corresponding to the demand of Equation 3 with the rate of exceedance $\nu_f = 0.0004 \text{ yr}^{-1}$.

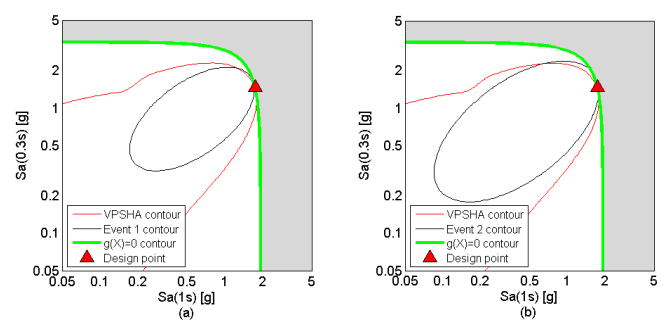


Figure 3: Simplification 2: single event distribution fitting; (a): event 1; (b): event 2.

3.3. Practical implications of the two simplifications for seismic demand estimations

Using both simplifications (i.e., the single period failure functions and the joint hazard from single

event scenarios), the computation of the seismic demand edp_f is further simplified by choosing the single period failure function corresponding to the spectral acceleration at which the single event *MRD* is fitted. In practice, this amounts to evaluating edp_f as the maximum of the seismic demands obtained with multiple CMS conditioned on the UHS amplitudes x_{if} . Using multiple CMS as described is thus implicitly accounting for the underlying joint spectral acceleration hazard.

In the developed example, edp_1 is thus evaluated using the *MRD* from event 1 with the failure function g_1 . Equivalently, edp_1 can be obtained using the CMS conditioned at $1s$ on x_{1f} based on event 1's magnitude and distance. Similarly, edp_2 , evaluated using the *MRD* from event 2 with the failure function g_2 , can be obtained using the CMS conditioned at $0.3s$ on x_{2f} based on event 2's magnitude and distance. Here, we obtain $edp_1 = 1.617$, and $edp_2 = 1.459$. An estimate of edp_f is finally computed by taking the maximum of the two seismic demands: we get $edp_f \approx 1.617$, which is almost equal to the value of 1.691 found in section 2.2 using the numerical integration of the full VP-SHA distribution.

4. INCORPORATION OF STRUCTURAL RESPONSE UNCERTAINTY

We first discuss a theoretical approach to account for structural response uncertainty in the previously described environmental contours. We then show an adaptation of the calibration of the CMS, conditioned on risk-targeted spectral acceleration amplitudes.

4.1. Theoretical considerations

In order to incorporate the presence of structural response uncertainty, the considered *EDP* will generally be expressed as a function of a level of response spectrum and a structural response uncertainty given that spectrum. $EDP|\mathbf{X}$ is now considered as a lognormal random variable, with fixed logarithmic standard deviation σ_Δ . This assumption is consistent with commonly observed response history analyses results (for instance, with drift demands (Shome and Cornell, 1999)). Due to this additional variability, we define a new type of per-

formance goal to estimate edp_f , where we target a specific level of performance given a fixed response spectrum, characterized by $\mathbf{X} = \mathbf{x}_d$:

$$P(EDP > edp_f | \mathbf{X} = \mathbf{x}_d) = p_d \quad (11)$$

with p_d a given probability. Target spectra are then calibrated by solving Equation 11 for \mathbf{x}_d , using the structural reliability techniques presented in this section. More details of this approach can be found in Loth (2014).

4.2. Adapted CMS calibration accounting for structural response uncertainty

Given v_f , p_d , and σ_Δ , the calibration of the CMS conditioned at T_i will be based on the corresponding risk-targeted spectral accelerations $Sa_{RT}(T_i)$, which can be obtained by solving Equation 11 assuming *EDP* to be a function of only $Sa(T_i)$. Note that this approach to compute a risk-targeted spectral acceleration is also suggested in section 21.2.1.2 of ASCE 7-10 (ASCE/SEI, 2010).

The next section will show an application of the use of these calibrated CMS to the estimation of *EDP* exceedance levels of a high rise structure.

5. EXAMPLE & COMPARISONS

5.1. Considered case study

In this section, we summarize results from realistic performance assessments of a MDOF structure based on the use of multiple CMS. The obtained results are validated against PSDA results developed for the same case study by Lin (2012).

The considered structure is a 20-story reinforced concrete moment frame designed for the FEMA P695 project (2009), in which it has the ID 1020. The structural model accounts for cyclic and in-cycle strength deterioration as well as stiffness deterioration. The building is assumed to be located at a real site in Palo Alto (latitude: 122.143° W, longitude: 37.442° N, shear wave velocity: $V_{s30} = 400$ m/s). It should be noted that this structure was designed for a different site, located in Northern Los Angeles (Haselton et al., 2010).

We will consider two different *EDP*'s: the maximum story drift ratio over all stories (*SDR*), and the peak floor acceleration at the 15th story ($PFA^{(15)}$). Given $v_f = 0.001$ yr⁻¹, we estimate sdr_f and

$pfa_f^{(15)}$ as theoretically defined by the demand levels exceeded with rate v_f .

5.2. Approach

The procedure used in this example was proposed in Loth (2014). For a calibration at $p_d = 0.5$, which is retained here, it consists in the following steps: (1) Identify a set of periods T_1, \dots, T_n relevant to the *EDP* of interest; (2) Obtain the hazard curves for $Sa(T_i)$ at the site, and compute the risk-targeted $Sa_{RT}(T_i)$ at each period T_i , for a chosen value for p_d ; (3) Compute the corresponding CMS $_i$ conditioned on $Sa_{RT}(T_i)$; (4) Select and scale ground motion records for each CMS $_i$; (5) Conduct response history analyses, and compute the geometric means of responses associated with each CMS; (6) Estimate edp_f as the maximum of the geometric means obtained with each CMS.

5.3. Results

For both *SDR* and $PFA^{(15)}$, the same set of periods of interest will be retained: the first three modal periods $T_1 = 2.63s$, $T_2 = 0.85s$, $T_3 = 0.46s$, and an elongated period of $T_L = 6.31s$ obtained from a pushover analysis (see Chapter 5 in Loth (2014)). Three modal periods were selected here because we expect $PFA^{(15)}$ to have significant high mode participation. Enough modal periods should generally be considered to account for sufficient mass participation (for instance, ASCE/SEI (2010) recommends 90% for response spectrum method), especially when considering higher mode dominated *EDP*'s. An elongated period was also included to account for the inelastic effect lengthening the period of the structure. The first four rows of Table 1 show the obtained *SDR* and $PFA^{(15)}$ values for the four considered CMS. We observe that the story drift generally appears to be first mode dominated, as CMS conditioned at T_1 leads to the highest demand ($sdr_f \approx 1.8\%$), while the other CMS provide lower values. In the case of $PFA^{(15)}$, the CMS leading to the highest responses is the one conditioned on the third modal period T_3 ($pfa_f^{(15)} \approx 0.40g$). This is an expected result, as floor accelerations of high rise buildings are often driven by higher mode effects.

Table 1: Summary of the response history analyses results (*SDR* and $PFA^{(15)}$) for the *EDP*-based assessment with $p_d = 0.5$.

	$T[s]$	<i>SDR</i> [%]	$PFA^{(15)}[g]$
	0.45	0.9	0.40
geomean	0.85	1.4	0.34
from CMS at	2.63	1.8	0.26
	6.31	1.5	0.25
<i>edp_f</i>		1.8	0.40

5.4. Comparison with PSDA

Lin's PSDA data provides values for sdr_f ranging from 1.6% to 1.9%¹, which is in good agreement with our results (1.8%). Similarly, for $PFA^{(15)}$, our estimate for $pfa_f^{(15)}$ is 0.40g, whereas Lin's estimates lie between 0.40g and 0.43g. This external analysis further confirms the accuracy of obtained values for sdr_f and $pfa_f^{(15)}$, and the adequacy of the proposed use of multiple CMS.

6. CONCLUSIONS

We have first described the definition and computation of joint mean rate density of spectral accelerations at multiple periods using vector-valued probabilistic seismic hazard analysis (VPSHA). While standard PSHA provides occurrence rates of a spectral acceleration at a single period, VPSHA allows the quantification of seismic demands influenced by spectral accelerations at multiple periods. A reliability assessment, based on environmental contours, of an example structure characterized with a known *EDP* function of spectral accelerations at two distinct periods was shown, using a numerical integration of the corresponding VPSHA distribution. We then proposed two key simplifications: (1) an approximation of the multivariate failure function with multiple univariate failure functions; (2) an approximation of the VPSHA distribution with the use of multiple single event scenarios. The joint application of these two simplifications is shown to be equivalent to the use of multiple CMS conditioned at periods of interest for the considered

¹Lin obtained slightly different values when integrating nonlinear response history analysis results using ground motion records selected from various target spectra.

structural demand. The incorporation of structural response uncertainty can be achieved by conditioning the CMS on risk-targeted spectral amplitudes. Results from a performance assessment of a 20-story building using these CMS are shown.

Future work may consist of analyzing joint MRD's from VPSHA corresponding to different sites with various seismic regimes, along with more complex structures and EDP's. This might result in the need to use a higher number of CMS. However, recent work (e.g., Carlton and Abrahamson, 2014; Loth, 2014) seems to indicate that a reduced number of "broadened" CMS (i.e., CMS with increased amplitudes over some period range) may be considered instead, which will reduce the effort involved with this approach.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

- ASCE/SEI (2010). *Minimum design loads for buildings and other structures*. ASCE 7-10; American Society of Civil Engineers.
- Baker, J. W. (2011). "Conditional mean spectrum: Tool for ground-motion selection." *Journal of Structural Engineering*, 137(3), 322–331.
- Barbosa, A. R. (2011). "Simplified vector-valued probabilistic seismic hazard and seismic demand analysis of a 13-story reinforced concrete frame-wall building." Ph.D. thesis, University of California, San Diego, California.
- Bazzurro, P. and Cornell, C. A. (1999). "Disaggregation of seismic hazard." *Bulletin of the Seismological Society of America*, 89(2), 501–520.
- Bazzurro, P. and Cornell, C. A. (2002). "Vector-valued probabilistic seismic hazard analysis (VPSHA)." *Proceedings of the 7th US National Conference on Earthquake Engineering*, 21–25.
- Bazzurro, P., Park, J., and Tothong, P. (2010). "Vector-valued probabilistic seismic hazard analysis of correlated ground motion parameters." *Report No. USGS award G09AP00135*.
- Carlton, B. and Abrahamson, N. (2014). "Issues and approaches for implementing conditional mean spectra in practice." *Bulletin of the Seismological Society of America*.
- Chiou, B. S. J. and Youngs, R. R. (2008). "An NGA model for the average horizontal component of peak ground motion and response spectra." *Earthquake Spectra*, 24(1), 173–215.
- Federal Emergency Management Administration, U. S. (2009). *Quantification of Building Seismic Performance Factors, FEMA P695*.
- Haselton, C. B., Liel, A. B., Deierlein, G. G., Dean, B. S., and Chou, J. H. (2010). "Seismic collapse safety of reinforced concrete buildings. I: Assessment of ductile moment frames." *Journal of Structural Engineering*, 137(4), 481–491.
- Haver, S. and Winterstein, S. R. (2009). "Environmental contour lines: A method for estimating long term extremes by a short term analysis." *Transactions of the Society of Naval Architects and Marine Engineers*, 116, 116–127.
- Lin, T. (2012). "Advancement of hazard-consistent ground motion selection methodology." Ph.D. thesis, Stanford University, California.
- Loth, C. (2014). "Multivariate ground motion intensity measure models, and implications for structural reliability assessment." Ph.D. thesis, Stanford University, California.
- Loth, C. and Baker, J. W. (2014). "Rational design spectra for structural reliability assessment using the response spectrum method." *Earthquake Spectra (in press)*.
- McGuire, R. K. (2004). *Seismic hazard and risk analysis*. Earthquake Engineering Research Institute.
- Melchers, R. E. (1999). *Structural reliability analysis and prediction*. John Wiley New York.
- Petersen, M. D., Frankel, A. D., Harmsen, S. C., Mueller, C. S., Haller, K. M., Wheeler, R. L., Wesson, R. L., Zeng, Y., Boyd, O. S., Perkins, D. M., et al. (2008). "Documentation for the 2008 update of the United States national seismic hazard maps." *Report No. 2008-1128*, U.S. Geological Survey Open-File.
- Shome, N. and Cornell, C. A. (1999). *Probabilistic seismic demand analysis of nonlinear structures*. Dept. of Civil and Environmental Engineering, Stanford Univ., Stanford, Calif.