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SPECTRAL VARIABILITY AND ITS RELATIONSHIP TO STRUCTURAL RESPONSE ESTIMATED FROM SCALED AND SPECTRUM-MATCHED GROUND MOTIONS

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ABSTRACT

Conditional spectral dispersion (*CSD*) is a measure of response spectrum variability that implicitly characterizes the variety of spectral shapes within a suite of ground motions. It is used here to explain the discrepancy between median structural demands estimated from different suites of scaled ground motions as well as those that have been spectrum-matched. Performing response history analyses with spectrum-matched ground motions is known to result in unconservatively biased median demand estimates in some cases. Herein, several suites of scaled ground motions with equivalent median intensities and varying levels of *CSD* are selected. A single suite of spectrum-matched ground motions is also created. These records are used to analyze inelastic single-degree-of-freedom and multiple-degree-of-freedom structural systems. A consistent trend among responses fully attributes the bias phenomenon to the asymmetric relationship between conditional spectral ordinates at periods affecting inelastic behavior and the resulting inelastic response, suggesting that no further explanation for the bias is needed.

Introduction

Quantification of the seismic performance of structures is a critical step in the design and analysis of our built environment. Response history analysis (RHA) is a computationally demanding procedure by which engineers may estimate structural seismic performance, its use becoming more prevalent in professional practice as computational power and understanding of both structural behavior and seismology improve. Prior to any assessment utilizing RHA, acceleration time histories (a.k.a. "records" or "ground motions") resulting from (or attempting to simulate) real earthquakes must be obtained.

Ground motion selection and modification (GMSM) is the collective field of study in which ground motions are selected from databases based on a combination of desired seismological characteristics, and are then sometimes modified. A common intensity measure (*IM*) used to describe ground motions is the response spectrum, denoted $S_a(T)$ at period T. Spectral shape – the relative amplitudes of spectral ordinates over a range of periods – has been shown to be useful for predicting inelastic responses, or "engineering demand parameters" (*EDP*s), such as interstory drift or ductility [e.g. 1,2]. This usefulness leads structural engineers often to require ground motions that have specific spectral shapes for design or analysis purposes

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[e.g. 3,4]. However, specific spectral shapes can be difficult to obtain due to limited record databases, especially as additional characteristics are specified.

Spectrum matching is a technique that modifies the response spectrum of an existing ground motion to provide a desired spectral shape [e.g. 5], effectively increasing the number of records with specific shape characteristics available to the engineer. It is also used to reduce spectral variability within a suite of ground motions at a range of periods, in turn reducing *EDP* variability and enabling efficient estimation of median demands.

A principal concern with the use of spectrum-matched records for RHA is that they are known to provide unconservatively biased response estimates relative to those obtained using comparable scaled ground motions [6-10]. Bias is defined as a ratio of geometric mean *EDP*s

$$bias = \frac{EDP_{geo}}{EDP_{geo,ref}} \tag{1}$$

where $EDP_{geo,ref}$ is the geometric mean *EDP* for a reference suite of ground motions (e.g. scaled records) and EDP_{geo} is the geometric mean *EDP* for a suite of ground motions of interest (e.g. spectrum-matched records).

Controversy remains over the presence and degree of such a bias [11-13]. Its identification depends on a number of factors. First, the degree of bias present tends to increase with the level of nonlinearity experienced by a system [e.g. 8, 10]. Second, due to the importance of spectral shape on nonlinear response, it is important to assess the presence of a bias using suites of ground motions with carefully controlled intensities *vis-à-vis* median spectral shape. If the median shape of each suite is different, it is difficult to separate this effect from others on the resulting inelastic structural response. Third, the variable nature of most *EDP*s makes lending statistical significance to moderate differences in response problematic.

Recent attempts to characterize the nature of the bias for spectrum-matched ground motions have not been able to account for the full magnitude of observed discrepancies. Carballo and Cornell [6] and Seifried [10] examine scaled ground motions normalized to a common spectral amplitude at a structure's fundamental period. They note an asymmetric relationship between a simplified measure of spectral shape for a given record and its resulting demand (i.e. *IM* versus *EDP*), but it is insufficient to describe the bias observed with spectrum-matched ground motions. This implies either that a more sufficient measure of spectral shape is required to capture the relationship or that an additional source also contributes to bias.

It is known that *EDP*s from a nonlinear structure are correlated with spectral ordinates at "effective" periods slightly longer than the structure's fundamental period [e.g. 14]. However, this relationship is poorly defined [e.g. 6,10,14]. This work considers a different measure of spectral shape to verify that an asymmetric relationship between *IM* and *EDP* is responsible for the bias from spectrum-matched records.

Conditional spectral dispersion

Conditional spectral dispersion (*CSD*) is introduced here as a measure that accounts completely for the degree of bias observed between different suites of ground motions. *CSD* is defined as the log-standard deviation of spectral ordinates at an effective period, conditional upon the normalization of the suite of ground motions to a structure's fundamental period, or

$$CSD = \sigma_{\ln S_a(T_{eff})} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left[\ln S_a\left(T_{eff}\right)_{geo} - \ln S_a\left(T_{eff}\right)_i\right]^2} \quad (2)$$

where $S_a(T_{eff})_{geo}$ is the geometric mean of spectral ordinates at effective period T_{eff}, and $S_a(T_{eff})_i$ is the spectral ordinate of the i^{th} of n ground motions at T_{eff}.

Herein, effective periods, are those at which energy present in ground motions will affect structural response. These include periods slightly longer than the fundamental period to account for inelastic deformation ("softening") or shorter than the fundamental period to account for additional modes that may contribute to response.

Although *CSD* explicitly describes the presence or absence of extreme spectral ordinates at a single period, it implicitly accounts for the shape of each spectrum because spectral amplitudes at nearby periods are correlated [e.g. 15]. It helps to describe the influence of an entire suite of ground motions as opposed to a single record. Large values of *CSD* imply that a wide range of spectral shapes and extreme spectral amplitudes are present, while small values of *CSD* imply more homogenous spectral shapes and ordinates are present. Therefore, selecting suites of ground motions to have the same median spectrum but varying levels of *CSD* will measure the impact of these different spectral shapes and spectral amplitudes at effective periods on structural response, on average, without having to account for different median intensities.

Scope

Carefully selected suites of ground motions, each with the same median spectrum but different levels of *CSD*, are used to analyze several inelastic single-degree-of-freedom (SDOF) systems and one inelastic multiple-degree-of-freedom (MDOF) system. For each system, five suites of scaled ground motions are chosen and conditioned on the fundamental period, and one common suite of spectrum-matched ground motions is utilized. Bias among the resulting responses is then examined.

Structural models

Inelastic SDOF systems

Non-degrading bilinear models have been used extensively in previous research to demonstrate the potential presence of a bias between spectrum-matched and scaled ground motion responses [6-10]. Their relatively simple behavior is similar to that of more complex systems where responses are dominated by a single mode, such as the inelastic MDOF model described below.

For an oscillator with elastic period T_{el}, the bilinear model is fully defined by yield displacement d_y, post yield stiffness ratio α, and mass (Fig. 1a). Yield displacement is determined as a fraction of the geometric mean of spectral displacement for a suite of ground motions, or

$$d_y = \frac{1}{R} \exp\left[\frac{1}{n}\sum_{i=1}^{n} \ln S_d(T_{el})_i\right] \tag{3}$$

where $S_d(T_{el})_i$ is spectral displacement for the i^{th} of n ground motions and R is a strength reduction factor. Geometric mean S_d is essentially constant for a given T_{el} from suite to suite in the analysis to follow, so d_y will be as well. Note that a unit mass is used here to reduce the dimensionality of the model. The *EDP* of interest with this system is ductility ratio, μ, given by

$$\mu = \frac{\max|d(t)|}{d_y} \tag{4}$$

where $d(t)$ is the inelastic displacement response history.

Structures with short or long fundamental periods are often affected differently by various ground motion characteristics, so one representative example of each (T_{el} = 0.33 sec and T_{el} = 2.0 sec) is analyzed. To expand the perspective of results, each of these systems is studied using two levels of post-yield stiffness, α={0, 0.10}, and three levels of strength reduction factor, R = {2, 4, 6}.

Inelastic MDOF system

The inelastic MDOF system used here is a 2-dimensional model of a 12-story, 3-bay reinforced concrete moment frame with first mode period T_1 = 2.01 seconds (Fig. 1b). This structure was modeled in *OpenSEES* [16] by Haselton and Deierlein [17] to conform to modern building codes, taking into account *P*-Δ effects, strain-softening, and cyclic deterioration behavior (Fig. 1c) that are critical for simulating large displacements or collapse.

The *EDP* of interest for this system is maximum interstory drift ratio among all stories (*MIDR*). Focus is placed on this parameter because it arises often in both design recommendations and performance-based assessments [e.g. 3,18].

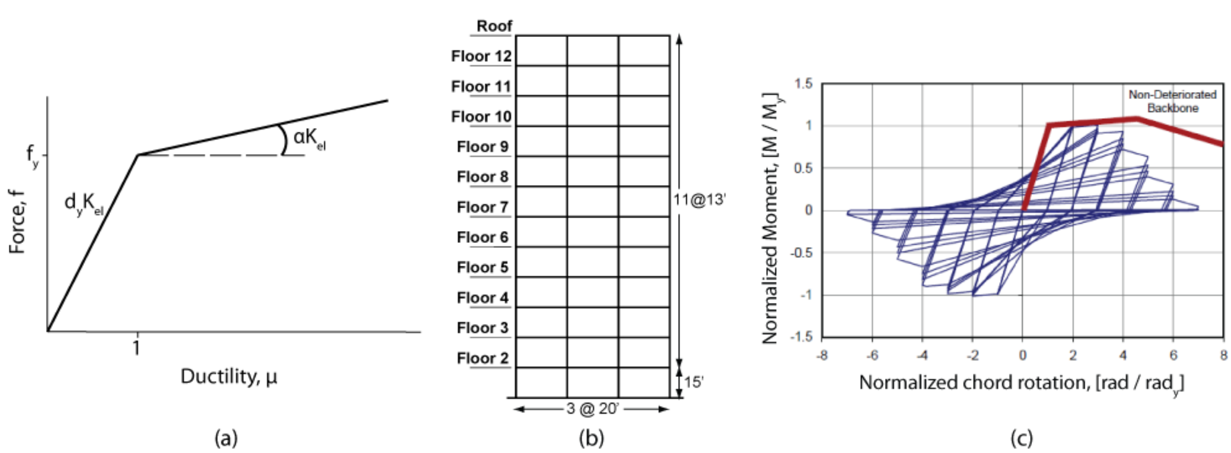

Figure 1. (a) Inelastic SDOF backbone curve; (b) schematic of inelastic MDOF frame; and (c) schematic moment-rotation behavior of MDOF joints (from [17]).

Ground motions

Ground motions are conditionally selected and scaled specific to the elastic period of each inelastic SDOF system. For both T=0.33 seconds and T=2.0 seconds, five suites are selected using the method of Jayaram et al. [19], which employs a greedy optimization algorithm to achieve a target median spectrum and variance. The target spectrum and variance for selection is based on a conditional spectrum (CS) consistent with a magnitude **M**=7 strike-slip earthquake at distance R=10 km and a shear wave velocity Vs^{30}=250 m/s as predicted by the Boore and Atkinson [20] ground motion prediction equation, denoted the "BA GMPE."

For a given fundamental period, each suite varies in both *CSD* and the number of ground motions they contain. The level of *CSD* for each suite is controlled by changing the target variance used in the selection algorithm. Note in the analysis below that as *CSD* is reduced, the dispersion of *EDP* is also reduced. The number of ground motions in each suite is set to provide roughly equivalent standard errors for estimation of the resulting median demand. A more complete description of these record sets is provided in Chapter 4 of Seifried [10]. Suites conditioned on $S_a(T$=0.33 sec$)$ are denoted $S1_{a\text{-}e}$ and those conditioned on $S_a(T$=2.0 sec$)$ are denoted $S2_{a\text{-}e}$. Suites $S2_{a\text{-}e}$ are illustrated in Figs. 2a-e.

These ground motions are selected from the NGA database [21] after first filtering records for **M** > 6 and maximum useable periods of at least 5 seconds. The tradeoff with applying any additional filters is that it becomes increasingly difficult to select a suite of scaled ground motions with a target median spectrum and variance as the target variance decreases. Fig. 2 demonstrates that adherence to the target median spectrum and variance is less ideal at very short periods, but these periods to not play a role in the SDOF analysis, and they are expected to have minimal contribution to *MIDR* for the MDOF analysis.

Spectrum-matched ground motions

A single suite of spectrum-matched ground motions, denoted *M1* and illustrated in Fig. 2f, is created to analyze each structure. *RSPMatch2009* [5] is used to match 25 records to the same **M**=7, R=10 km, Vs^{30} = 250 m/s BA GMPE target spectrum from the selection process above. Each spectrum is matched at 75 evenly log-spaced periods from 0.18 to 8 seconds. A more complete description of the spectrum-matching procedure is provided in [5, 10]. Appendix A contains the indices of the 25 out of 50 records from [10] that are used in this analysis.

CSD in each suite

CSD is calculated for each suite at a single T_{eff} in Eq. 2, therefore the absolute values of *CSD* in Fig. 3, where $T_{eff} = 2.5T_{el}$, depend on this choice. However, note that because the spectral variance in each suite of scaled ground motions is controlled for a range of periods, relative values of *CSD* among the suites will not change, so any trends in the analysis to follow will not change for other T_{eff} in the controlled range. Also, due to tolerances in the matching process, *CSD* for the spectrum-matched suite is not exactly equal to zero.

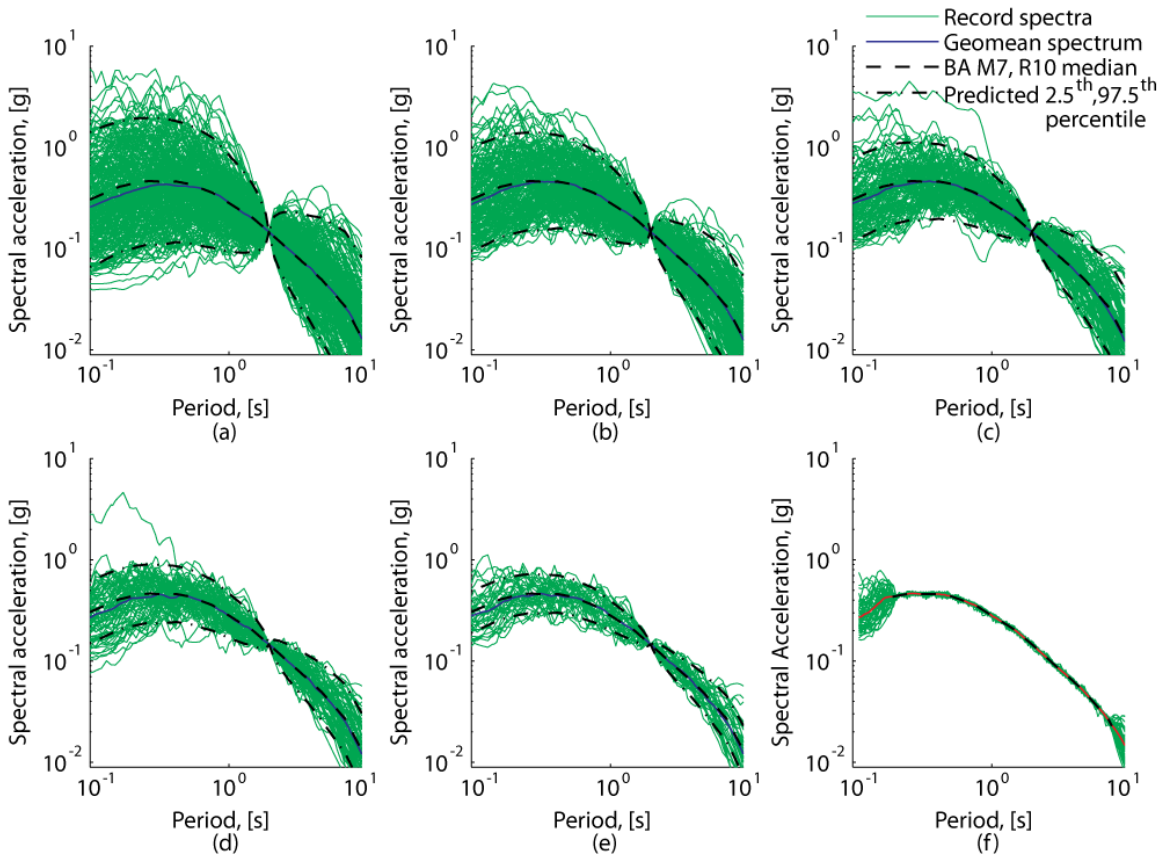

Figure 2. Ground motion suites conditioned at T=2.0 sec and based on a CS with **M**=7, R=10 km, Vs^{30}=250 m/s. Each has approximately the same median spectrum, while spectral variability (σ) is modified from the BA GMPE prediction by (a) 1.2σ with 250 records, (b) 1.0σ with 180 records, (c) 0.8σ with 120 records, (d) 0.6σ with 70 records, and (e) 0.4σ with 35 records. Spectrum-matched records are shown in (f).

Analysis

Inelastic SDOF systems

Suites $S1_{a\text{-}e}$, $S2_{a\text{-}e}$, and $M1$ are used to analyze the inelastic SDOF systems at each combination of T_{el}, α, and R. Bias of μ is calculated using suites $S1_b$ and $S2_b$ as the reference suites in Eq. 1 for T_{el}=0.33 sec and T_{el}=2.0 sec, respectively. Bias and standard errors are shown in Fig. 3, where suites $S1_a$ and $S2_a$, which have the most CSD, are on the right of each subfigure, and suite $M1$, with the least CSD, is on the left of each subfigure. The bias always equals 1.0 for suites $S1_b$ and $S2_b$ because they are being compared with themselves.

In general, the four plots in Fig. 3 display a consistent trend between bias and CSD. There is some variation in this trend, but this is expected from the limited sample size. Additionally, the trend among responses from scaled suites aligns with the bias observed for the spectrum-matched suite. The apparent slight offset in absolute values of CSD among each R for an individual suite is artificially introduced to allow the error bars to be differentiated.

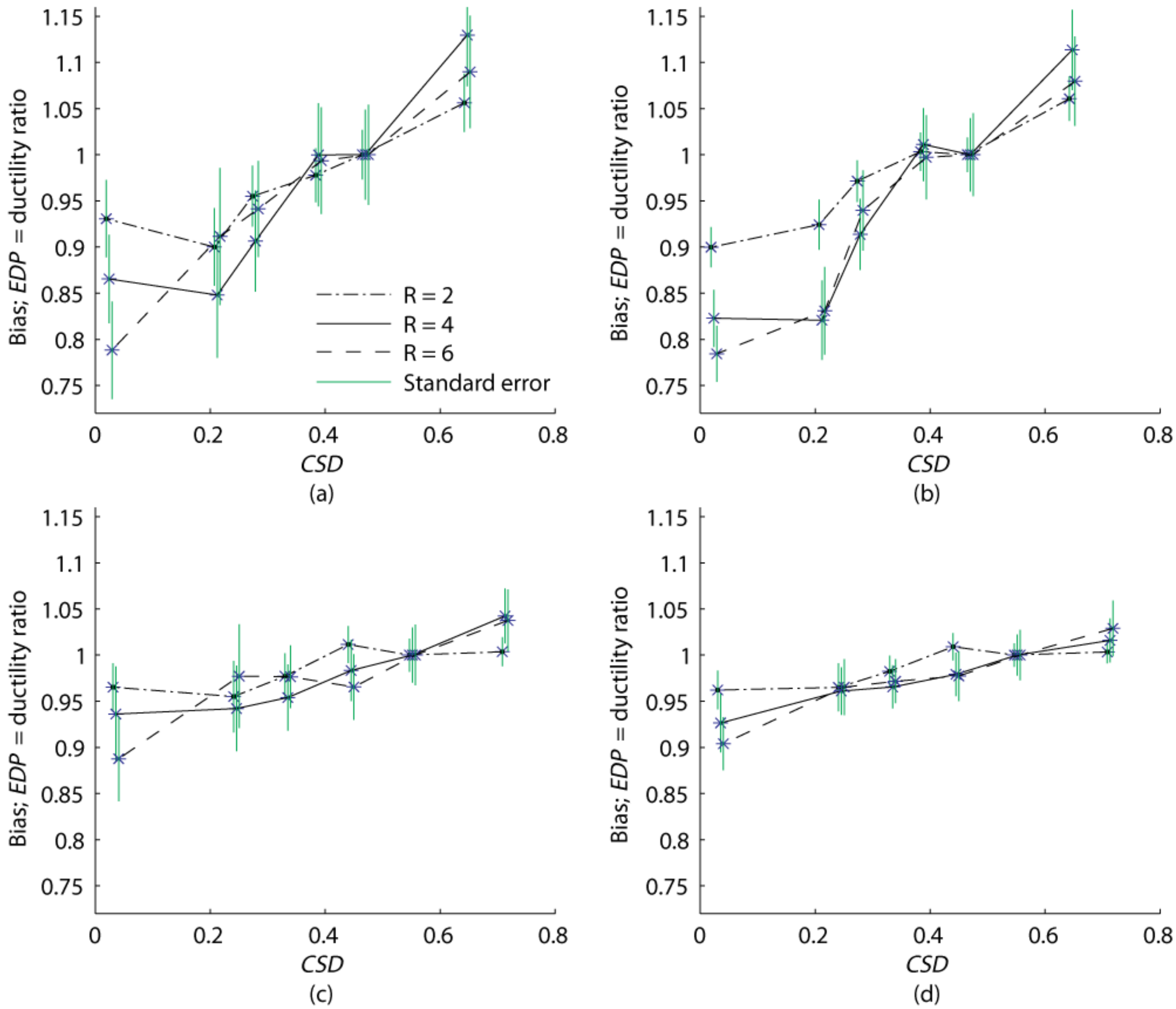

Figure 3. Bias versus conditional spectral dispersion (*CSD*) for each inelastic SDOF system: (a) T_{el}=0.33 sec, α = 0; (b) T_{el}=0.33 sec, α = 0.1; (c) T_{el}=2.0 sec, α = 0; (d) T_{el}=2.0 sec, α = 0.1. *CSD* is evaluated by Eq. 2 at T_{eff}=2.5T_{el}. Each point represents bias from geometric mean μ. Suite *M1* is on the left and suite $S1_a$ or $S2_a$ is on the right.

Table 1. Summary of structural response results from inelastic SDOF analysis.

	T_{el}=0.33 sec, α=0			T_{el}=0.33 sec, α=0.1				T_{el}=2.0 sec, α=0			T_{el}=2.0 sec, α=0.1		
	R=2	*R*=4	*R*=6	*R*=2	*R*=4	*R*=6		*R*=2	*R*=4	*R*=6	*R*=2	*R*=4	*R*=6
$S1_a$	2.70	9.92	19.7	2.30	6.43	11.6	$S2_a$	1.99	4.61	7.55	1.85	3.64	5.70
$S1_b$	2.56	8.78	18.1	2.16	5.78	10.8	$S2_b$	1.99	4.42	7.28	1.85	3.58	5.54
$S1_c$	2.50	8.78	18.0	2.17	5.84	10.7	$S2_c$	2.01	4.35	7.03	1.86	3.51	5.41
$S1_d$	2.44	7.96	17.0	2.10	5.28	10.1	$S2_d$	1.94	4.22	7.10	1.81	3.46	5.38
$S1_e$	2.30	7.45	16.5	2.00	4.74	8.95	$S2_e$	1.90	4.17	7.11	1.78	3.44	5.34
M1	2.38	7.60	14.3	1.95	4.75	8.45	*M1*	1.92	4.14	6.46	1.78	3.32	5.00

The geometric mean ductility ratios used to create Fig. 3 are reported in Table 1. Despite the large degree of bias (bias ≈ 0.8-0.9 in many cases) and trends in Fig. 3, the level of ductility experienced by some of these structures is sometimes unrealistic ($\mu \approx$ 15-20 for T_{el}=0.33 sec and R=6). A model with more realistic behavior, the MDOF system, is therefore analyzed below.

Inelastic MDOF system

The 12-story MDOF model analyzed here is designed such that the median-level intensities of suites $S2_{a\text{-}e}$ do not impose much demand. Therefore, suites *M1* and $S2_{a\text{-}e}$ are scaled up by a factor of 1.5 (and denoted *M1** and $S2_{a\text{-}e}$*) to achieve a higher degree of inelastic behavior. This intensity level results in some collapses for the two most variable suites of scaled ground motions (see Table 2). When a ground motion causes the structure to collapse, the realization of *MIDR* for that record is artificially (and slightly conservatively) set to the peak observed level of *MIDR* from the remaining ground motions in the suite that did not cause collapse. This enables the median to continue to be estimated as the geometric mean without ignoring the collapse cases, and also allows estimation of standard error.

Bias for *MIDR* is calculated relative to suite $S2_b$* using Eq. 1 and plotted versus *CSD* in Fig. 4. As with the SDOF analysis, a trend is observed between bias and *CSD* among responses from scaled ground motions that is consistent with responses from spectrum-matched ground motions. Suite *M1** is on the left of Fig. 4 with the least *CSD*, while suite $S2_a$* is on the right with the most *CSD*. Also similar to the SDOF analysis, there is some degree of (expected) variation in the trend. A further summary of results is provided in Table 2, where the reduction in dispersion of *MIDR* ($\sigma_{\ln MIDR}$) as *CSD* decreases is evident. Note that the combination of changing $\sigma_{\ln MIDR}$ and changing numbers of ground motions leads to roughly consistent standard errors for each suite in Fig. 4 (and for each subfigure of Fig. 3).

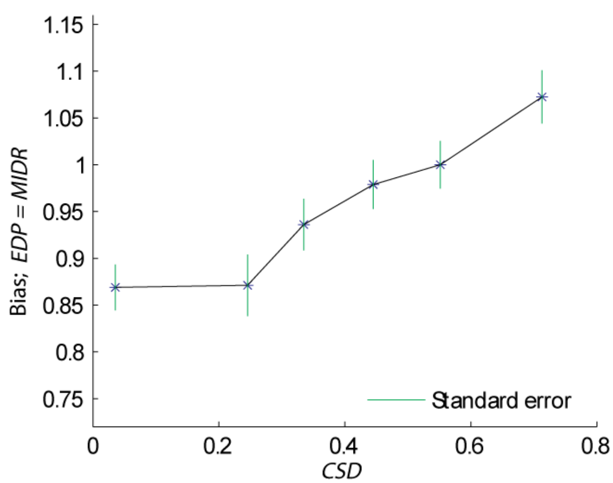

Figure 4. Bias versus *CSD* for the 12-story MDOF system. *CSD* is evaluated by Eq. 2 at T_{eff}=2.5T_{el}. Each point represents bias from geometric mean *MIDR*. Suite *M1** is on the left and suite $S2_a$* is on the right.

Table 2. Summary of structural response results from inelastic MDOF analysis.

	$S2_a$*	$S2_b$*	$S2_c$*	$S2_d$*	$S2_e$*	*M1**
Fraction of collapses	**8**/250	**2**/170	**0**/120	**0**/70	**0**/35	**0**/25
$MIDR_{geo}$	0.0112	0.0104	0.0102	0.0098	0.0091	0.0091
$\sigma_{\ln MIDR}$	0.45	0.34	0.29	0.23	0.19	0.17

Conclusions

Four inelastic SDOF systems and one inelastic MDOF system are studied to determine the cause of a response bias for geometric mean *EDP*s resulting from scaled and spectrum-matched ground motions. RHA is performed on each structural model using five suites of scaled ground motions with equivalent median intensities and different levels of *CSD* at a range of effective periods, as well as one suite of spectrum-matched ground motions. The resulting *EDP*s reveal a trend between bias and *CSD* among the scaled suites that is consistent with the bias present for the spectrum-matched suite. Therefore, *CSD* completely accounts for this bias, which reinforces that an asymmetric relationship between *EDP* and *IM* is solely responsible for it. Thus, as *CSD* is reduced either through spectrum-matching or careful record selection, extreme spectral ordinates and their corresponding disproportionate responses are lost, and geometric mean *EDP* is reduced.

There are two main implications of these findings. First, the process of spectrum matching itself is not responsible for the observed bias between *EDP*s resulting from scaled and spectrum-matched ground motions. Second, efficient GMSM techniques should consider the demonstrated effect of spectral variability on the central tendency of response. Current ground motion selection practice assumes that reducing spectral variability only serves to reduce *EDP* variability and not mean or median response.

These conclusions are subject to a number of limitations. First, the structural models and *EDP*s considered are representative of single-mode-dominated systems with moderate fundamental periods. The similarity in behavior of structures or *EDP*s that are controlled more by multiple or higher modes or that have very short or very long fundamental periods is not known. Second, the ground motions that were used have been coarsely filtered and do not explicitly consider some effects known to be important to structural response (e.g. duration or directivity). Finally, this analysis only considers time-domain spectrum matching, but other methods are also available. However, if additional models, *EDP*s, ground motion properties, or matching techniques are desired to be studied in future research, a framework has been established here to aid that work.

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Appendix

Indices of spectrum-matched ground motions from Chapter 2 of Seifried [10] that are used in this analysis: [1,2,3,6,7,9,11,12,14,15,22,26,27,28,30,31,32,33,38,42,43,44,46,49,50].

References

1. Iervolino, I. and Cornell, C.A. Record selection for nonlinear seismic analysis of structures. *Earthquake Spectra* 2005, 21(3):685-713.
2. Baker, J.W. and Cornell, C.A. Spectral shape, epsilon and record selection. *Earthquake Engineering and Structural Dynamics* 2006; 35(9): 1077-1095.
3. ASCE/SEI. *Minimum Design Loads for Buildings and Other Structures,* ASCE 7-10. American Society of Civil Engineers, 2010.
4. Risk Engineering, Inc. *Technical basis for revision of regulatory guidance on design ground motions: hazard- and risk-consistent ground motion spectra guidelines,* NUREG/CR-6728. U.S. Nuclear Regulatory Commission Report, 2001.
5. Al Atik, L. and Abrahamson, N. An improved method for nonstationary spectral matching. *Earthquake Spectra* 2010, 26(3):601-617.
6. Carballo, J.E. and Cornell, C.A. *Probabilistic seismic demand analysis: Spectrum matching and design,* Report No. RMS-41, Reliability of Marine Structures Program, Stanford University, Stanford, CA, 2000.
7. Bazzurro, P. and Luco, N. Do scaled and spectrum-matched near-source records produce biased nonlinear structural responses? *8th U.S. National Conference on Earthquake Engineering*, San Francisco, CA, 2006.
8. Iervolino, I., De Luca, F. and Cosenza, E. Spectral shape-based assessment of SDOF nonlinear response to real, adjusted, and artificial accelerograms. *Engineering Structures* 2010. 32(9):2776-2792.
9. Huang, Y.-N., Whittaker, A.S., Luco, N. and Hamburger, R.O. Scaling earthquake ground motions for performance-based assessment of buildings. *Journal of Structural Engineering* 2011, 137(3):311-321.
10. Seifried, A.E. Response spectrum compatibilization and its impact on structural response assessment. *Ph.D. Thesis*, Dept. of Civil and Environmental Engineering, Stanford University, Stanford, CA, 2013.
11. Hancock, J., Bommer, J.J. and Stafford, P.J. Numbers of scaled and matched accelerograms required for inelastic dynamic analyses. *Earthquake Engineering and Structural Dynamics* 2008, 37(14):1585-1607.
12. Heo, Y., Kunnath, S. and Abrahamson, N. Amplitude-scaled versus spectrum-matched ground motions for seismic performance assessment. *Journal of Structural Engineering* 2011, 137(3):278-288.
13. Grant, D.N. and Diaferia, R. Assessing adequacy of spectrum-matched ground motions for response history analysis. *Earthquake Engineering and Structural Dynamics* 2013, 42(9):1265-1280.
14. Iwan, W.D. Estimating inelastic response spectra from elastic response spectra. *Earthquake Engineering and Structural Dynamics* 1980, 8:375-388.
15. Baker, J.W. and Jayaram, N. Correlation of spectral acceleration values from NGA ground motion models. *Earthquake Spectra* 2008, 24(1):299-317
16. Open System for Earthquake Engineering Simulation (OpenSEES). Pacific Earthquake Engineering Research Center, 2009. Available from: http//opensees.berkeley.edu.
17. Haselton, C.B. and Deierlein, G.G. *Assessing seismic collapse safety of modern reinforced concrete moment frame buildings,* PEER Report 2007/08, Pacific Earthquake Engineering Research Center, Berkeley, CA, 2007.
18. ATC. *Seismic performance assessment of buildings,* ATC-58. Applied Technology Council, 2012.
19. Jayaram, N., Lin, T. and Baker J.W. A computationally efficient ground motion selection algorithm for matching a target response spectrum mean and variance. *Earthquake Spectra* 2011, 27(3):797-815.
20. Boore, D.M. and Atkinson G.M. Ground-motion prediction equations for the average horizontal component of PGA, PGV, and 5% damped PSA at spectral periods between 0.01s and 10.0s. *Earthquake Spectra* 2008, 24(1):99-138.
21. Chiou, B., Darragh, R., Gregor, N. and Silva, W. NGA project strong-motion database. *Earthquake Spectra* 2008, 24(1):23-44