# Spectral Variability and its Relationship to Structural Response Estimated from Scaled and Spectrum-Matched Ground Motions

A. E. Seifried<sup>1</sup> and J. W. Baker<sup>2</sup>

## ABSTRACT

Conditional spectral dispersion (*CSD*) is a measure of response spectrum variability that implicitly characterizes the variety of spectral shapes within a suite of ground motions. It is used here to explain the discrepancy between median structural demands estimated from different suites of scaled and spectrum-matched ground motions. Performing response history analyses with spectrum-matched ground motions is known to result in unconservatively biased median demand estimates in some cases. Herein, several suites of scaled ground motions with equivalent median intensities and varying levels of *CSD* are selected. A single suite of spectrum-matched ground motions is also created. These records are used to analyze the responses of inelastic single-degree-of-freedom and first-mode-dominated multiple-degree-of-freedom structural systems. Collapse capacities are also examined. A consistent trend between *CSD* and resulting median responses indicates that the bias phenomenon can be fully explained by an asymmetric relationship between conditional spectral ordinates at periods affecting inelastic response.

# **INTRODUCTION**

Quantification of the seismic performance of structures is a critical step in the design and analysis of our built environment. Response history analysis (RHA) is a computationally demanding procedure by which engineers can estimate structural seismic performance, its use becoming more prevalent in professional practice as computational power and understanding of both structural behavior and seismology improve. Prior to any assessment utilizing RHA, acceleration time histories (a.k.a. "records" or "ground motions") resulting from (or attempting to simulate) real earthquakes must be obtained.

<sup>&</sup>lt;sup>1</sup>Senior Project Engineer, Lettis Consultants International, Inc., Boulder, CO 80305, seifried@lettisci.com

<sup>&</sup>lt;sup>2</sup>Associate Professor, Dept. of Civil Engineering, Stanford University, Stanford, CA 94305

Ground motion selection and modification (GMSM) is the collective field of study in which ground motions are selected from databases contingent on desired seismological characteristics, and are then sometimes modified. A common intensity measure (*IM*) used to describe ground motions is spectral acceleration, denoted  $S_a(T)$  at period *T*. Spectral shape – a measure of the relative amplitudes of spectral ordinates over a range of periods – has been shown to be useful for predicting structural responses, or "engineering demand parameters" (*EDP*s), such as interstory drift or ductility (e.g., Iervolino and Cornell, 2005; Baker and Cornell, 2006). This usefulness leads structural engineers often to require ground motions that have specific spectral shapes for design or analysis purposes (e.g., ASCE, 2010; McGuire et al., 2001). However, specific spectral shapes can be difficult to obtain due to limited record databases, especially as additional seismological characteristics are specified.

Spectrum matching is a technique that modifies an acceleration time history such that its response spectrum is changed to provide a desired spectral shape (e.g., Al Atik and Abrahamson, 2010), effectively increasing the number of records with specific shape characteristics available to the engineer. It is also used to reduce spectral variability within a suite of ground motions at a range of periods, in turn reducing *EDP* variability to enable more efficient estimation of median demands (e.g., Hancock et al. 2008). Recent work by Carlson et al. (2014) has quantified the effect of matching on a number of other ground motion characteristics in addition to spectral ordinates.

A principal concern with the use of spectrum-matched records for RHA is that they may provide unconservatively biased response estimates relative to those obtained using comparable scaled ground motions (Carballo and Cornell, 2000; Bazzurro and Luco, 2006; Iervolino et al., 2010; Huang et al., 2011; Seifried, 2013), though not all studies indicate such a bias (Hancock et al., 2008; Heo et al., 2011; Grant and Diaferia, 2013). Bias is defined here as a ratio of geometric mean *EDP*s

$$bias = \frac{EDP_{geo}}{EDP_{geo,ref}} \tag{1}$$

where  $EDP_{\text{geo,ref}}$  is the geometric mean EDP for a reference suite of ground motions (e.g., scaled records) and  $EDP_{\text{geo}}$  is the geometric mean EDP for a suite of ground motions of interest (e.g., spectrum-matched records). Note that true bias is calculated with respect to the true expected response, but in this work  $EDP_{\text{geo,ref}}$  is used to approximate the true expected EDP.

The geometric mean is equivalent to the median for lognormal distributions, and it is used because lognormal distributions have been shown to model many EDPs reasonably well (Aslani and Miranda, 2005). The distribution of log(EDP) is therefore normal, which has many convenient statistical properties.

Identification of any bias depends on a number of factors. First, the degree of bias tends to increase with the level of nonlinearity experienced by a system (e.g., Iervolino, 2010; Seifried, 2013). Second, due to the importance of spectral shape on nonlinear response, it is important to assess the presence of a bias using suites of ground motions with carefully controlled intensities *vis-à-vis* equivalent spectral shape. Given the importance of spectral shape to system response, if the median shape of each suite is different, it is difficult to separate the effects of spectral shape and the matching procedure on the resulting inelastic structural responses. Third, the variable nature of most *EDP*s makes it difficult to distinguish between significant differences in response and artifacts resulting from uncertain estimates.

Recent attempts to characterize the nature of the bias for spectrum-matched ground motions have not been able to account for the full magnitude of observed discrepancies. Carballo and Cornell (2000) examine scaled ground motions normalized to a common spectral amplitude at a structure's fundamental period. They note an asymmetric relationship between a simplified measure of spectral shape for a given record and its resulting demand: disproportionately large *EDP*s were observed for an increase in the shape parameter compared to *EDP*s observed for an equivalent decrease in the shape parameter, which indicates a skewed conditional distribution of *EDP* and *IM*. However, this parameter is insufficient to describe the bias observed for responses to spectrum-matched ground motions relative to scaled or unscaled ground motions. This implies either that a more sufficient measure of spectral shape is required to capture the relationship, or that an additional source also contributes to bias.

It is known that *EDP*s from a nonlinear structure are correlated with spectral ordinates at "effective" periods longer than the structure's fundamental period (e.g., Iwan, 1980). However, this relationship is poorly defined (e.g., Carballo and Cornell, 2000; Seifried, 2013; Iwan, 1980). This work considers a different measure of spectral shape to verify that an asymmetric relationship between *IM* and *EDP* is responsible for the bias observed with spectrum-matched records, and also confirms that a response bias is not limited to this type of ground motion modification.

## CONDITIONAL SPECTRAL DISPERSION

Conditional spectral dispersion (*CSD*) is introduced here as the log-standard deviation of spectral ordinates at an effective period, conditional upon the normalization of the ground motion suite to a single spectral value at a structure's fundamental period,  $S_a(T_1)$ , or

$$CSD = \sigma_{\ln S_a(T_{\text{eff}})} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left[ \ln S_a(T_{\text{eff}})_i - \ln S_a(T_{\text{eff}})_{\text{geo}} \right]^2}$$
(2)

where  $S_a(T_{eff})_{geo}$  is the geometric mean of spectral ordinates at effective period  $T_{eff}$ , and *i* denotes the *i*<sup>th</sup> of *n* ground motions. The conditioning on  $S_a(T_1)$  is omitted from the notation for brevity, however this conditioning is essential, as a change to  $S_a(T_1)$  can have a strong influence on the resulting structural response without changing the value of *CSD*.

Herein, effective periods are those at which energy present in ground motions will affect structural response. This includes periods slightly longer than the fundamental period to account for inelastic deformation ("softening") or those shorter than the fundamental period to account for additional modes that may contribute to *EDP* levels, where applicable.

*CSD* explicitly describes the presence or absence of extreme spectral ordinates at a single period, and it also implicitly accounts for the shape of each spectrum because spectral amplitudes at nearby periods are correlated (e.g., Baker and Jayaram, 2008). Note also that it describes an entire suite of ground motions as opposed to a single record. Large values of *CSD* imply that a wide range of spectral shapes and extreme spectral amplitudes are present, while small values of *CSD* imply that more homogenous spectral shapes and ordinates are present. Therefore, selecting multiple suites of ground motions to have the same median spectrum but varying levels of *CSD* will measure the impact of these different spectral shapes and spectral amplitudes at effective periods on structural response, on average, without having to account for the effect of different median intensities.

## SCOPE

Carefully selected suites of ground motions, each with the same median response spectrum but different levels of *CSD*, are used to analyze several inelastic single-degree-of-freedom (SDOF) systems and one inelastic multiple-degree-of-freedom (MDOF) system. For each system, five suites of scaled ground motions are chosen and conditioned on the fundamental period, and one common suite of spectrum-matched ground motions is utilized. Bias among the resulting

responses from each suite is then examined, including tests for statistical significance of any observed differences in geometric mean *EDP*. The collapse capacity of the MDOF system in relation to *CSD* is also investigated.

# STRUCTURAL MODELS

#### **INELASTIC SDOF SYSTEMS**

Non-degrading bilinear models have been used extensively in previous research to demonstrate the potential presence of a bias between *EDP*s obtained from spectrum-matched and scaled ground motions (Carballo and Cornell, 2000; Bazzurro and Luco, 2006; Iervolino et al., 2010; Huang et al., 2011; Seifried, 2013). Their relatively simple behavior is similar to that of more complex systems with *EDP*s dominated by a single mode, such as the inelastic MDOF model described below.

For an oscillator with elastic period,  $T_{el}$ , the bilinear model is fully defined by yield displacement,  $d_y$ , post yield stiffness ratio,  $\alpha$ , and mass (Fig. 1a). Yield displacement is determined here as a fraction of the spectral displacement for a suite of ground motions, or

$$d_{\mathcal{Y}} = \frac{1}{R} S_d(T_{\rm el})_{\rm geo} \tag{3}$$

where  $S_d(T_{el})_{geo}$  is the geometric mean spectral displacement from a suite of ground motions at  $T_{el}$ , and *R* is a strength reduction factor. A unit mass is used to reduce the dimensionality of the model. The *EDP* of interest with this system is ductility ratio,  $\mu$ , given by

$$\mu = \frac{\max|d(t)|}{d_{\mathcal{Y}}} \tag{4}$$

where d(t) is the inelastic displacement response history.

Structures with short or long fundamental periods may be affected differently by various ground motion characteristics, so two representative examples ( $T_{\rm el} = 0.33$  sec and  $T_{\rm el} = 2.0$  sec) are analyzed. To consider the impact of other structural properties, each of these systems is studied using two levels of post-yield stiffness,  $\alpha = \{0, 0.10\}$ , and three levels of



**Figure 1.** (a) Inelastic SDOF backbone curve; (b) schematic of MDOF frame; and (c) schematic moment-rotation behavior of MDOF joints. Note: (c) is from Haselton and Deierlein (2007).

strength reduction factor,  $R = \{2, 4, 6\}$ . Note that a post-yield stiffness of 0.10 is not likely to be encountered in real structural components: a ductility of ten (which is not uncommon with this type of system) will result in a force equal to twice the yield strength. The goal of including this value along with the alternative of zero is to identify potential bias over a broad range of this parameter, as well as because it has been used in other studies (Bazzurro and Luco, 2006).

# INELASTIC MDOF SYSTEM

The inelastic MDOF system studied here is a 2-dimensional model of a 12-story, 3-bay reinforced concrete moment frame with its first three modal periods  $T_1 = 2.01$ ,  $T_2 = 0.68$ , and  $T_3 = 0.39$  seconds (Fig. 1b). This structure was modeled in *OpenSEES* (2009) by Haselton and Deierlein (2007) to conform to modern building codes, taking into account *P*- $\Delta$  effects, strain-softening, and cyclic deterioration behavior (Fig. 1c) that are critical for simulating large displacements, element rotations, or collapse.

The *EDPs* of interest for this system are story drift ratio (*SDR*), the maximum story displacement divided by story height observed over all stories, and collapse capacity ( $S_a(T_1)_{col}$ ), the  $S_a(T_1)$  value at which the structure achieves dynamic instability during RHA. These parameters both arise often in design recommendations and performance-based assessments (e.g., ASCE, 2010; ATC, 2012).

#### **GROUND MOTIONS**

#### SCALED GROUND MOTIONS

Multiple suites of single-component records are selected to have equivalent median response spectra but varying levels of CSD. Spectral shape is given primary importance in the ground motion selection due to its influence on structural response. For each of the two SDOF systems (with elastic periods of T=0.33 seconds and T=2.0 seconds), five suites of records are selected using the method of Jayaram et al. (2011), which employs an optimization algorithm to match both a target median spectrum and target spectral variance over a range of periods. The target spectrum and variance are based on a conditional spectrum (CS; Baker and Cornell, 2006) consistent with a magnitude M=7 strike-slip earthquake at distance R=10 km and site condition  $V_s^{30}$ =250 m/s, as predicted by the Boore and Atkinson (2008) ground motion prediction equation and correlations from Baker and Jayaram (2008), denoted the "BA GMPE." Ground motions are selected from the NGA database (Chiou et al., 2008), considering only candidate records with M > 6, maximum useable periods of at least 5 seconds, and scale factors of 10 or less. Other seismological parameters are not explicitly considered, though they will be reflected to some degree by the shape of the target response spectrum. Suites conditioned on  $S_a(T=0.33 \text{ sec})$  are denoted  $SI_{a-e}$  (not shown), while those conditioned on  $S_a(T=2.0 \text{ sec})$  are denoted  $S2_{a-e}$  and illustrated in Figs. 2a-e. The conditional dispersion used as a target for selection in suites  $SX_a$  through  $SX_e$  is set using factors of 1.2, 1.0, 0.8, 0.6, and 0.4, respectively, applied to the BA GMPE prediction.

One of the goals of this analysis is to determine whether any observed difference in median response between suites of ground motions is statistically significant using a 1-sided t-test. Given the variability of *EDP*s and the degree of differences observed below, many ground motions are required to achieve this. Note in the analysis below that as *CSD* is reduced, the dispersion of *EDP* is also reduced. The number of ground motions in each suite is set to provide roughly equivalent standard errors (SE) of the resulting median demand according to

$$n \approx \left(\frac{\sigma_{\ln EDP}}{SE}\right)^2 \tag{5}$$

The large number of records included in the selected suites precludes more rigorous filtering criteria. The tradeoff with applying additional seismological characteristic restrictions

is that it becomes increasingly difficult to achieve the target median spectrum and variance, especially as the target variance decreases. Fig. 2 demonstrates that adherence to target values is less ideal at very short periods, but energy at these periods is inconsequential to the SDOF analysis, and is expected to have minimal contribution to *SDR* and collapse capacity for the MDOF analysis.

The selected suites are not fully independent of one another in that records are allowed to appear in multiple suites. This is another tradeoff between the number of records in each suite and fit to the target spectrum. However, because of the large number of records, and because the records are randomly selected after relatively few characteristic filters, a systematic bias due to duration, directivity, magnitude, etc., is not expected to influence the results of this analysis.



**Figure 2.** Ground motion suites conditioned at T=2.0 sec and based on a CS with M=7, R=10 km,  $V_s^{30}=250$  m/s. Each has approximately the same median spectrum, while spectral variability ( $\sigma$ ) is modified from the BA GMPE prediction by (a)  $1.2\sigma$  with 250 records, (b)  $1.0\sigma$  with 180 records, (c)  $0.8\sigma$  with 120 records, (d)  $0.6\sigma$  with 70 records, and (e)  $0.4\sigma$  with 35 records. Spectrum-matched ground motions are shown in (f).

## **SPECTRUM-MATCHED GROUND MOTIONS**

A single suite of spectrum-matched ground motions is created to analyze each structure. Twenty five seed records with a good initial fit to the target spectrum between periods of 0.1 and 10 seconds are selected, considering only NGA database records with magnitudes between M=6.5 and M=7.5, distances less than or equal to 60 km, and maximum useable periods of at least 5 sec. The time domain spectrum matching procedure of *RSPMatch2009* (Al Atik and Abrahamson, 2010) is used to match each record to the same M=7, R=10 km,  $Vs^{30} = 250$  m/s BA GMPE median spectrum from the selection process above. Spectra are matched at 75 evenly log-spaced periods from 0.18 to 8 seconds. The procedure is applied in 4 iterations using the improved tapered cosine wavelet (Al Atik and Abrahamson, 2010) in progressively wider period bands. Each resulting time series is checked to ensure that it has realistic time domain properties and energy distribution. The resulting suite is denoted *M1* and illustrated in Fig. 2f.

#### CSD IN EACH SUITE

*CSD* is calculated for each suite at a single  $T_{eff}$  using Eq. 2. The absolute values of *CSD* in Fig. 3 depend on the choice of  $T_{eff}$ , which is  $2.5T_{el}$ . However, spectral variability in each suite of ground motions is controlled for a range of periods in the record selection process, so relative values of *CSD* among the suites will not change if  $T_{eff}$  is selected as some other value. Therefore, any trends in the analysis to follow do not depend on the specific value of  $T_{eff}$  as long as it is in the range of periods controlled by the selection process. Also, due to tolerances in the matching procedure, *CSD* for the spectrum-matched suite is not exactly equal to zero, as it would be if all spectral ordinates were identical at  $T_{eff}$ .



**Figure 3.** Bias in estimated ductility versus conditional spectral dispersion (*CSD*) for each inelastic SDOF system. *CSD* is evaluated by Eq. 2 at  $T_{\text{eff}}=2.5T_{\text{el}}$ .

## ANALYSIS

#### **INEALSTIC SDOF SYSTEMS**

Suites  $SI_{a-e}$ ,  $S2_{a-e}$ , and MI are used to analyze the inelastic SDOF systems at each combination of  $T_{el}$ ,  $\alpha$ , and R. Bias of  $\mu$  is calculated using suites  $SI_b$  and  $S2_b$  as the reference suites in Eq. 1 for  $T_{el}=0.33$  sec and  $T_{el}=2.0$  sec, respectively, because they correspond to the unmodified CS obtained from the BA GMPE. Bias and standard errors are shown in Fig. 3, where suites  $SI_a$ and  $S2_a$ , which have the largest *CSD*, are on the right of each subfigure, and suite MI, which has the smallest *CSD*, is on the left of each subfigure. Geometric mean ductility ( $\mu_{geo}$ ) and bias for each suite are provided in Tables 1 and 2, respectively. Bias is equal to 1.0 for suites  $SI_b$ and  $S2_b$  because the numerator and denominator of Eq. 1 are from the same suite. The apparent slight offset in absolute values of *CSD* among each *R* value for an individual suite is artificially introduced to allow the standard error bars to be differentiated.

	$T_{\rm el}=0.33  {\rm sec},  \alpha=0$			$T_{\rm el}=0.33  {\rm sec},  \alpha=0.1$				$T_{\rm el}=2.0$ sec, $\alpha=0$			$T_{\rm el}=2.0  {\rm sec},  \alpha=0.1$		
	<i>R</i> =2	<i>R</i> =4	<i>R</i> =6	R=2	<i>R</i> =4	<i>R</i> =6	_	<i>R</i> =2	<i>R</i> =4	<i>R</i> =6	R=2	<i>R</i> =4	<i>R</i> =6
$SI_a$	2.70	9.92	19.7	2.30	6.43	11.6	$S2_a$	1.99	4.61	7.55	1.85	3.64	5.70
$SI_b$	2.56	8.78	18.1	2.16	5.78	10.8	$S2_b$	1.99	4.42	7.28	1.85	3.58	5.54
$SI_c$	2.50	8.78	18.0	2.17	5.84	10.7	$S2_c$	2.01	4.35	7.03	1.86	3.51	5.41
$SI_d$	2.44	7.96	17.0	2.10	5.28	10.1	$S2_d$	1.94	4.22	7.10	1.81	3.46	5.38
$SI_e$	2.30	7.45	16.5	2.00	4.74	8.95	$S2_e$	1.90	4.17	7.11	1.78	3.44	5.34
Ml	2.38	7.60	14.3	1.95	4.75	8.45	M1	1.92	4.14	6.46	1.78	3.32	5.00

**Table 1.** Geometric mean  $\mu$  from inelastic SDOF analysis.

**Table 2.** SDOF ductility bias, relative to suites  $S1_b$  and  $S2_b$ . **Bold** values are statistically significant to a level of 0.05 using a 1-sided t-test. *Italicized* values are statistically significant to a level of 0.10.

	$T_{\rm el}=0.33  {\rm sec},  \alpha=0$			$T_{\rm el}=0.33$ sec, $\alpha=0.1$				$T_{\rm el}=2.0 \text{ sec}, \alpha=0$			$T_{\rm el}=2.0  {\rm sec},  \alpha=0.1$		
	<i>R</i> =2	<i>R</i> =4	R=6	R=2	<i>R</i> =4	<i>R</i> =6		<i>R</i> =2	<i>R</i> =4	<i>R</i> =6	<i>R</i> =2	<i>R</i> =4	<i>R</i> =6
$SI_a$	1.06	1.13	1.09	1.06	1.11	1.08	$S2_a$	1.00	1.04	1.04	1.00	1.02	1.03
$SI_b$	1.00	1.00	1.00	1.00	1.00	1.00	$S2_b$	1.00	1.00	1.00	1.00	1.00	1.00
$SI_c$	0.98	1.00	0.99	1.00	1.01	1.00	$S2_c$	1.01	0.98	0.97	1.01	0.98	0.98
$SI_d$	0.96	0.91	0.94	0.97	0.91	0.94	$S2_d$	0.98	0.95	0.98	0.98	0.97	0.97
$SI_e$	0.90	0.85	0.91	0.92	0.82	0.83	$S2_e$	0.96	0.94	0.98	0.97	0.96	0.97
Ml	0.93	0.87	0.79	0.90	0.82	0.78	Ml	0.97	0.94	0.89	0.96	0.93	0.90

In general, the four plots in Fig. 3 display a consistent trend between bias and *CSD* over all combinations of parameters, though the degree of bias varies. Bias tends to be closest to 1.0 when R=2 and deviates further from 1.0 as R increases. This agrees with previous research (e.g., Luco and Bazzurro, 2006; Seifried, 2013), though it is not as clear here. Bias also appears to depend on the elastic period of the system, but is less influenced by  $\alpha$ . Additionally, the trend among responses from the scaled suites aligns with the bias observed for the spectrum-matched suite.

Note that even though many ground motions are used, some variation in the trend from Fig. 3 is expected due to the limited sample size. Tests for statistical significance are performed to indicate whether the trend is meaningful given these variations. Bias tends to be statistically significant for responses to ground motion suites at opposite extremes of *CSD*, and this significance is stronger for the system with the shorter  $T_{el}$ .

As mentioned above, this type of model has been used in other research to analyze 11

whether a response bias is exists for spectrum matched ground motions relative to scaled motions. While the bias approaches 0.8-0.9 for some cases, the level of ductility experienced by some of these models, as well as the associated peak post-yield force when  $\alpha = 0.1$ , is sometimes unrealistic (e.g.,  $\mu \approx 15-20$  for  $T_{el}=0.33$  sec and R=6). Therefore, these models should be viewed through the context of those studies. Results presented here demonstrate that a consistent bias is present with spectrum matched ground motions. Fig. 3 suggests that the cause of this bias is related to the *CSD* of a suite of records.

The asymmetry in responses noted by Carballo and Cornell (2000) and Seifried (2013) is illustrated by the nonlinear relationship shown in Fig. 4. Ground motions with larger conditional  $S_a(T_{eff})$  tend to produce ductilities disproportionately larger than those with smaller conditional  $S_a(T_{eff})$ , which is emphasized by the regression line included in each subfigure. Therefore, a greater value of *CSD* encompasses a greater range of the nonlinearity between *IM* and *EDP*, which in turn results in a greater median ductility. In reality,  $S_a$  at more than a single period longer than  $T_{el}$  will influence the peak response of the SDOF systems, so Fig. 4 is a simplified representation of the relationship between response and spectral shape, but it offers insight into why a trend is observed for geometric mean response as *CSD* changes. The figure also suggests that either  $\mu$  or  $S_a(T_{eff})$  are not strictly distributed lognormally.



**Figure 4.** Responses from SDOF systems versus conditional spectral ordinates. R=6 and  $\alpha=0.1$  for the system in each subfigure.  $T_{\text{eff}}$  is 2.5 times longer than  $T_{\text{el}}$  in each case.

The subtle nature of the bias observed between suites of scaled ground motions with similar levels of *CSD* offers an explanation for why this phenomenon has gone mostly unnoticed: even with large, carefully selected suites of ground motions, statistical significance is difficult to ascertain. However, even though the greatest degree of bias and strongest statistical significance is observed at extreme ductilities experienced by systems with extreme structural properties ( $\alpha$ =0.1), bias is still generally present in all analysis cases.

## **INELASTIC MDOF SYSTEM**

The 12-story MDOF model analyzed here is designed such that the median-level intensities of suites  $S2_{a-e}$  do not impose significant inelastic deformations. Therefore, suites M1 and  $S2_{a-e}$  are scaled up by a factor of 1.5 (and denoted  $M1^*$  and  $S2_{a-e}^*$ ) to achieve a higher degree of inelastic behavior. Recall that each of these scaled suites is conditioned on a period of 2.0 seconds, which approximately equals the fundamental period of the 12-story model. Although the rescaled intensity level is no longer associated with the BA GMPE scenario described above, the ground motion suites still all have consistent median spectra and so serve to evaluate relative structural responses. The rescaled ground motions result in some collapses for the two most variable suites of scaled ground motions (see Table 3). When a ground motion causes collapse, that realization of SDR is set to the peak observed level of SDR from the remaining ground motions in the suite that did not cause collapse. This enables the median to continue to be estimated as the geometric mean without ignoring collapse cases, and also allows estimation of standard error.

Bias for *SDR* is calculated relative to suite  $S2_b^*$  using Eq. 1 and plotted versus *CSD* in Fig. 5. Summary statistics are provided in Table 3. As with the SDOF analysis, a trend is observed between bias and *CSD* among responses from scaled ground motions that is consistent with responses from spectrum-matched ground motions. In Table 3, a reduction in dispersion of *SDR* ( $\sigma_{\text{InSDR}}$ ) as *CSD* decreases is evident. Statistical significance for the bias is also indicated in Table 3.

**Table 3.** Summary of structural response results from inelastic MDOF analysis. **Bold** values are statistically significant to a level of 0.05 using a 1-sided t-test. *Italicized* values are statistically significant to a level of 0.10.

	$S2_a*$	$S2_b*$	$S2_c*$	$S2_d*$	$S2_e*$	Ml*	
Fraction of collapses	<b>8</b> /250	<b>2</b> /170	<b>0</b> /120	<b>0</b> /70	0/35	0/25	
SDR <sub>geo</sub>	0.0112	0.0104	0.0102	0.0098	0.0091	0.0091	
$\sigma_{\ln SDR}$	0.45	0.34	0.29	0.23	0.19	0.17	
bias	1.08	-	0.98	0.94	0.88	0.88	



Figure 5. Bias versus CSD for the 12-story MDOF system. CSD is evaluated by Eq. 2 at  $T_{eff}=2.5T_{el}$ .

# **COLLAPSE CAPACITY OF MDOF SYSTEM**

The 12-story MDOF is also investigated for the effects of CSD on  $S_a(T_1)_{col}$  using suites M1 and  $S2_{a-e}$ . Incremental dynamic analysis (IDA; Vamvatsikos and Cornell, 2002) is performed and the resulting collapse-level spectral intensities at the fundamental period of the structure are recorded. Summary statistics of these values for each suite is reported in Table 4. Bias is calculated relative to suite  $S2_b$  using Eq. 1 and plotted versus CSD in Fig. 6. As with each analysis above, a trend is observed between bias and CSD among responses from scaled ground motions that is consistent with responses from spectrum-matched ground motions. Also similar to the above analyses, there is some degree of variation in the trend. Note that in this case a bias greater than 1.0 implies that on average the ground motions reach greater intensities before

causing collapse (i.e., a bias > 1 is unconservative in this case). In Table 4, the reduction in dispersion of  $S_a(T_1)_{col}$  as *CSD* decreases is evident. Statistical significance for the bias is also indicated in Table 4.

This analysis of  $S_a(T_1)_{col}$  expands the observation of a response bias related to *CSD* beyond the peak-oriented EDPs of  $\mu$  and *SDR* to a cumulative damage measure. These results support the intuition that if *SDR* is reduced by decreasing *CSD*, on average, then collapse will also be delayed by decreasing *CSD*, on average.

**Table 4.** Summary of collapse capacity estimated from IDA performed with the MDOF system. **Bold** values are statistically significant to a level of 0.05 using a 1-sided t-test. *Italicized* values are statistically significant to a level of 0.10.



**Figure 6.** Bias versus CSD for the 12-story MDOF system. CSD is evaluated by Eq. 2 at  $T_{\text{eff}}=2.5T_{\text{el}}$ .

#### CONCLUSIONS

The effect of *CSD* on structural responses is investigated. A number of structural models are analyzed using five suites of scaled ground motions with equivalent median intensities and different levels of *CSD* at a range of effective periods, as well as one suite of spectrum-matched ground motions. For all models and *EDP*s studied, a similar trend is observed between bias and *CSD* among the scaled suites that is consistent with the bias present for the spectrum-matched suite. *CSD* completely accounts for the observed bias, which reinforces that an asymmetric relationship between *EDP* and *IM* is responsible for it.

The presented results agree with the bias reported for spectrum-matched ground motions in previous research: a bias is present, consistently unconservative and tends to increase with ductility. This research builds on previous work by demonstrating statistical significance of the bias, expanding results to a cumulative damage measure, and revealing an underlying trend with the spectral variability of scaled ground motions. This trend is often subtle, which offers a possible explanation for why it has not explicitly been noted previously. There also appears to be some dependence on structural properties, which may contribute to the variation in conclusions from other research.

Due to the dependence of the investigated bias on structural model characteristics and *EDP*, this work does not show the bias to be systematic or predictable based on *CSD* alone. This implies that a generic correction factor based on *CSD* to counteract any bias, while ideal for practitioners, may not be appropriate. However, the viability of such factors based on additional structural characteristics is not precluded here. Short of a solution of that type, the median response level of a particular *EDP* for a particular structural system at a target *CSD* value could be estimated through the analysis of multiple suites of ground motions. It is also worth noting that not all combinations of model and EDP are guaranteed to result in a bias.

There are two main implications of these findings. First, the process of spectrum matching itself is not responsible for the observed bias between *EDP*s resulting from scaled and spectrum-matched ground motions. Second, efficient GMSM techniques should consider the demonstrated effect of conditional spectral variability on the central tendency of response, especially procedures that seek to minimize spectral variability. Current ground motion selection practice assumes that reducing spectral variability serves to reduce *EDP* variability

without introducing bias, but the above results indicate that this is incorrect.

Note also that bias is computed above with respect to geometric mean *EDP*, while design standards often consider arithmetic means. The arithmetic mean is larger than the geometric mean of lognormally distributed data, and the difference between these two measures increases as variability increases. This implies that if bias were computed using the arithmetic mean of responses from scaled ground motions that retain *CSD* and from spectrum matched ground motions that remove *CSD*, it may be even greater than the results presented here.

These conclusions are subject to a number of limitations. First, the structural models and *EDP*s considered are representative of single-mode-dominated systems with moderate fundamental periods. Second, the ground motions that were used have been coarsely filtered and do not explicitly consider some effects known to be important to structural response (e.g., duration or directivity), though they are accounted for to some extent through spectral shape, and the large number of considered records should avoid undue influence of these characteristics. Finally, this analysis only considers spectrum matching in the time-domain, although other procedures are available. However, if additional models, *EDP*s, ground motion properties, or matching techniques are desired to be studied in future research, a framework has been established here to aid that work.

## ACKNOWLEDGEMENTS

We thank Curt Haselton for providing the MDOF structural model and Peter Stafford for constructive comments that improved the manuscript. This work was supported in part by the National Science Foundation under NSF grant number CMMI 0726684. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation. We also thank the Shah Family Fellowship and John A. Blume Fellowship for providing additional financial support for this work.

#### REFERENCES

Al Atik, L. and Abrahamson, N. An improved method for nonstationary spectral matching. *Earthquake Spectra* 2010, 26(3):601-617.

ASCE/SEI. *Minimum Design Loads for Buildings and Other Structures*, ASCE 7-10. American Society of Civil Engineers, 2010.

- Aslani, H. and Miranda, E. Probability-based seismic response analysis. *Engineering Structures* 2005, 27(8):1151-1163.
- ATC. Seismic performance assessment of buildings, ATC-58. Applied Technology Council, 2012.
- Baker, J.W. and Cornell, C.A. Spectral shape, epsilon and record selection. *Earthquake Engineering & Structural Dynamics* 2006; 35(9): 1077-1095.
- Baker, J.W. and Jayaram, N. Correlation of spectral acceleration values from NGA ground motion models. *Earthquake Spectra* 2008, 24(1):299-317.
- Bazzurro, P. and Luco, N. Do scaled and spectrum-matched near-source records produce biased nonlinear structural responses? *δ<sup>th</sup> U.S. National Conference on Earthquake Engineering*, San Francisco, CA, 2006.
- Boore, D.M. and Atkinson G.M. Ground-motion prediction equations for the average horizontal component of PGA, PGV, and 5% damped PSA at spectral periods between 0.01s and 10.0s. *Earthquake Spectra* 2008, 24(1):99-138.
- Carballo, J.E. and Cornell, C.A. *Probabilistic seismic demand analysis: Spectrum matching and design,* Report No. RMS-41, Reliability of Marine Structures Program, Stanford University, Stanford, CA, 2000.
- Carlson, C. Zekkos, D. and Athanasopoulos-Zekkos A. Predictive Equations to Quantify the Impact of Spectral Matching on Ground Motion Characteristics. *Earthquake Spectra* In-Press.
- Chiou, B., Darragh, R., Gregor, N. and Silva, W. NGA project strong-motion database. *Earthquake Spectra* 2008, 24(1):23-44.
- Grant, D.N. and Diaferia, R. Assessing adequacy of spectrum-matched ground motions for response history analysis. *Earthquake Engineering & Structural Dynamics* 2013, 42(9):1265-1280.
- Hancock, J., Bommer, J.J. and Stafford, P.J. Numbers of scaled and matched accelerograms required for inelastic dynamic analyses. *Earthquake Engineering & Structural Dynamics* 2008, 37(14):1585-1607.
- Haselton, C.B. and Deierlein, G.G. Assessing seismic collapse safety of modern reinforced concrete moment frame buildings, PEER Report 2007/08, Pacific Earthquake Engineering Research Center, Berkeley, CA, 2007.
- Heo, Y., Kunnath, S. and Abrahamson, N. Amplitude-scaled versus spectrum-matched ground motions for seismic performance assessment. *Journal of Structural Engineering* 2011, 137(3):278-288.
- Huang, Y.-N., Whittaker, A.S., Luco, N. and Hamburger, R.O. Scaling earthquake ground motions for performance-based assessment of buildings. *Journal of Structural Engineering* 2011, 137(3):311-321.
- Iervolino, I. and Cornell, C.A. Record selection for nonlinear seismic analysis of structures. *Earthquake Spectra* 2005, 21(3):685-713.
- Iervolino, I., De Luca, F. and Cosenza, E. Spectral shape-based assessment of SDOF nonlinear response to real, adjusted, and artificial accelerograms. *Engineering Structures* 2010. 32(9):2776-2792.
- Iwan, W.D. Estimating inelastic response spectra from elastic response spectra. *Earthquake Engineering & Structural Dynamics* 1980, 8:375-388.
- Jayaram, N., Lin, T. and Baker J.W. A computationally efficient ground motion selection algorithm for matching a target response spectrum mean and variance. *Earthquake Spectra* 2011, 27(3):797-815.
- McGuire, R.K., Silva, W.J., Costantino, C.J. Technical basis for revision of regulatory guidance on design ground motions: hazard- and risk-consistent ground motion spectra guidelines, NUREG/CR-6728. U.S. Nuclear Regulatory Commission Report, 2001.
- Open System for Earthquake Engineering Simulation (OpenSEES). Pacific Earthquake Engineering Research Center, 2009. Available from: http://opensees.berkeley.edu.
- Seifried, A.E. Response spectrum compatibilization and its impact on structural response assessment. *Ph.D. Thesis*, Dept. of Civil and Environmental Engineering, Stanford University, Stanford, CA, 2013.
- Vamvatsikos, D. and Cornell, C.A. Incremental dynamic analysis, *Earthquake Engineering & Structural Dynamics* 2002, 31(3):491-514.